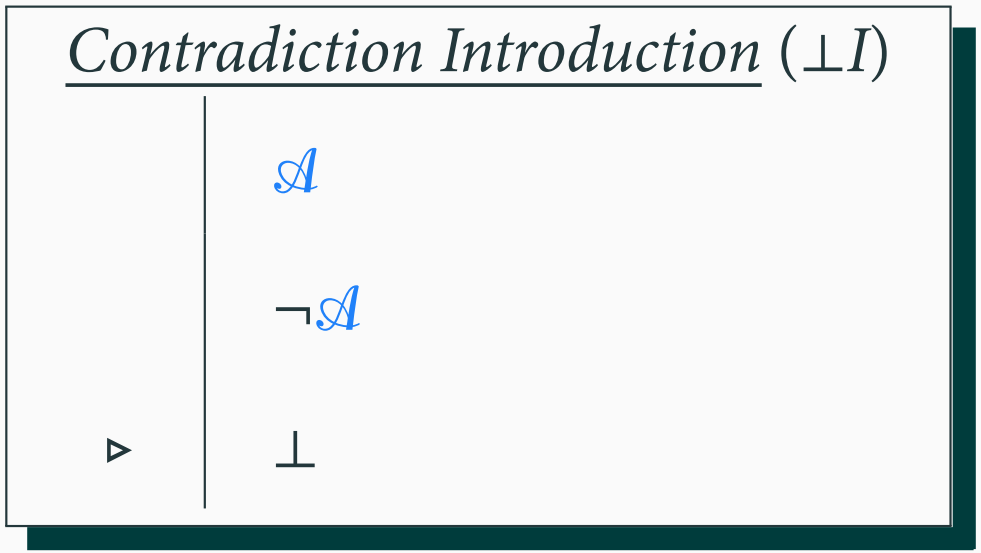
Natural Deduction Proofs for SL

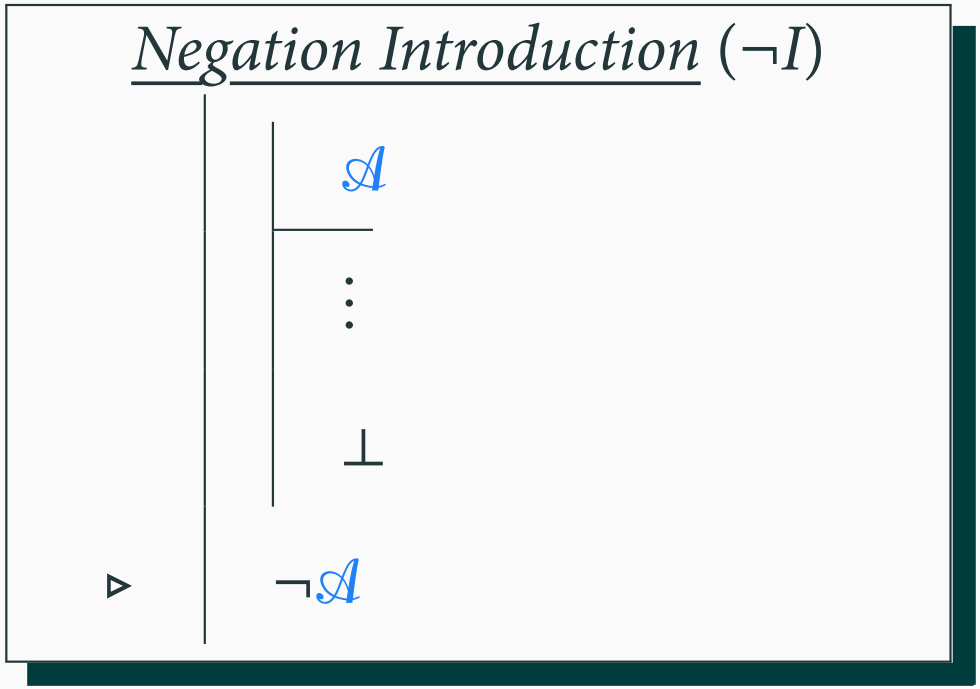
Day 3

1	$A \wedge \neg A$	Ass. ($\neg I$)
2	A	$\wedge E$ 1
3	$\neg A$	$\wedge E$ 1
4	$\bot$	$\bot I$ 2, 3
5	$\neg(A \wedge \neg A)$	$\neg I$ 1–4

Contradiction Introduction



Negation Introduction



$P \vdash \neg\neg P$

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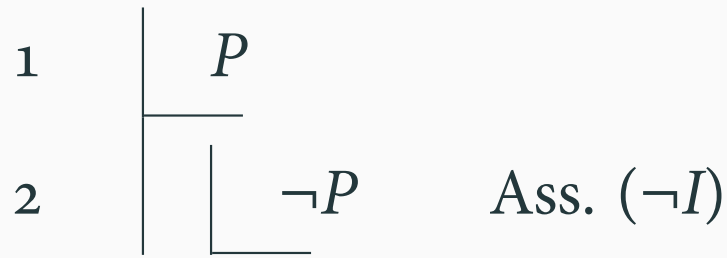
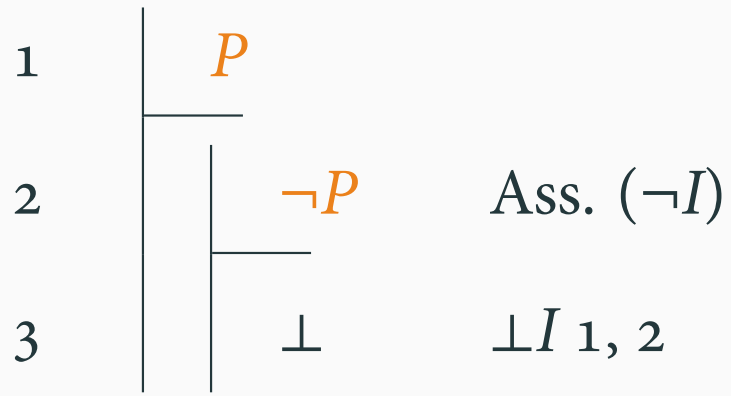
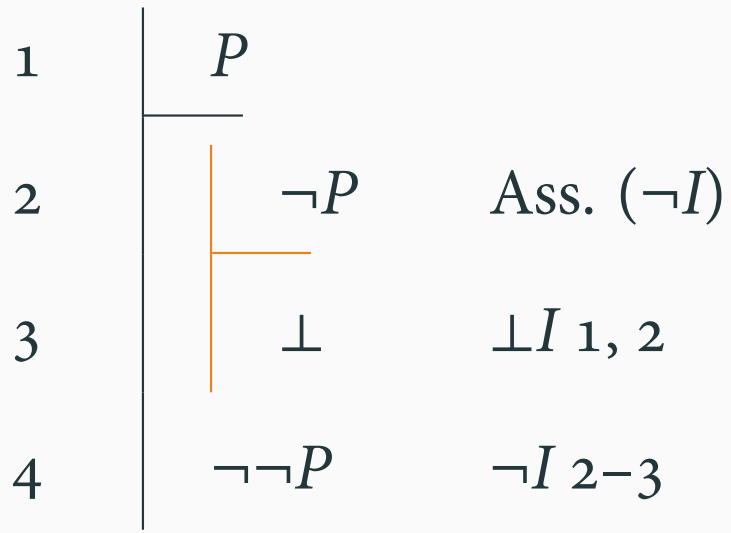


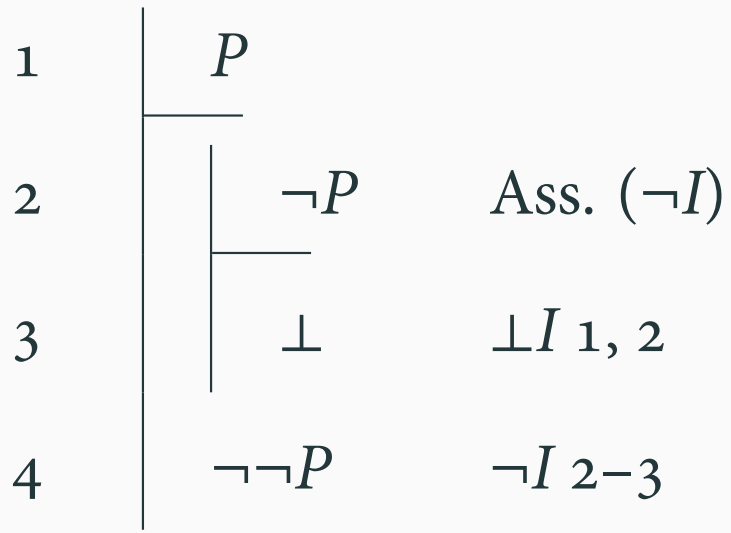
$P \vdash \neg\neg P$



$P \vdash \neg\neg P$



$P \vdash \neg\neg P$



$X \wedge Y \vdash \neg(\neg Y \wedge X)$

1 $X \wedge Y$

$X \wedge Y \vdash \neg(\neg Y \wedge X)$

$X \wedge Y \vdash \neg(\neg Y \wedge X)$

1	$X \wedge Y$	
2	Y	$\wedge E$ 1
3	$\neg Y \wedge X$	Ass. ($\neg I$)

$X \wedge Y \vdash \neg(\neg Y \wedge X)$

1	$X \wedge Y$	
2	Y	$\wedge E$ 1
3	$\neg Y \wedge X$	Ass. ($\neg I$)
4	$\neg Y$	$\wedge E$ 3

$X \wedge Y \vdash \neg(\neg Y \wedge X)$

1	$X \wedge Y$	
2	Y	$\wedge E$ 1
3	$\neg Y \wedge X$	Ass. ($\neg I$)
4	$\neg Y$	$\wedge E$ 3
5	$\bot$	$\bot I$ 2, 4

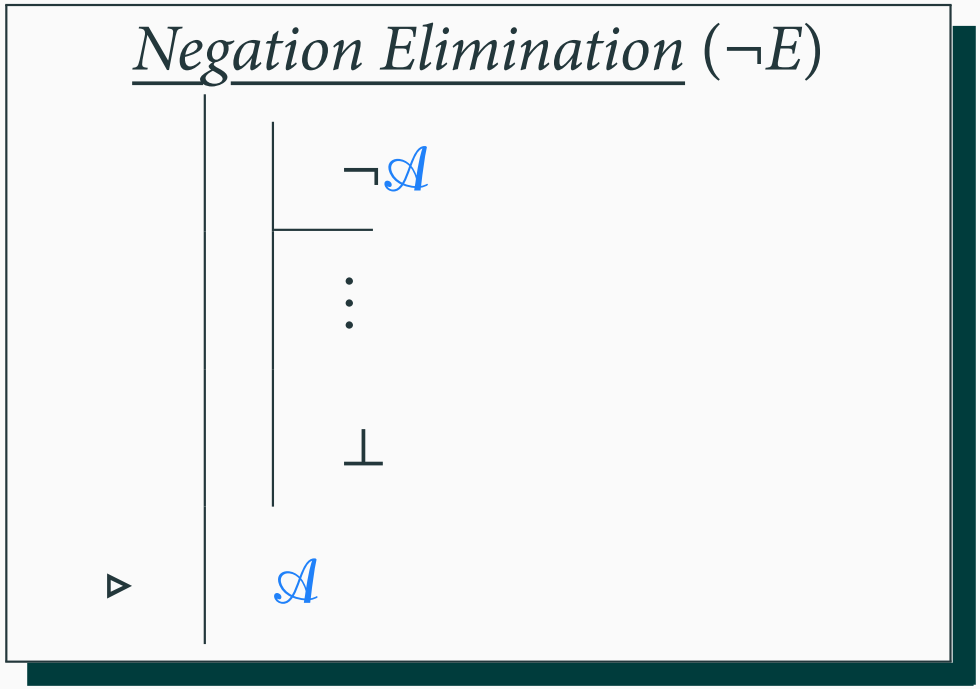
$X \wedge Y \vdash \neg(\neg Y \wedge X)$

1	$X \wedge Y$	
2	Y	$\wedge E$ 1
3	$\neg Y \wedge X$	Ass. ($\neg I$)
4	$\neg Y$	$\wedge E$ 3
5	$\bot$	$\bot I$ 2, 4
6	$\neg(\neg Y \wedge X)$	$\neg I$ 3–5

$X \wedge Y \vdash \neg(\neg Y \wedge X)$

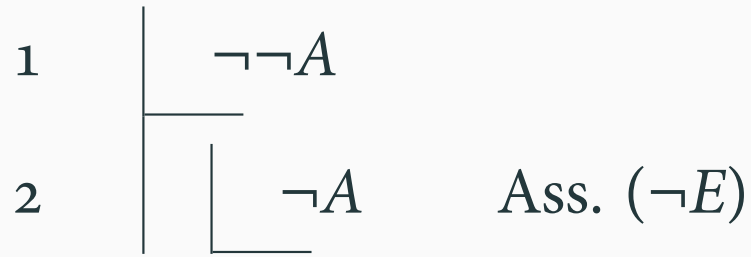
1	$X \wedge Y$	
2	Y	$\wedge E$ 1
3	$\neg Y \wedge X$	Ass. ($\neg I$)
4	$\neg Y$	$\wedge E$ 3
5	$\bot$	$\bot I$ 2, 4
6	$\neg(\neg Y \wedge X)$	$\neg I$ 3–5

Negation Elimination

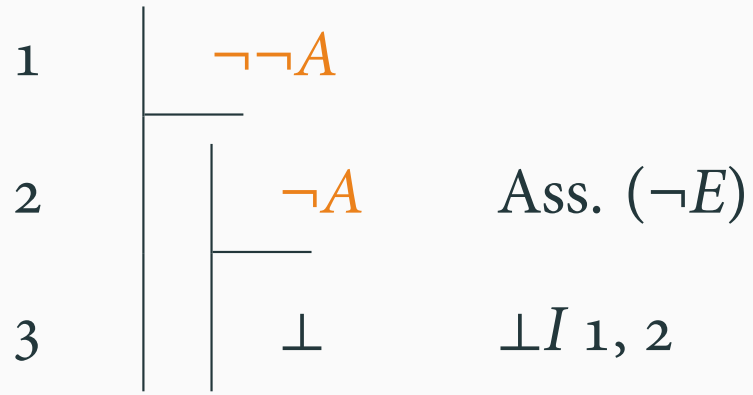


$\neg\neg A \quad \vdash \quad A$

$\neg\neg A \vdash A$



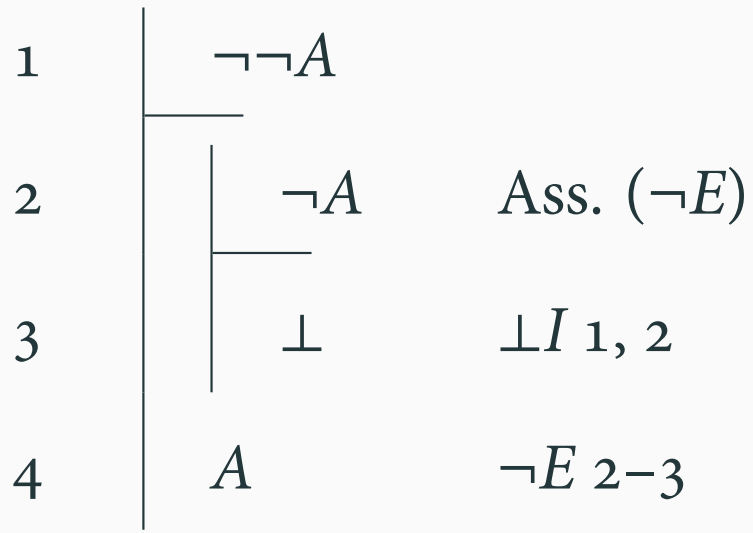
$\neg\neg A \vdash A$



$\neg\neg A \vdash A$

1	$\neg\neg A$	
2	$\neg A$	Ass. ($\neg E$)
3	$\bot$	$\bot I$ 1, 2
4	A	$\neg E$ 2–3

$\neg\neg A \quad \vdash \quad A$



$\vdash \quad G \longleftrightarrow \neg\neg G$

1	G	Ass. ($\longleftrightarrow I$)

$\vdash \quad G \longleftrightarrow \neg\neg G$

1	G	Ass. $(\longleftrightarrow I)$
2	$\neg G$	Ass. $(\neg I)$

$\vdash G \longleftrightarrow \neg\neg G$

1	G	Ass. ($\longleftrightarrow I$)
2	$\quad \neg G$	Ass. ($\neg I$)
3	$\quad \bot$	$\bot I$ 1, 2

$\vdash G \longleftrightarrow \neg\neg G$

1	G	Ass. ($\longleftrightarrow I$)
2	$\quad \neg G$	Ass. ($\neg I$)
3	$\quad \bot$	$\bot I$ 1, 2
4	$\neg\neg G$	$\neg I$ 2, 3

$\vdash \quad G \longleftrightarrow \neg\neg G$

1	G	Ass. ($\longleftrightarrow I$)
2	$\quad \neg G$	Ass. ($\neg I$)
3	$\quad \bot$	$\bot I$ 1, 2
4	$\neg\neg G$	$\neg I$ 2, 3
5	$\neg\neg G$	Ass. ($\longleftrightarrow I$)

$\vdash \quad G \longleftrightarrow \neg\neg G$

1	G	Ass. $(\longleftrightarrow I)$
2	$\quad \neg G$	Ass. $(\neg I)$
3	$\quad \bot$	$\bot I$ 1, 2
4	$\neg\neg G$	$\neg I$ 2, 3
5	$\neg\neg G$	Ass. $(\longleftrightarrow I)$
6	$\quad \neg G$	Ass. $(\neg E)$

$\vdash G \longleftrightarrow \neg\neg G$

1	G	Ass. ($\longleftrightarrow I$)
2	$\quad \neg G$	Ass. ($\neg I$)
3	$\quad \bot$	$\bot I$ 1, 2
4	$\neg\neg G$	$\neg I$ 2, 3
5	$\neg\neg G$	Ass. ($\longleftrightarrow I$)
6	$\quad \neg G$	Ass. ($\neg E$)
7	$\quad \bot$	$\bot I$ 5, 6

$\vdash \; G \longleftrightarrow \neg\neg G$

1	G	Ass. ($\longleftrightarrow I$)
2	$\quad \neg G$	Ass. ($\neg I$)
3	$\quad \bot$	$\bot I$ 1, 2
4	$\neg\neg G$	$\neg I$ 2, 3
5	$\neg\neg G$	Ass. ($\longleftrightarrow I$)
6	$\quad \neg G$	Ass. ($\neg E$)
7	$\quad \bot$	$\bot I$ 5, 6
8	G	$\neg E$ 6–7

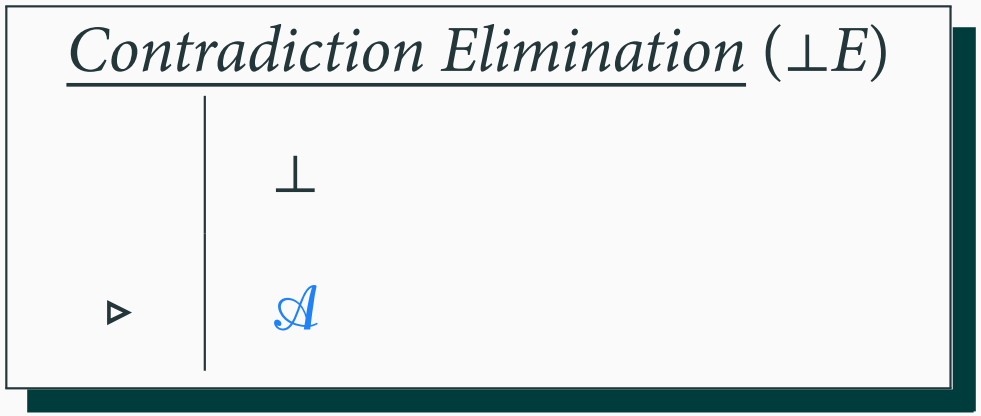
$\vdash G \longleftrightarrow \neg\neg G$

1	G	Ass. ($\longleftrightarrow I$)
2	$\quad \neg G$	Ass. ($\neg I$)
3	$\quad \bot$	$\bot I$ 1, 2
4	$\neg\neg G$	$\neg I$ 2, 3
5	$\neg\neg G$	Ass. ($\longleftrightarrow I$)
6	$\quad \neg G$	Ass. ($\neg E$)
7	$\quad \bot$	$\bot I$ 5, 6
8	G	$\neg E$ 6–7
9	$G \longleftrightarrow \neg\neg G$	$\longleftrightarrow I$ 1–4, 5–8

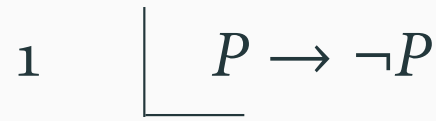
$\vdash \; G \longleftrightarrow \neg\neg G$

1	G	Ass. ($\longleftrightarrow I$)
2	$\quad \neg G$	Ass. ($\neg I$)
3	$\quad \bot$	$\bot I$ 1, 2
4	$\neg\neg G$	$\neg I$ 2, 3
5	$\neg\neg G$	Ass. ($\longleftrightarrow I$)
6	$\quad \neg G$	Ass. ($\neg E$)
7	$\quad \bot$	$\bot I$ 5, 6
8	G	$\neg E$ 6–7
9	$G \longleftrightarrow \neg\neg G$	$\longleftrightarrow I$ 1–4, 5–8

Contradiction Elimination



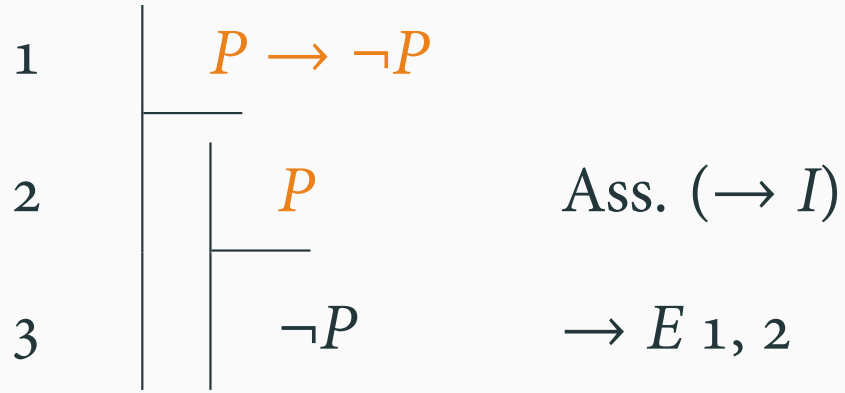
$P \to \neg P \quad \vdash \quad P \to Q$



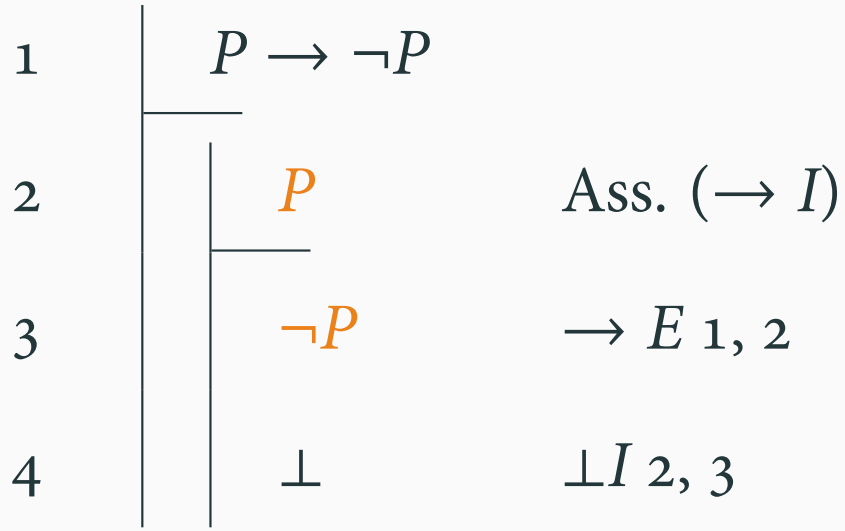
$P \to \neg P \quad \vdash \quad P \to Q$

1	$P \to \neg P$	
2	P	Ass. $(\to I)$

$P \to \neg P \quad \vdash \quad P \to Q$



$P \to \neg P \quad \vdash \quad P \to Q$



$P \to \neg P \quad \vdash \quad P \to Q$

1	$P \to \neg P$	
2	$\quad P$	Ass. ($\to I$)
3	$\quad \neg P$	$\to E$ 1, 2
4	$\quad \bot$	$\bot I$ 2, 3
5	$\quad Q$	$\bot E$ 4

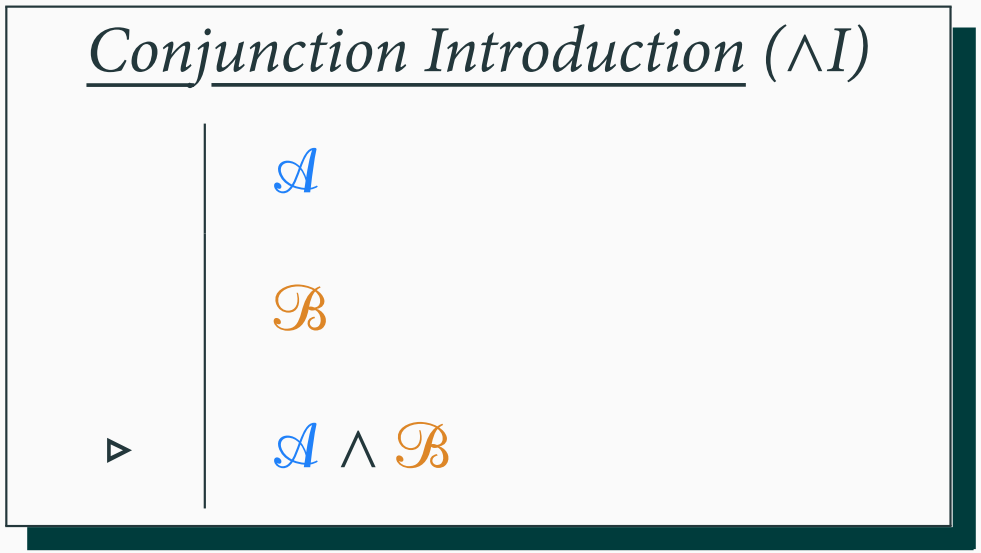
$P \to \neg P \quad \vdash \quad P \to Q$

1	$P \to \neg P$	
2	P	Ass. ($\to I$)
3	$\neg P$	$\to E$ 1, 2
4	$\bot$	$\bot I$ 2, 3
5	Q	$\bot E$ 4
6	$P \to Q$	$\to I$ 2–5

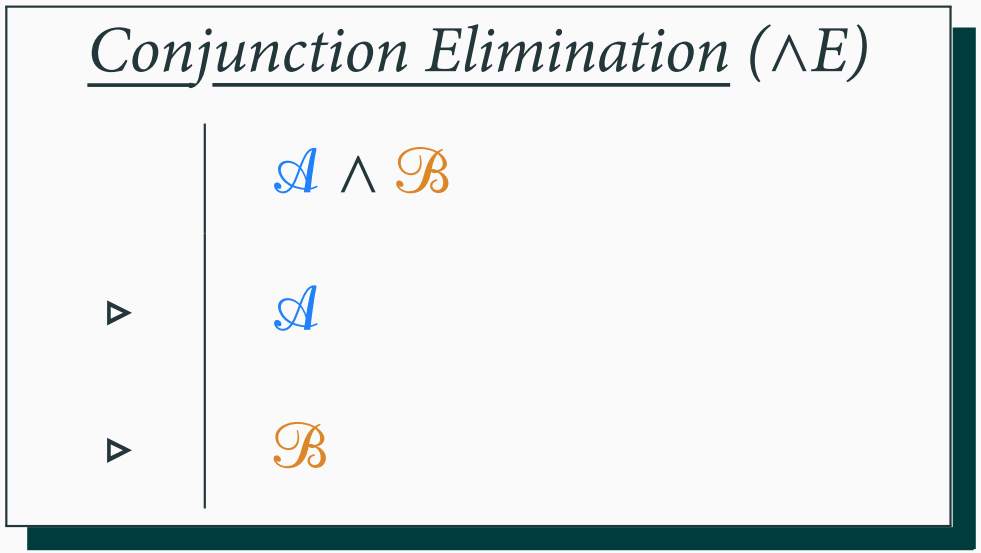
$P \to \neg P \quad \vdash \quad P \to Q$

1	$P \to \neg P$	
2	$\quad P$	Ass. ($\to I$)
3	$\quad \neg P$	$\to E$ 1, 2
4	$\quad \bot$	$\bot I$ 2, 3
5	$\quad Q$	$\bot E$ 4
6	$P \to Q$	$\to I$ 2–5

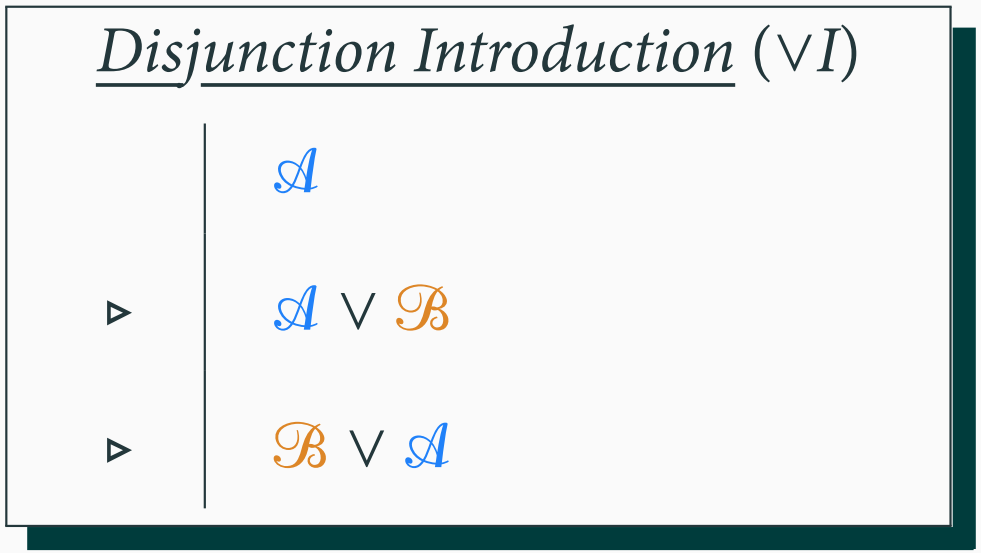
Conjunction Introduction



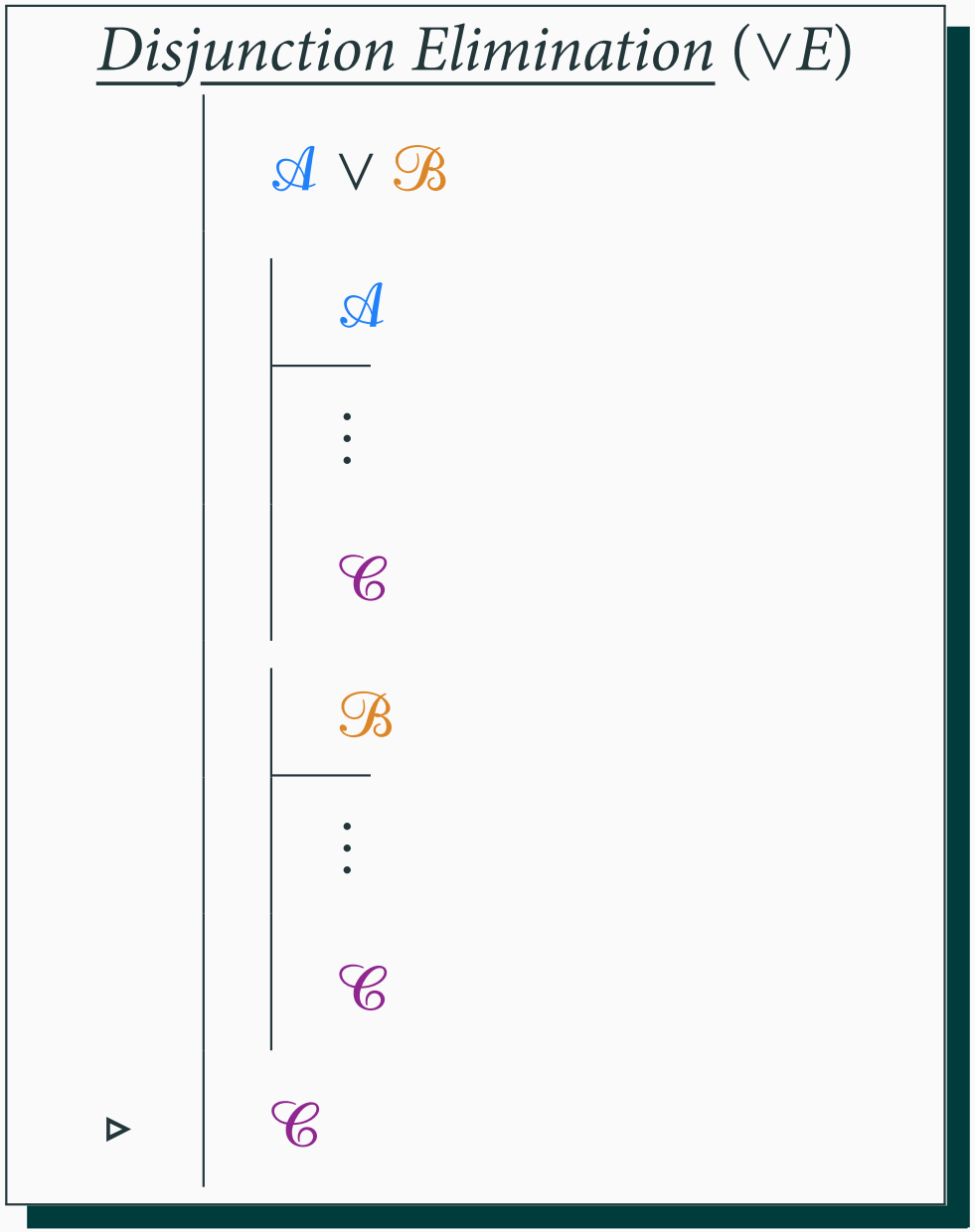
Conjunction Elimination



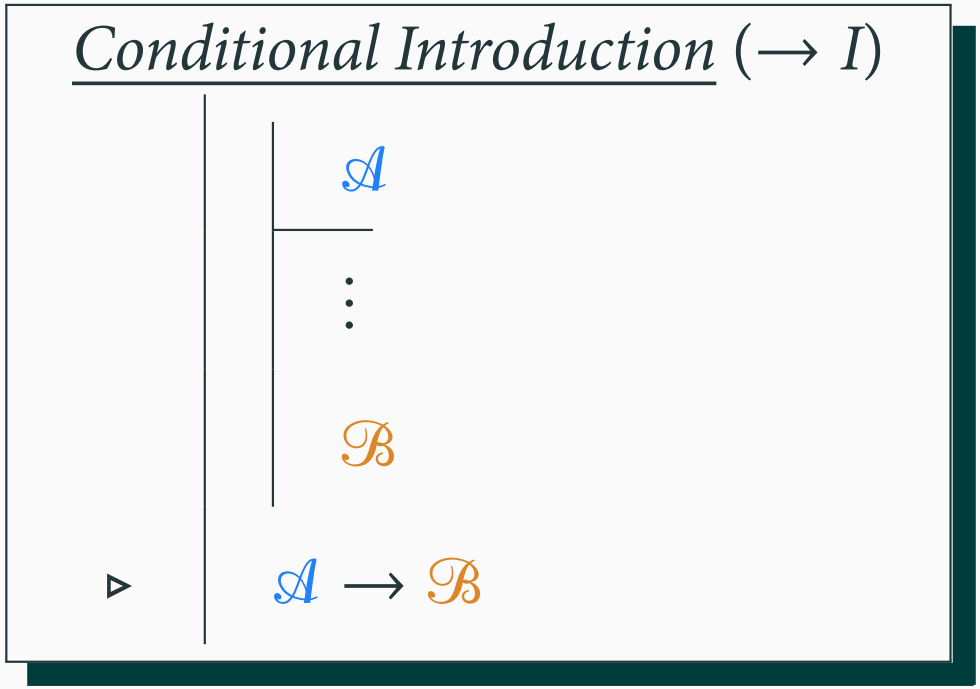
Disjunction Introduction



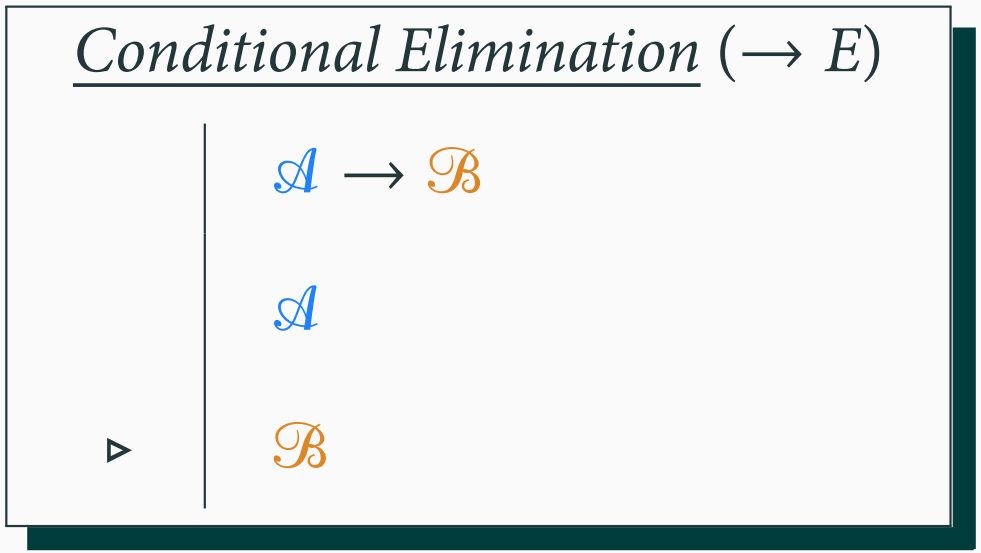
Disjunction Elimination



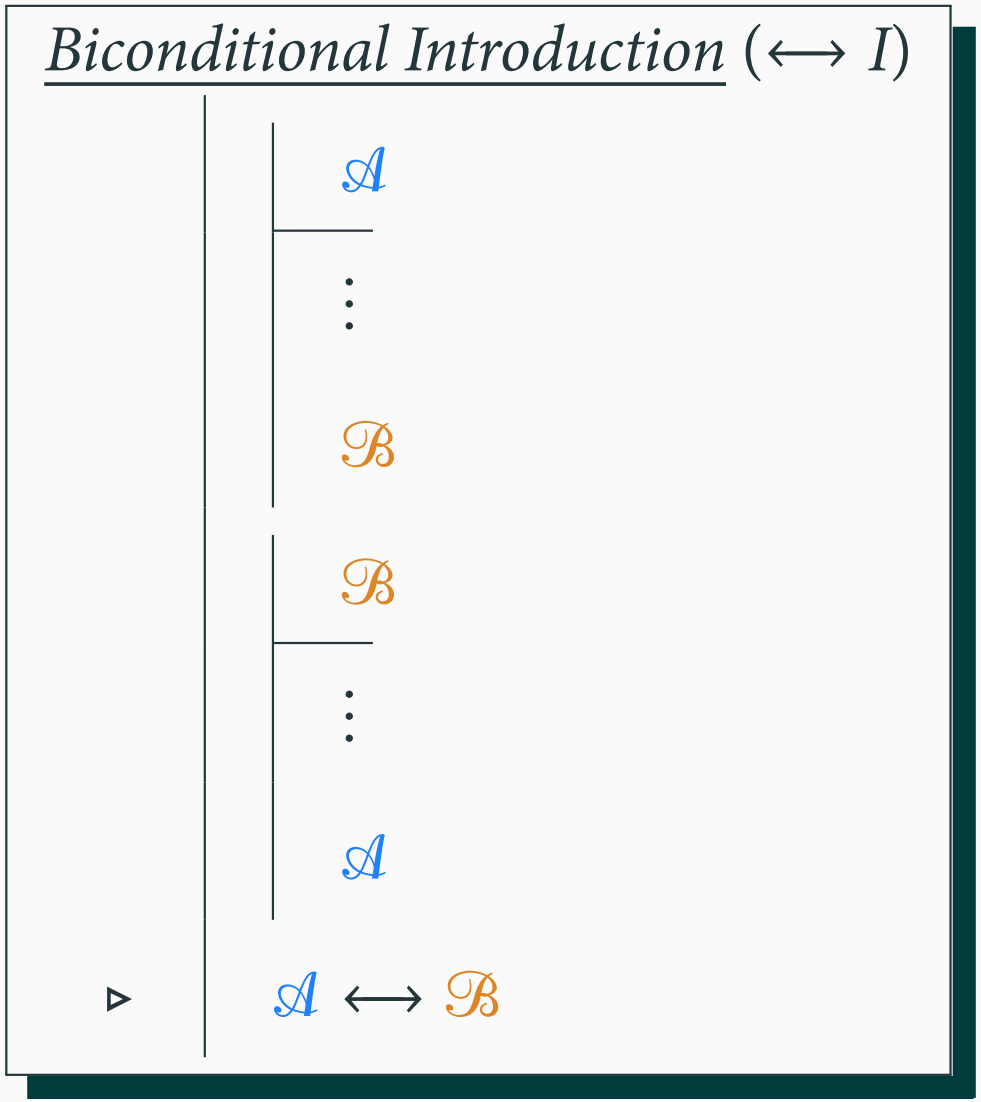
Conditional Introduction



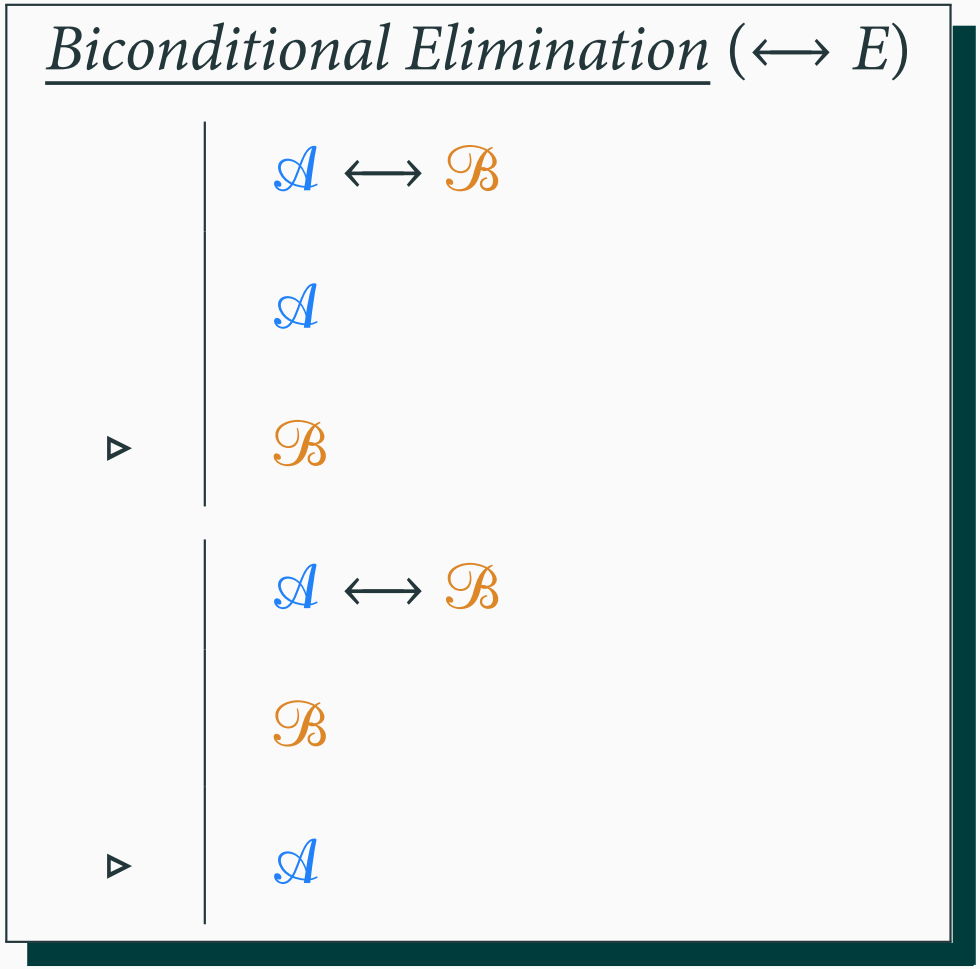
Conditional Elimination



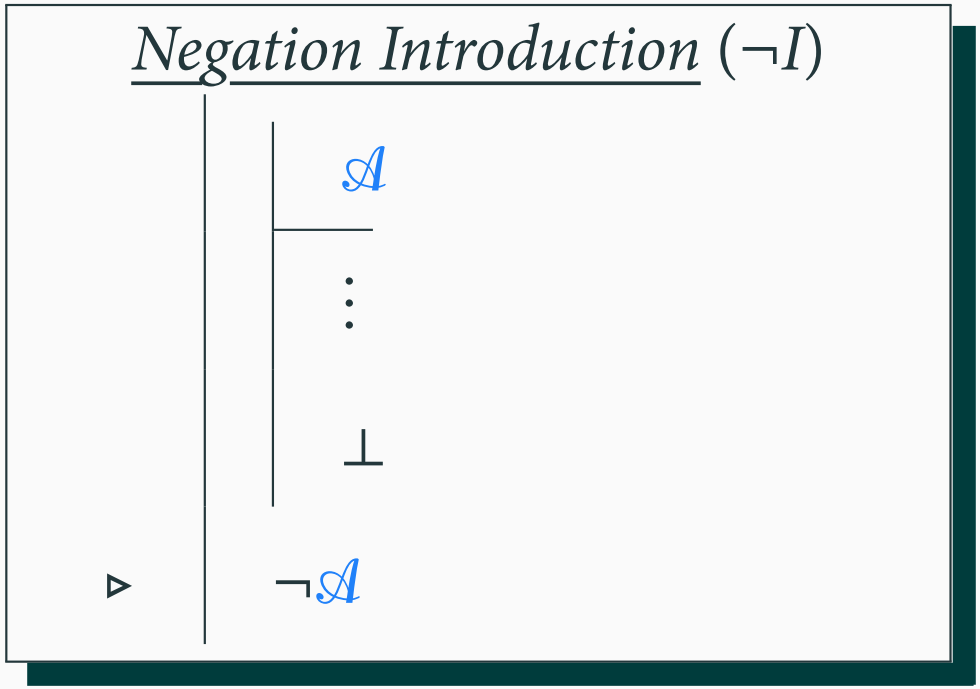
Biconditional Introduction



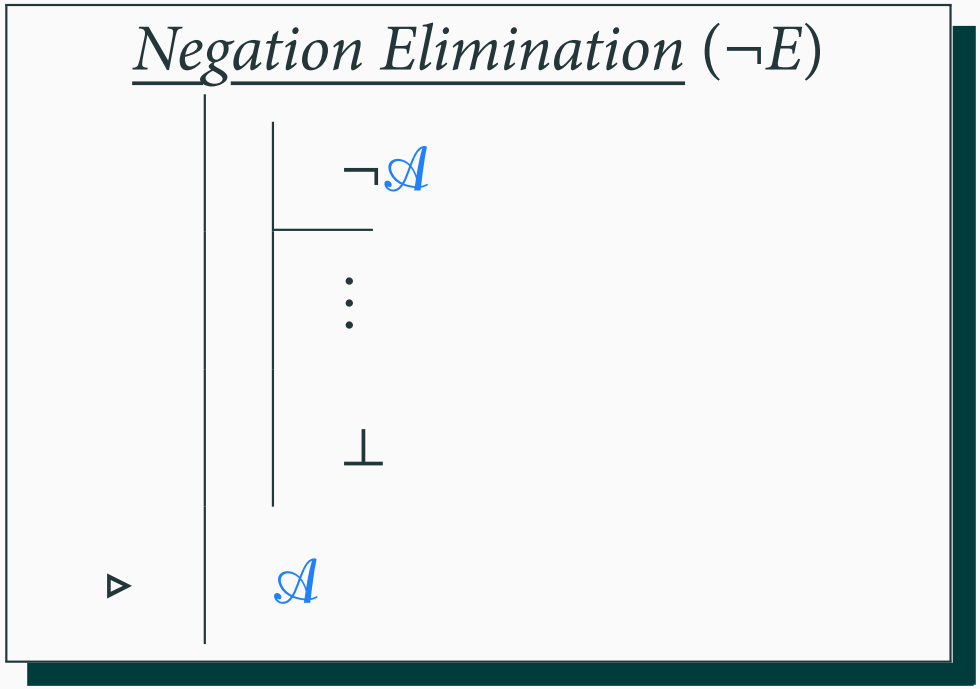
Biconditional Elimination



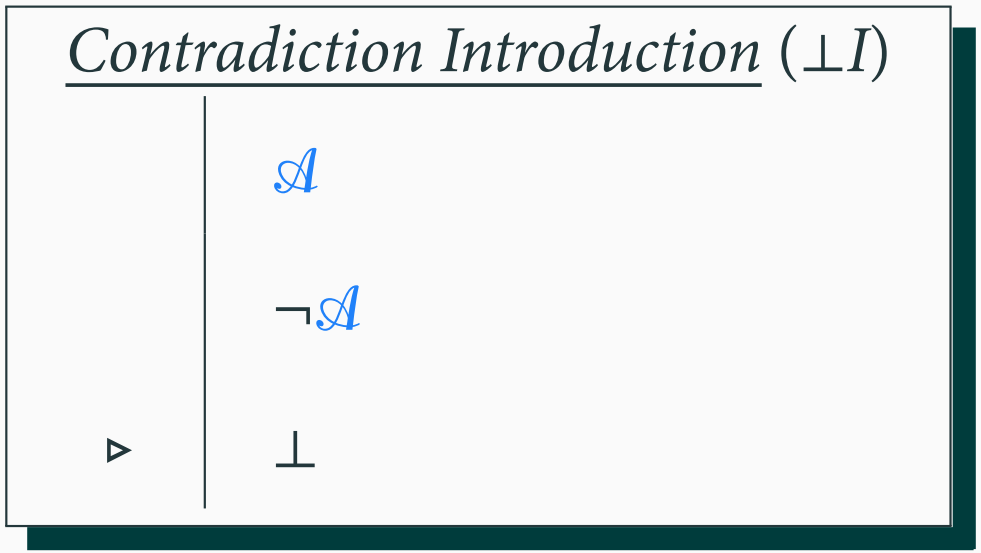
Negation Introduction



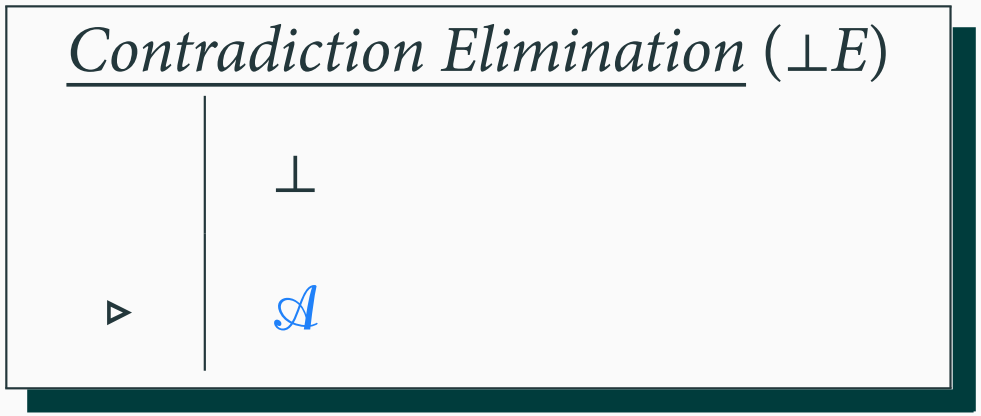
Negation Elimination



Contradiction Introduction



Contradiction Elimination



Outline

Proof Strategies

Proof Strategies

- Constructing a legal natural deduction proof requires *creativity*.

Proof Strategies

- Constructing a legal natural deduction proof requires *creativity*.
- There's no algorithm or step-by-step instructions I can give you.

Proof Strategies

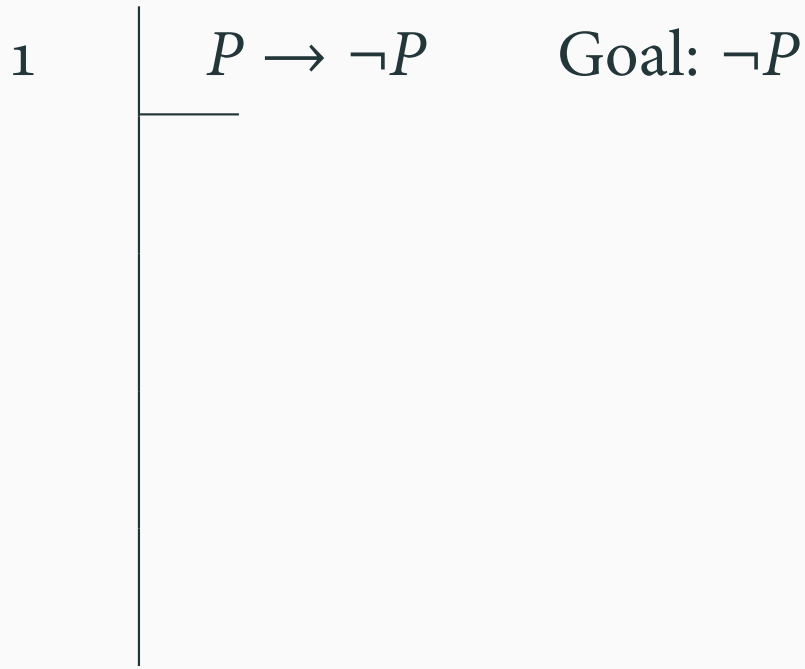
- Constructing a legal natural deduction proof requires *creativity*.
- There's no algorithm or step-by-step instructions I can give you.
- But I have some tips.

Proof Strategies

- Tip #0: come up with a ‘big picture’ strategy

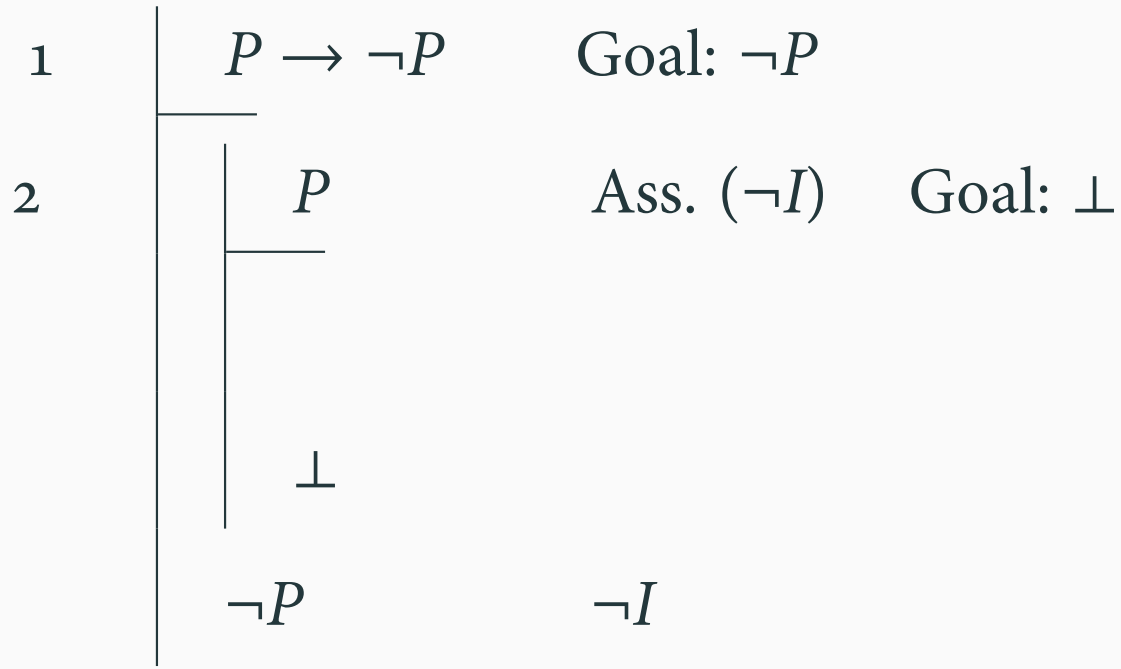
Proof Strategies

- Tip #1: Try to use the introduction rule for the main operator of the sentence you want to write down.



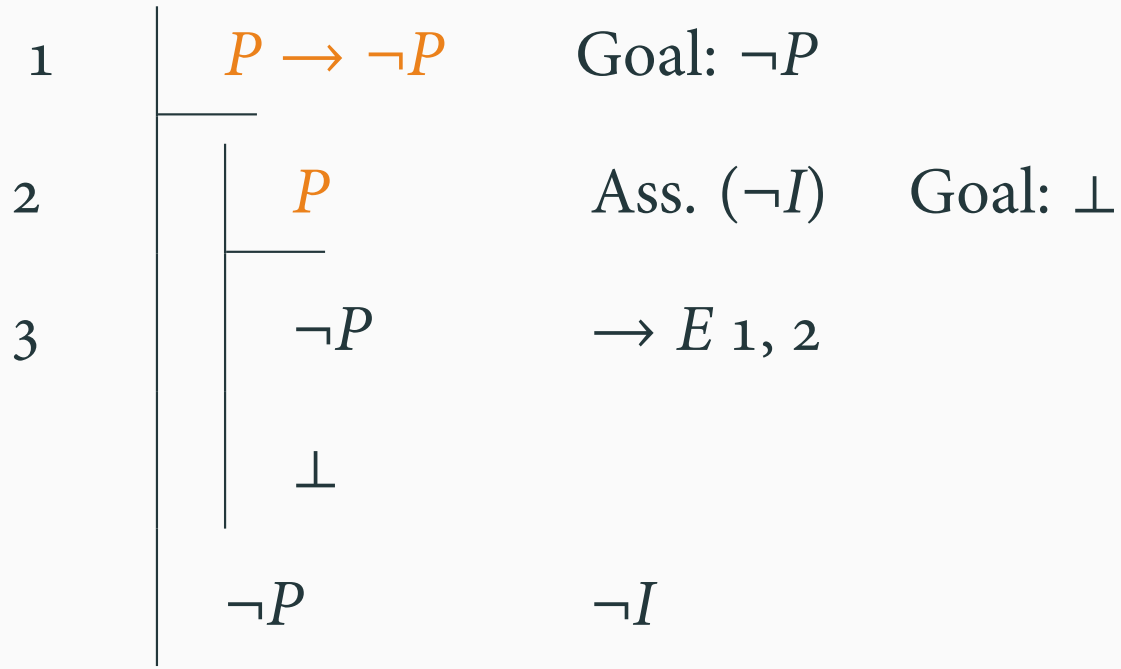
Proof Strategies

- Tip #1: Try to use the introduction rule for the main operator of the sentence you want to write down.



Proof Strategies

- Tip #1: Try to use the introduction rule for the main operator of the sentence you want to write down.



Proof Strategies

- Tip #1: Try to use the introduction rule for the main operator of the sentence you want to write down.

1	$P \rightarrow \neg P$	Goal: $\neg P$	
2	P	Ass. ($\neg I$)	Goal: $\bot$
3	$\neg P$	$\rightarrow E$ 1, 2	
4	$\bot$	$\bot I$ 2, 3	
	$\neg P$	$\neg I$	

Proof Strategies

- Tip #1: Try to use the introduction rule for the main operator of the sentence you want to write down.

1	$P \rightarrow \neg P$	Goal: $\neg P$	
2	P	Ass. ($\neg I$)	Goal: $\bot$
3	$\neg P$	$\rightarrow E$ 1, 2	
4	$\bot$	$\bot I$ 2, 3	
5	$\neg P$	$\neg I$ 2–5	

Proof Strategies

- Tip #1: Try to use the introduction rule for the main operator of the sentence you want to write down.

1	$P \rightarrow \neg P$	Goal: $\neg P$	
2	$\quad P$	Ass. ($\neg I$)	Goal: $\bot$
3	$\quad \neg P$	$\rightarrow E$ 1, 2	
4	$\quad \bot$	$\bot I$ 2, 3	
5	$\neg P$	$\neg I$ 2–5	

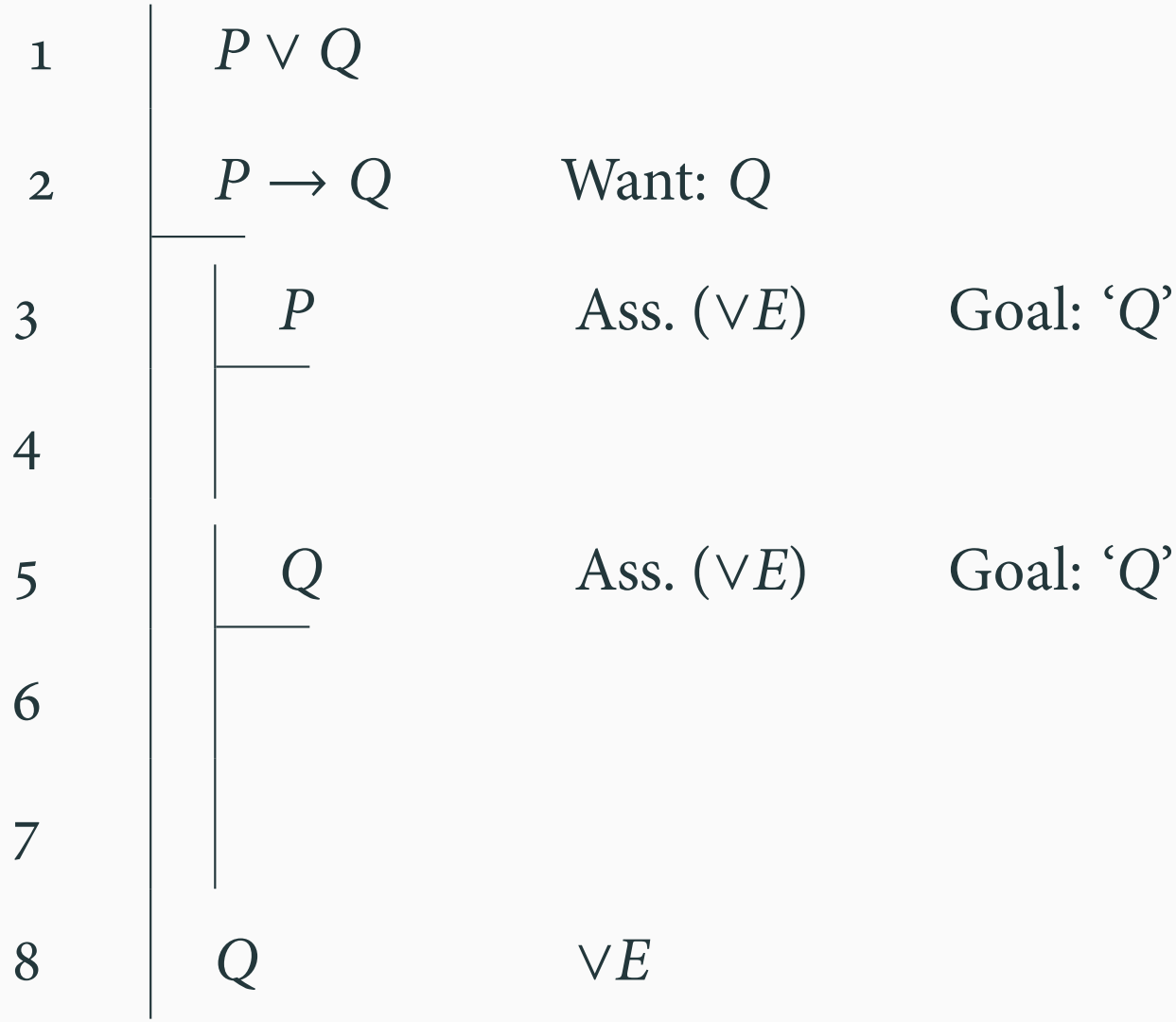
Proof Strategies

- Tip #2: Try to use the elimination rule for the main operator of a sentence you have written down.

Proof Strategies

1	$P \vee Q$	
2	$P \rightarrow Q$	Want: Q

Proof Strategies



Proof Strategies

1	$P \vee Q$		
2	$P \rightarrow Q$	Want: Q	
3	P	Ass. ($\vee E$)	Goal: 'Q'
4	Q	$\rightarrow E$ 2, 3	
5	Q	Ass. ($\vee E$)	Goal: 'Q'
6			
7			
8	Q	$\vee E$	

Proof Strategies

1	$P \vee Q$		
2	$P \rightarrow Q$	Want: Q	
3	$\quad P$	Ass. ($\vee E$)	Goal: 'Q'
4	$\quad Q$	$\rightarrow E$ 2, 3	
5	$\quad Q$	Ass. ($\vee E$)	Goal: 'Q'
6	$\quad Q \wedge Q$	$\wedge I$ 5	
7			
8	Q	$\vee E$	

Proof Strategies

1	$P \vee Q$		
2	$P \rightarrow Q$	Want: Q	
3	$\quad P$	Ass. ($\vee E$)	Goal: 'Q'
4	$\quad Q$	$\rightarrow E$ 2, 3	
5	$\quad Q$	Ass. ($\vee E$)	Goal: 'Q'
6	$\quad Q \wedge Q$	$\wedge I$ 5	
7	$\quad Q$	$\wedge E$ 6	
8	Q	$\vee E$	

Proof Strategies

1	$P \vee Q$		
2	$P \rightarrow Q$	Want: Q	
3	$\quad P$	Ass. ($\vee E$)	Goal: 'Q'
4	$\quad Q$	$\rightarrow E$ 2, 3	
5	$\quad Q$	Ass. ($\vee E$)	Goal: 'Q'
6	$\quad Q \wedge Q$	$\wedge I$ 5	
7	$\quad Q$	$\wedge E$ 6	
8	Q	$\vee E$ 1, 3–4, 5–7	

Proof Strategies

1	$P \vee Q$		
2	$P \rightarrow Q$	Want: Q	
3	$\quad P$	Ass. ($\vee E$)	Goal: 'Q'
4	$\quad Q$	$\rightarrow E$ 2, 3	
5	$\quad Q$	Ass. ($\vee E$)	Goal: 'Q'
6	$\quad Q \wedge Q$	$\wedge I$ 5	
7	$\quad Q$	$\wedge E$ 6	
8	Q	$\vee E$ 1, 3–4, 5–7	

Proof Strategies

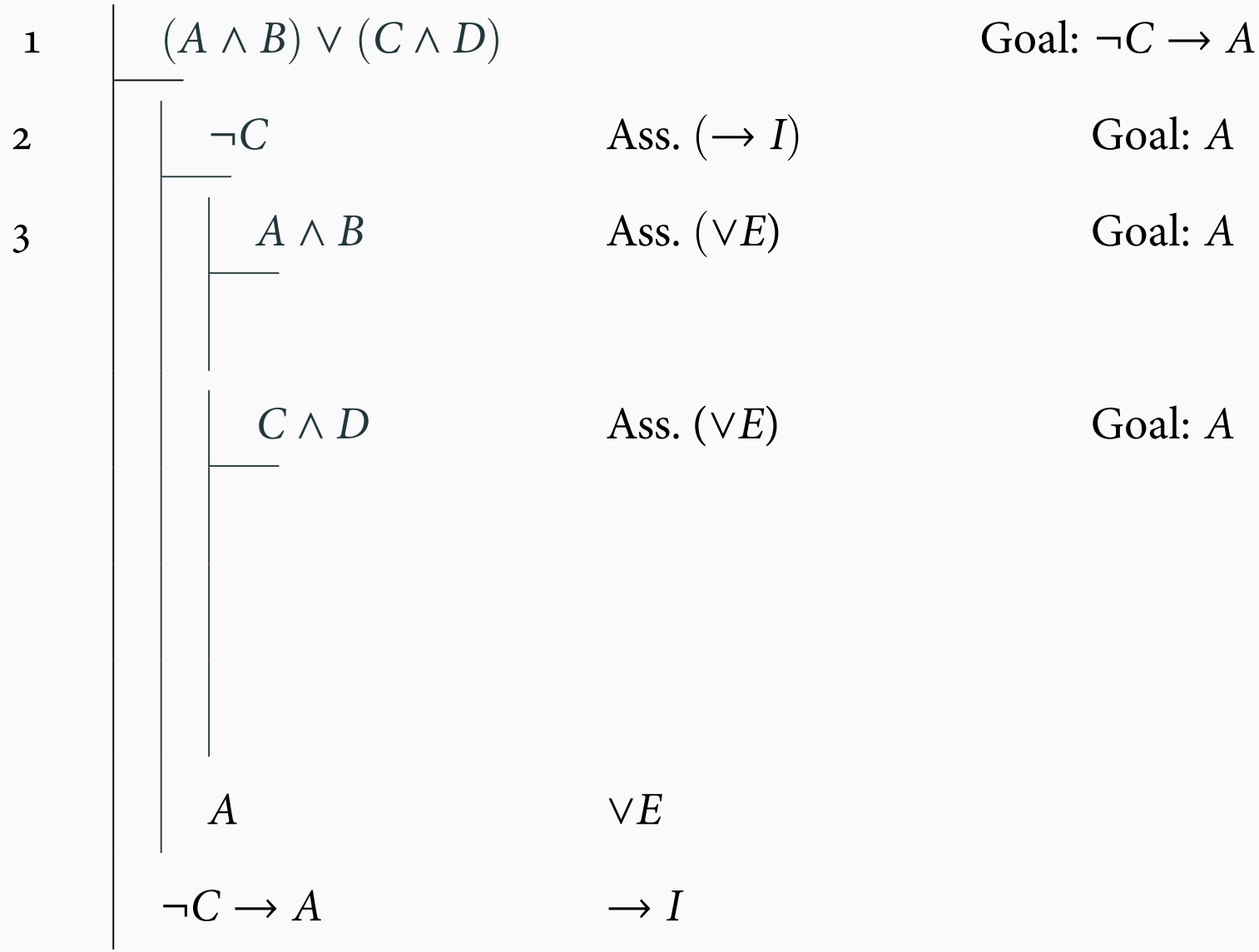
- Tip #3: Form sub-goals.

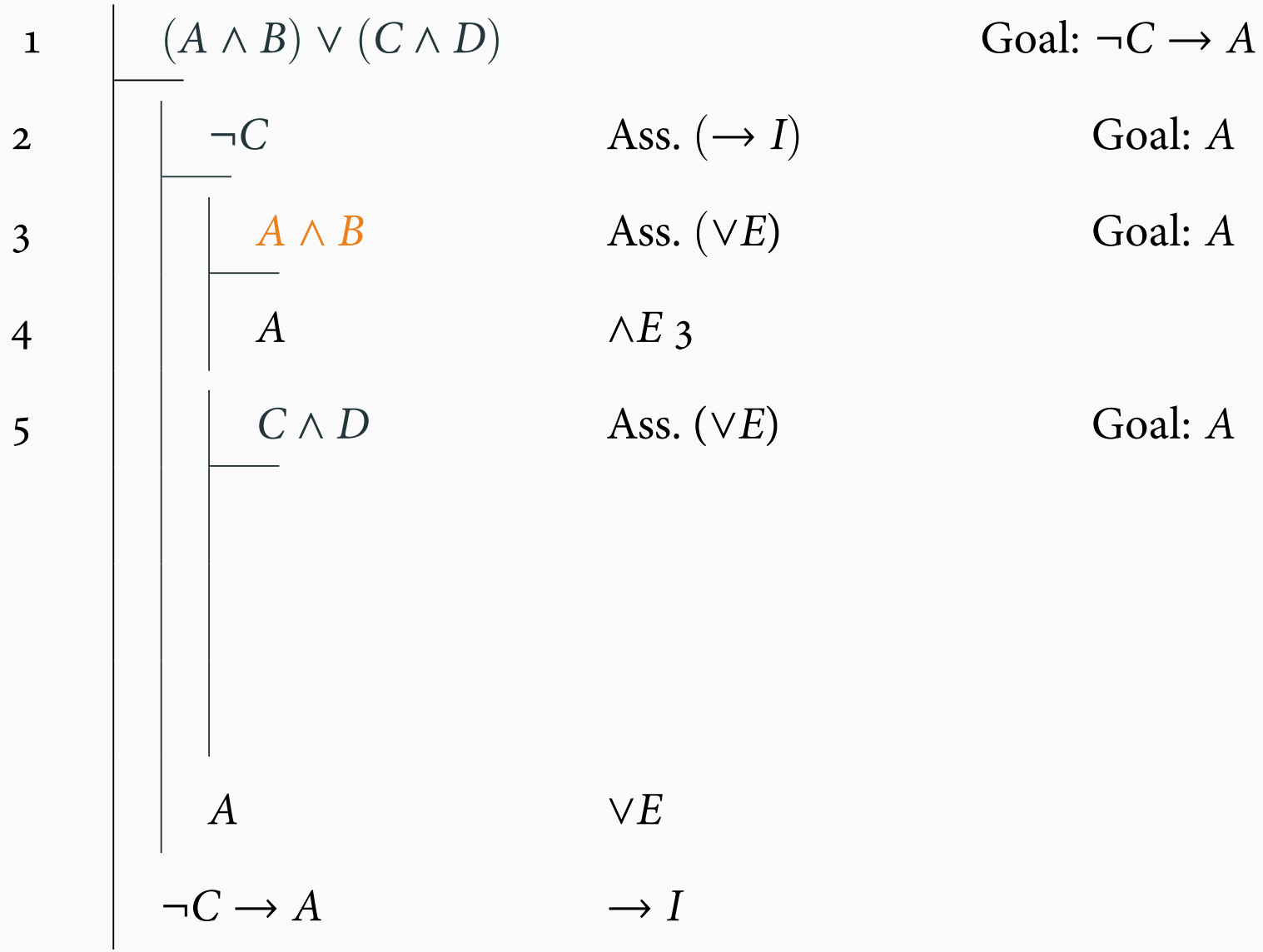
Proof Strategies

- Tip #3: Form sub-goals.
  - Use the sub-goals to structure your thinking about the problem, turning one hard problem into several easier problems.

1 | $(A \wedge B) \vee (C \wedge D)$ Goal: $\neg C \rightarrow A$

1	$(A \wedge B) \vee (C \wedge D)$		Goal: $\neg C \rightarrow A$
2	$\neg C$	Ass. $(\rightarrow I)$	Goal: A
	A		
	$\neg C \rightarrow A$	$\rightarrow I$	


1
$(A \wedge B) \vee (C \wedge D)$
Goal: $\neg C \to A$
2
$\neg C$
Ass. $(\to I)$
Goal: A
3
$A \wedge B$
Ass. $(\vee E)$
Goal: A
$C \wedge D$
Ass. $(\vee E)$
Goal: A
A
$\vee E$
$\neg C \to A$
$\to I$


1
$(A \wedge B) \vee (C \wedge D)$
Goal: $\neg C \rightarrow A$
2
$\neg C$
Ass. $(\rightarrow I)$
Goal: A
3
$A \wedge B$
Ass. $(\vee E)$
Goal: A
4
A
$\wedge E$ 3
5
$C \wedge D$
Ass. $(\vee E)$
Goal: A
A
$\vee E$
$\neg C \rightarrow A$
$\rightarrow I$

1	$(A \wedge B) \vee (C \wedge D)$		Goal: $\neg C \to A$
2	$\neg C$	Ass. $(\to I)$	Goal: A
3	$A \wedge B$	Ass. $(\vee E)$	Goal: A
4	A	$\wedge E$ 3	
5	$C \wedge D$	Ass. $(\vee E)$	Goal: A
6	C	$\wedge E$ 5	
	A	$\vee E$	
	$\neg C \to A$	$\to I$	

1	$(A \wedge B) \vee (C \wedge D)$		Goal: $\neg C \to A$
2	$\neg C$	Ass. $(\to I)$	Goal: A
3	$A \wedge B$	Ass. $(\vee E)$	Goal: A
4	A	$\wedge E$ 3	
5	$C \wedge D$	Ass. $(\vee E)$	Goal: A
6	C	$\wedge E$ 5	
7	$\bot$	$\bot I$ 2, 6	
	A	$\vee E$	
	$\neg C \to A$	$\to I$	

1	$(A \land B) \lor (C \land D)$		Goal: $\neg C \to A$
2	$\quad \neg C$	Ass. $(\to I)$	Goal: A
3	$\quad\quad A \land B$	Ass. $(\lor E)$	Goal: A
4	$\quad\quad A$	$\land E$ 3	
5	$\quad\quad C \land D$	Ass. $(\lor E)$	Goal: A
6	$\quad\quad C$	$\land E$ 5	
7	$\quad\quad \bot$	$\bot I$ 2, 6	
8	$\quad\quad A$	$\bot E$ 7	
	$\quad A$	$\lor E$	
	$\neg C \to A$	$\to I$	

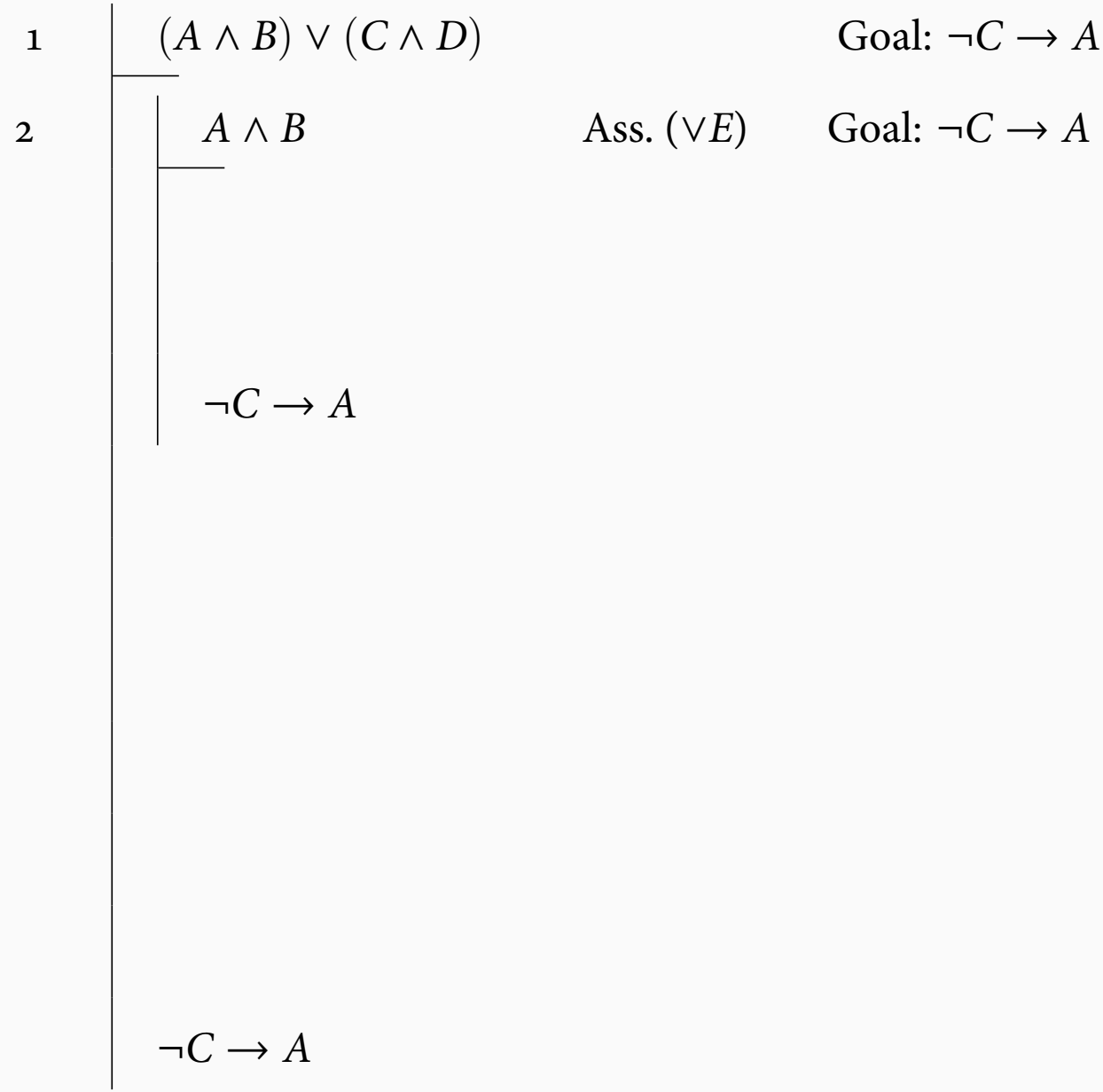
1	$(A \land B) \lor (C \land D)$		Goal: $\neg C \to A$
2	$\neg C$	Ass. $(\to I)$	Goal: A
3	$A \land B$	Ass. $(\lor E)$	Goal: A
4	A	$\land E$ 3	
5	$C \land D$	Ass. $(\lor E)$	Goal: A
6	C	$\land E$ 5	
7	$\bot$	$\bot I$ 2, 6	
8	A	$\bot E$ 7	
9	A	$\lor E$ 1, 3–4, 5–8	
	$\neg C \to A$	$\to I$	

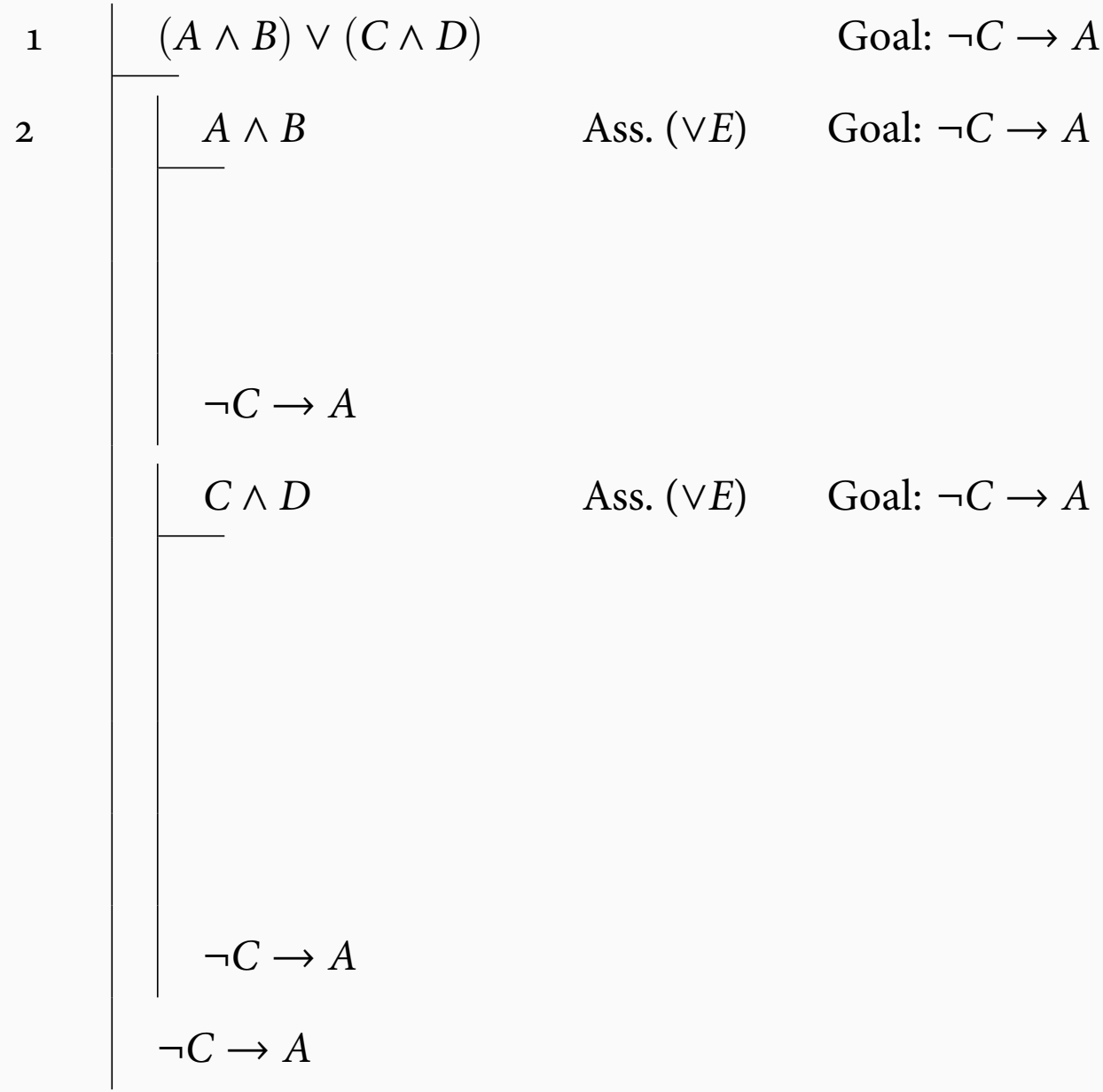
1	$(A \land B) \lor (C \land D)$		Goal: $\neg C \to A$
2	$\neg C$	Ass. $(\to I)$	Goal: A
3	$A \land B$	Ass. $(\lor E)$	Goal: A
4	A	$\land E$ 3	
5	$C \land D$	Ass. $(\lor E)$	Goal: A
6	C	$\land E$ 5	
7	$\bot$	$\bot I$ 2, 6	
8	A	$\bot E$ 7	
9	A	$\lor E$ 1, 3–4, 5–8	
10	$\neg C \to A$	$\to I$ 2–9	

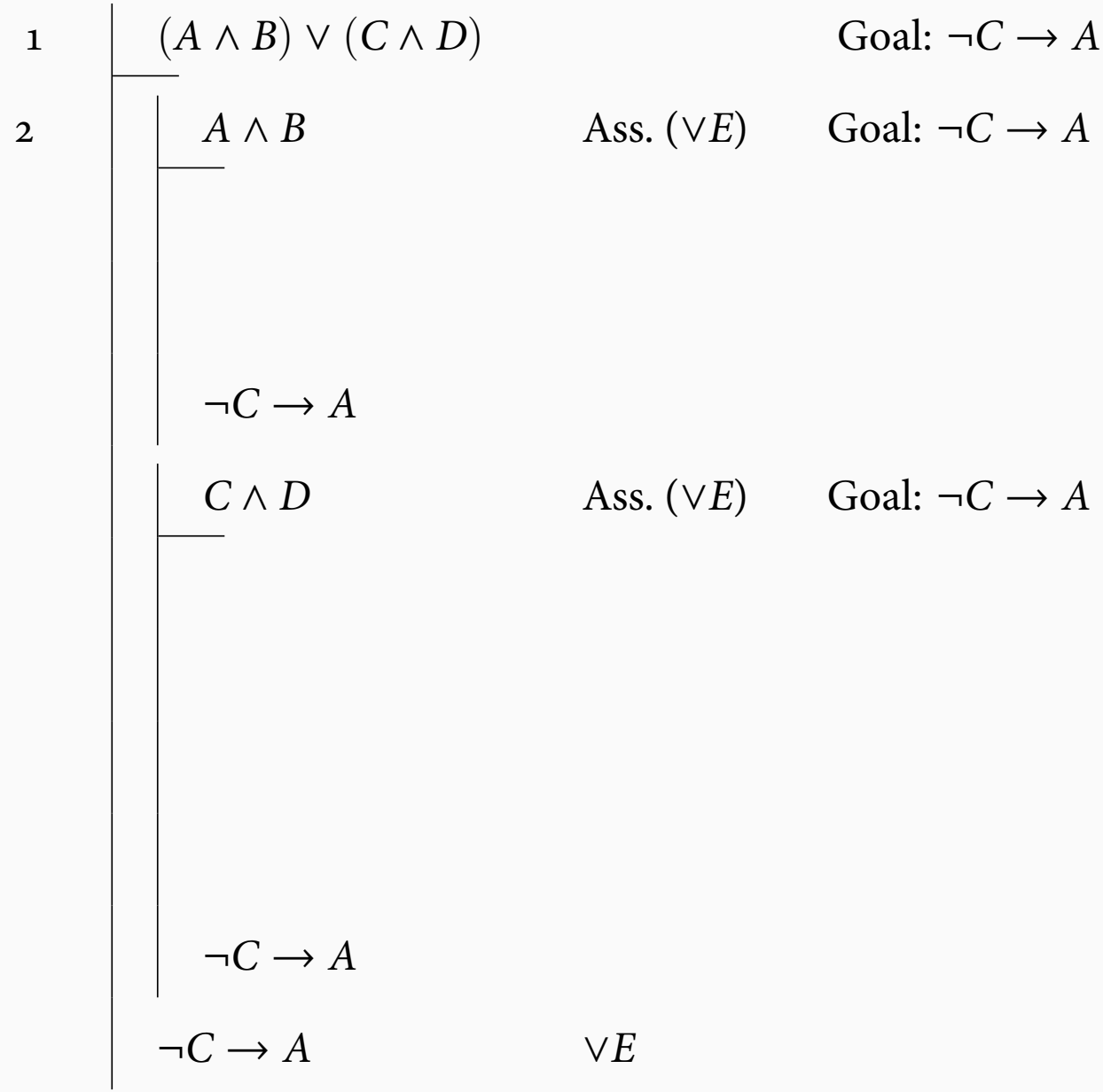
1	$(A \wedge B) \vee (C \wedge D)$		Goal: $\neg C \to A$
2	$\quad \neg C$	Ass. $(\to I)$	Goal: A
3	$\quad\quad A \wedge B$	Ass. $(\vee E)$	Goal: A
4	$\quad\quad A$	$\wedge E$ 3	
5	$\quad\quad C \wedge D$	Ass. $(\vee E)$	Goal: A
6	$\quad\quad C$	$\wedge E$ 5	
7	$\quad\quad \bot$	$\bot I$ 2, 6	
8	$\quad\quad A$	$\bot E$ 7	
9	$\quad A$	$\vee E$ 1, 3–4, 5–8	
10	$\neg C \to A$	$\to I$ 2–9	

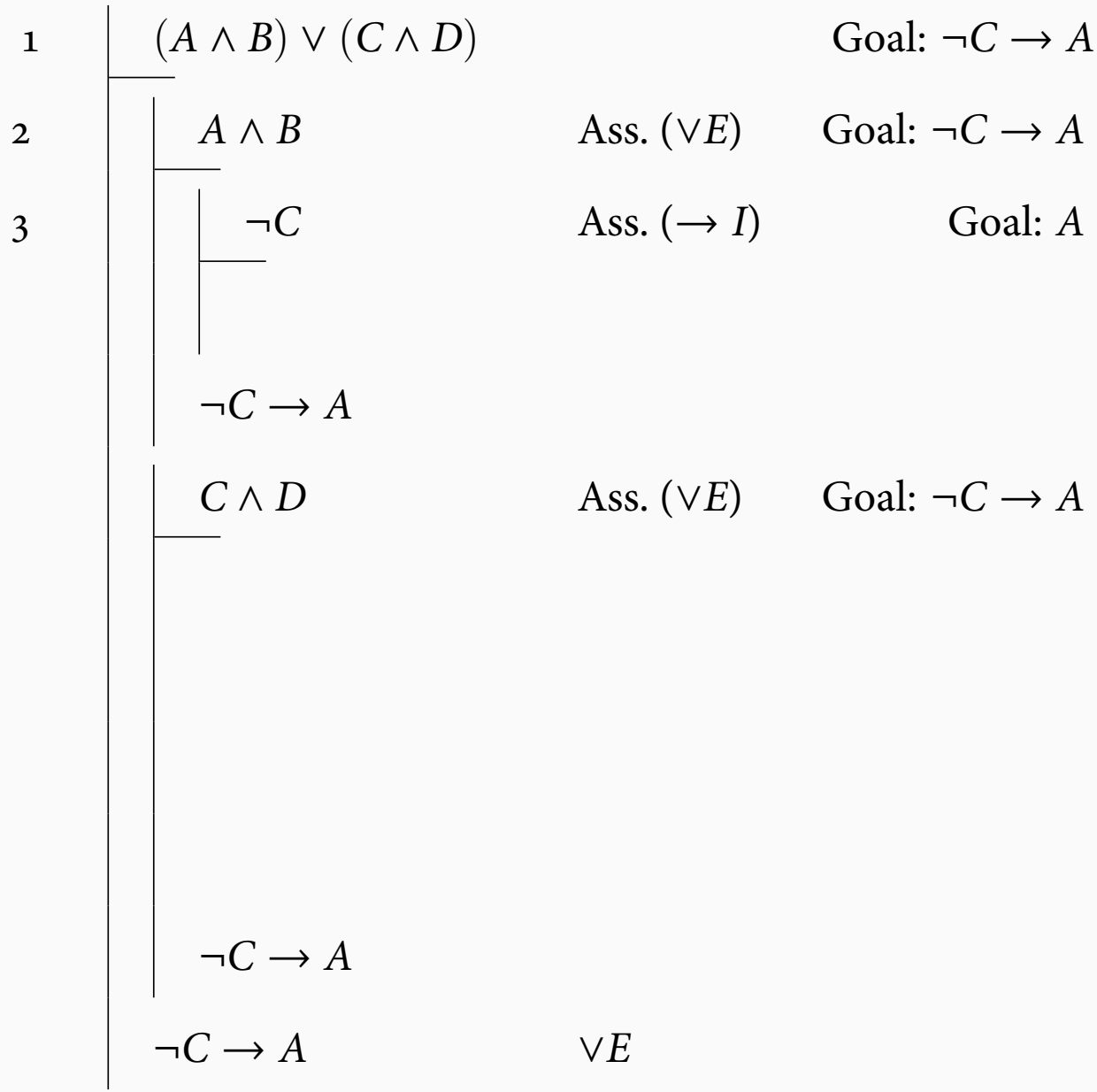
1 | $(A \wedge B) \vee (C \wedge D)$ Goal: $\neg C \rightarrow A$

$\neg C \rightarrow A$


1
$(A \wedge B) \vee (C \wedge D)$
Goal: $\neg C \to A$
2
$A \wedge B$
Ass. $(\vee E)$
Goal: $\neg C \to A$
$\neg C \to A$
$\neg C \to A$


1
$(A \wedge B) \vee (C \wedge D)$
Goal: $\neg C \to A$
2
$A \wedge B$
Ass. $(\vee E)$
Goal: $\neg C \to A$
$\neg C \to A$
$C \wedge D$
Ass. $(\vee E)$
Goal: $\neg C \to A$
$\neg C \to A$
$\neg C \to A$


1
$(A \wedge B) \vee (C \wedge D)$
Goal: $\neg C \to A$
2
$A \wedge B$
Ass. ($\vee E$)
Goal: $\neg C \to A$
$\neg C \to A$
$C \wedge D$
Ass. ($\vee E$)
Goal: $\neg C \to A$
$\neg C \to A$
$\neg C \to A$
$\vee E$


1
$(A \wedge B) \vee (C \wedge D)$
Goal: $\neg C \rightarrow A$
2
$A \wedge B$
Ass. ($\vee E$)
Goal: $\neg C \rightarrow A$
3
$\neg C$
Ass. ($\rightarrow I$)
Goal: A
$\neg C \rightarrow A$
$C \wedge D$
Ass. ($\vee E$)
Goal: $\neg C \rightarrow A$
$\neg C \rightarrow A$
$\neg C \rightarrow A$
$\vee E$

1	$(A \wedge B) \vee (C \wedge D)$		Goal: $\neg C \rightarrow A$
2	$A \wedge B$	Ass. ($\vee E$)	Goal: $\neg C \rightarrow A$
3	$\neg C$	Ass. ($\rightarrow I$)	Goal: A
4	A	$\wedge E$ 2	
	$\neg C \rightarrow A$		
	$C \wedge D$	Ass. ($\vee E$)	Goal: $\neg C \rightarrow A$
	$\neg C \rightarrow A$		
	$\neg C \rightarrow A$	$\vee E$	

1	$(A \wedge B) \vee (C \wedge D)$		Goal: $\neg C \rightarrow A$
2	$A \wedge B$	Ass. ($\vee E$)	Goal: $\neg C \rightarrow A$
3	$\neg C$	Ass. ($\rightarrow I$)	Goal: A
4	A	$\wedge E$ 2	
5	$\neg C \rightarrow A$	$\rightarrow I$ 3–4	
6	$C \wedge D$	Ass. ($\vee E$)	Goal: $\neg C \rightarrow A$
	$\neg C \rightarrow A$		
	$\neg C \rightarrow A$	$\vee E$	

1	$(A \wedge B) \vee (C \wedge D)$		Goal: $\neg C \rightarrow A$
2	$A \wedge B$	Ass. ($\vee E$)	Goal: $\neg C \rightarrow A$
3	$\neg C$	Ass. ($\rightarrow I$)	Goal: A
4	A	$\wedge E$ 2	
5	$\neg C \rightarrow A$	$\rightarrow I$ 3–4	
6	$C \wedge D$	Ass. ($\vee E$)	Goal: $\neg C \rightarrow A$
7	$\neg C$	Ass. ($\rightarrow I$)	Goal: A
	A		
	$\neg C \rightarrow A$	$\rightarrow I$	
	$\neg C \rightarrow A$	$\vee E$	

1	$(A \wedge B) \vee (C \wedge D)$		Goal: $\neg C \rightarrow A$
2	$A \wedge B$	Ass. ($\vee E$)	Goal: $\neg C \rightarrow A$
3	$\neg C$	Ass. ($\rightarrow I$)	Goal: A
4	A	$\wedge E$ 2	
5	$\neg C \rightarrow A$	$\rightarrow I$ 3–4	
6	$C \wedge D$	Ass. ($\vee E$)	Goal: $\neg C \rightarrow A$
7	$\neg C$	Ass. ($\rightarrow I$)	Goal: A
8	C	$\wedge E$ 6	
	A		
	$\neg C \rightarrow A$	$\rightarrow I$	
	$\neg C \rightarrow A$	$\vee E$	

1	$(A \wedge B) \vee (C \wedge D)$		Goal: $\neg C \rightarrow A$
2	$A \wedge B$	Ass. ($\vee E$)	Goal: $\neg C \rightarrow A$
3	$\neg C$	Ass. ($\rightarrow I$)	Goal: A
4	A	$\wedge E$ 2	
5	$\neg C \rightarrow A$	$\rightarrow I$ 3–4	
6	$C \wedge D$	Ass. ($\vee E$)	Goal: $\neg C \rightarrow A$
7	$\neg C$	Ass. ($\rightarrow I$)	Goal: A
8	C	$\wedge E$ 6	
9	$\perp$	$\perp I$ 7, 8	
	A		
	$\neg C \rightarrow A$	$\rightarrow I$	
	$\neg C \rightarrow A$	$\vee E$	

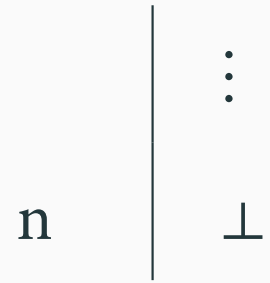
1	$(A \wedge B) \vee (C \wedge D)$		Goal: $\neg C \rightarrow A$
2	$A \wedge B$	Ass. ($\vee E$)	Goal: $\neg C \rightarrow A$
3	$\neg C$	Ass. ($\rightarrow I$)	Goal: A
4	A	$\wedge E$ 2	
5	$\neg C \rightarrow A$	$\rightarrow I$ 3–4	
6	$C \wedge D$	Ass. ($\vee E$)	Goal: $\neg C \rightarrow A$
7	$\neg C$	Ass. ($\rightarrow I$)	Goal: A
8	C	$\wedge E$ 6	
9	$\bot$	$\bot I$ 7, 8	
10	A	$\bot E$ 9	
	$\neg C \rightarrow A$	$\rightarrow I$	
	$\neg C \rightarrow A$	$\vee E$	

1	$(A \wedge B) \vee (C \wedge D)$		Goal: $\neg C \rightarrow A$
2	$\quad A \wedge B$	Ass. ($\vee E$)	Goal: $\neg C \rightarrow A$
3	$\quad\quad \neg C$	Ass. ($\rightarrow I$)	Goal: A
4	$\quad\quad A$	$\wedge E$ 2	
5	$\quad \neg C \rightarrow A$	$\rightarrow I$ 3–4	
6	$\quad C \wedge D$	Ass. ($\vee E$)	Goal: $\neg C \rightarrow A$
7	$\quad\quad \neg C$	Ass. ($\rightarrow I$)	Goal: A
8	$\quad\quad C$	$\wedge E$ 6	
9	$\quad\quad \bot$	$\bot I$ 7, 8	
10	$\quad\quad A$	$\bot E$ 9	
11	$\quad \neg C \rightarrow A$	$\rightarrow I$ 7–10	
	$\neg C \rightarrow A$	$\vee E$	

1	$(A \wedge B) \vee (C \wedge D)$		Goal: $\neg C \rightarrow A$
2	$A \wedge B$	Ass. ($\vee E$)	Goal: $\neg C \rightarrow A$
3	$\neg C$	Ass. ($\rightarrow I$)	Goal: A
4	A	$\wedge E$ 2	
5	$\neg C \rightarrow A$	$\rightarrow I$ 3–4	
6	$C \wedge D$	Ass. ($\vee E$)	Goal: $\neg C \rightarrow A$
7	$\neg C$	Ass. ($\rightarrow I$)	Goal: A
8	C	$\wedge E$ 6	
9	$\bot$	$\bot I$ 7, 8	
10	A	$\bot E$ 9	
11	$\neg C \rightarrow A$	$\rightarrow I$ 7–10	
12	$\neg C \rightarrow A$	$\vee E$ 1, 2–5, 6–11	

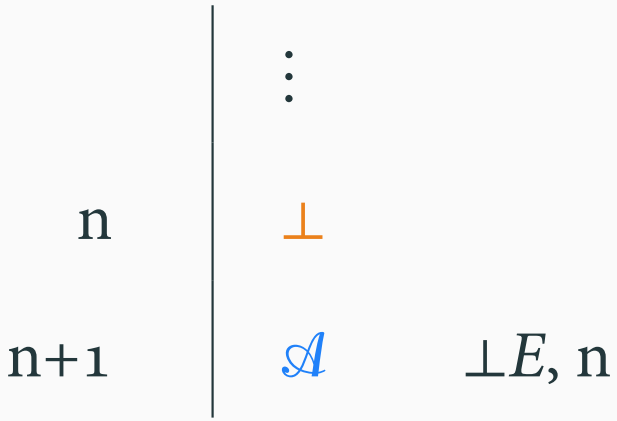
Proof Strategies

- Tip #4: Keep in mind: once you have a contradiction, $\bot$, you can get *anything you want*



Proof Strategies

- Tip #4: Keep in mind: once you have a contradiction, $\bot$, you can get *anything you want*



Proof Strategies

- Tip #5: Think about meaning.

Proof Strategies

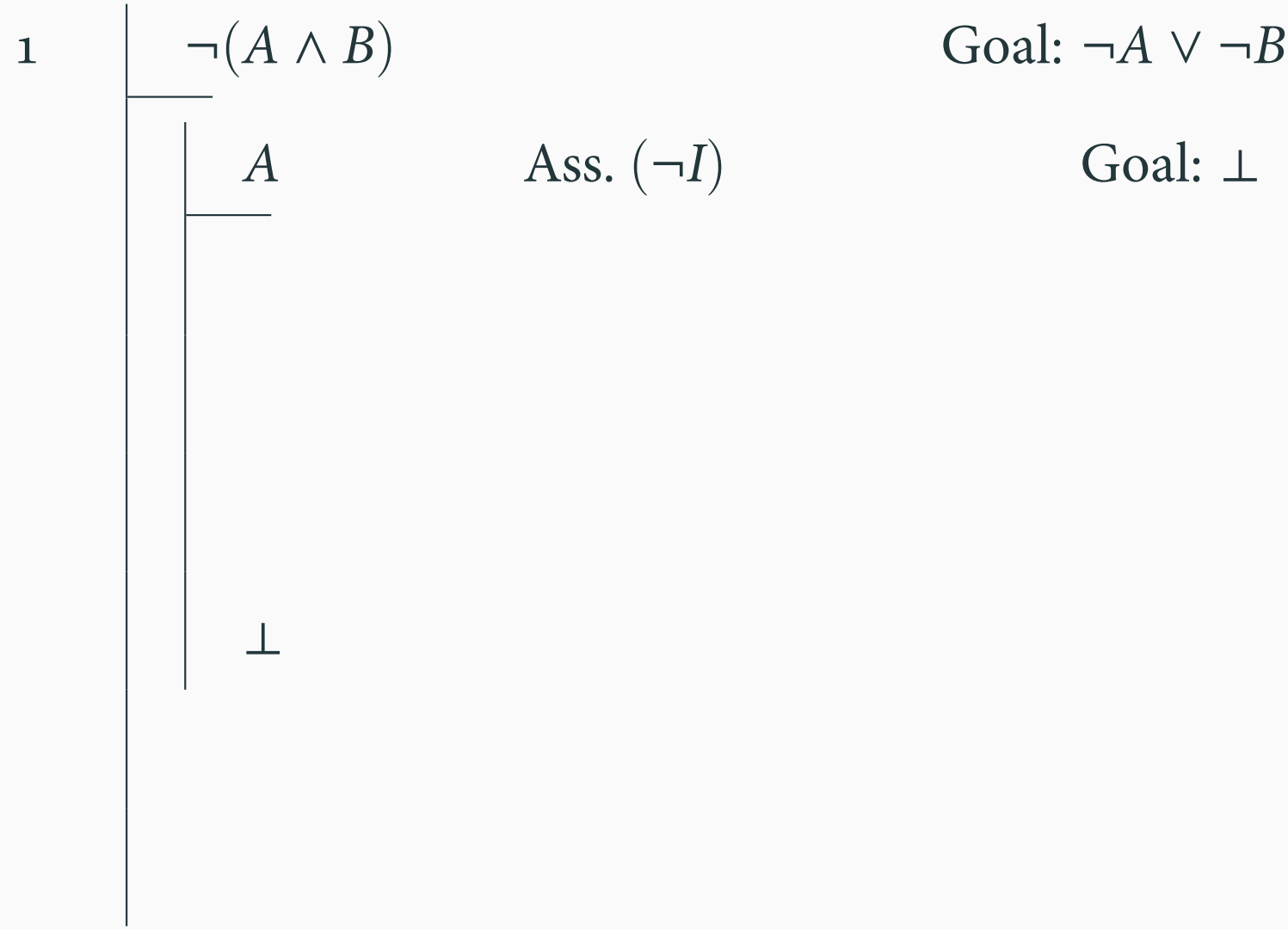
- Tip #5: Think about meaning.
  - ▹ If a strategy requires you to derive $\mathscr{B}$ from $\mathscr{A}$, first consider whether $\mathscr{B}$ *follows from* $\mathscr{A}$. If not, abandon that strategy.

$\neg(A \wedge B) \quad \vdash \quad \neg A \vee \neg B$

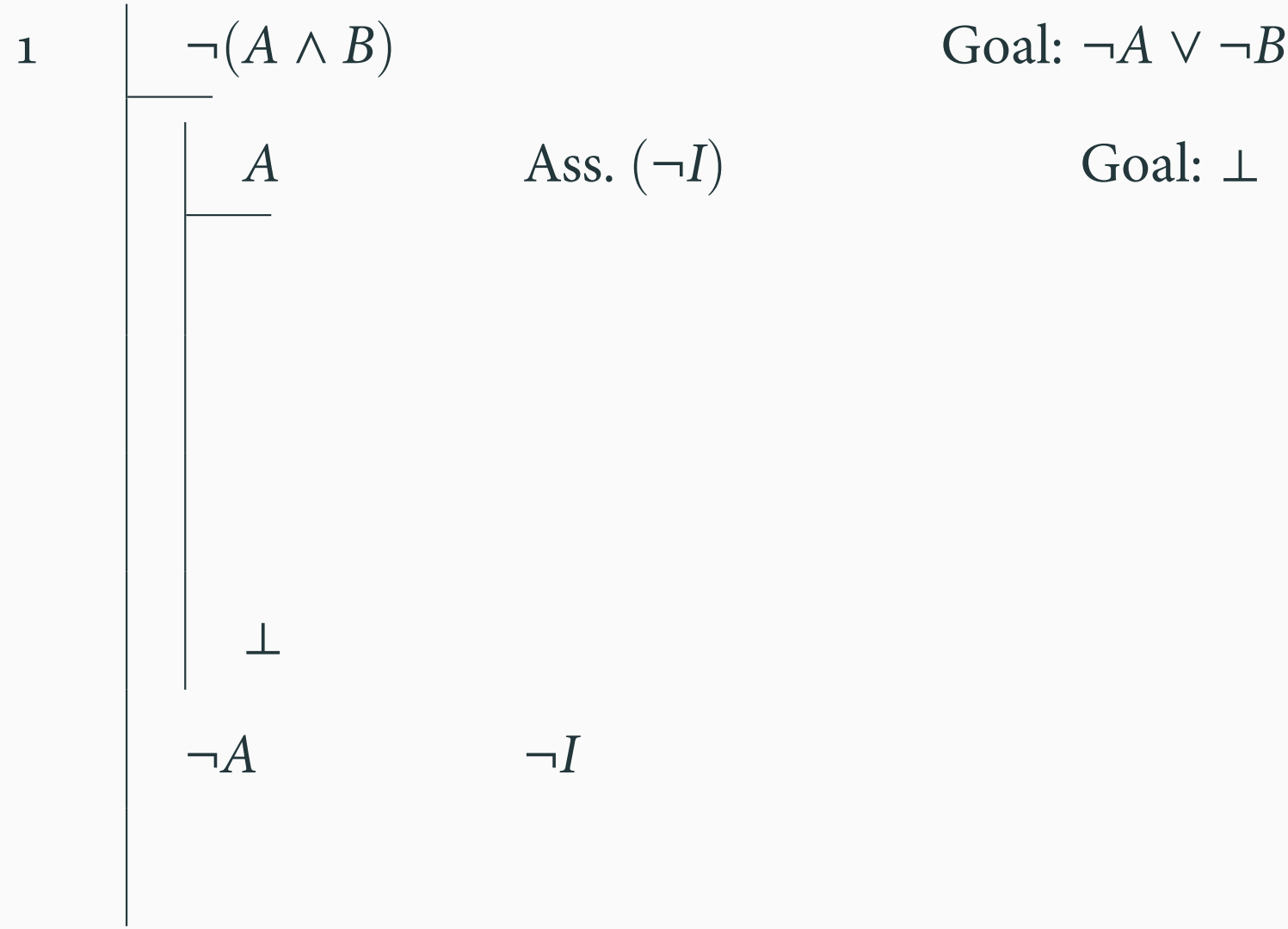
1 | $\neg(A \wedge B)$

Goal: $\neg A \vee \neg B$

$\neg(A \wedge B) \quad \vdash \quad \neg A \vee \neg B$



$\neg(A \wedge B) \quad \vdash \quad \neg A \vee \neg B$



$\neg(A \wedge B) \quad \vdash \quad \neg A \vee \neg B$

1	$\neg(A \wedge B)$		Goal: $\neg A \vee \neg B$
	A	Ass. $(\neg I)$	Goal: $\bot$
	$\bot$		
	$\neg A$	$\neg I$	
	$\neg A \vee \neg B$	$\vee I$	

Proof Strategies

- Tip #6: If you see no other strategies, try negation elimination

Proof Strategies

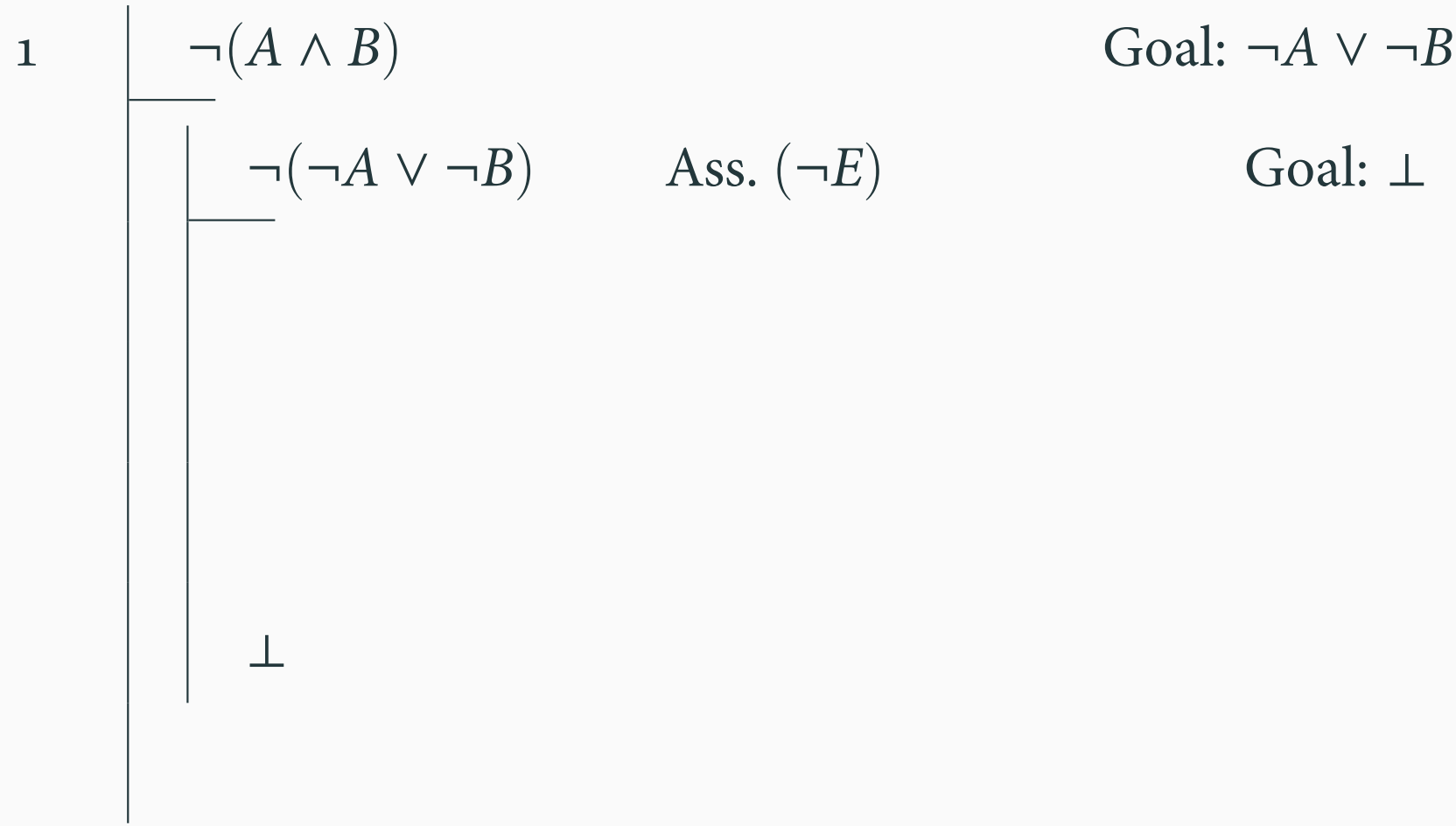
- Tip #6: If you see no other strategies, try negation elimination
- ▹ If anything works, negation elimination will.

$\neg(A \wedge B) \quad \vdash \quad \neg A \vee \neg B$

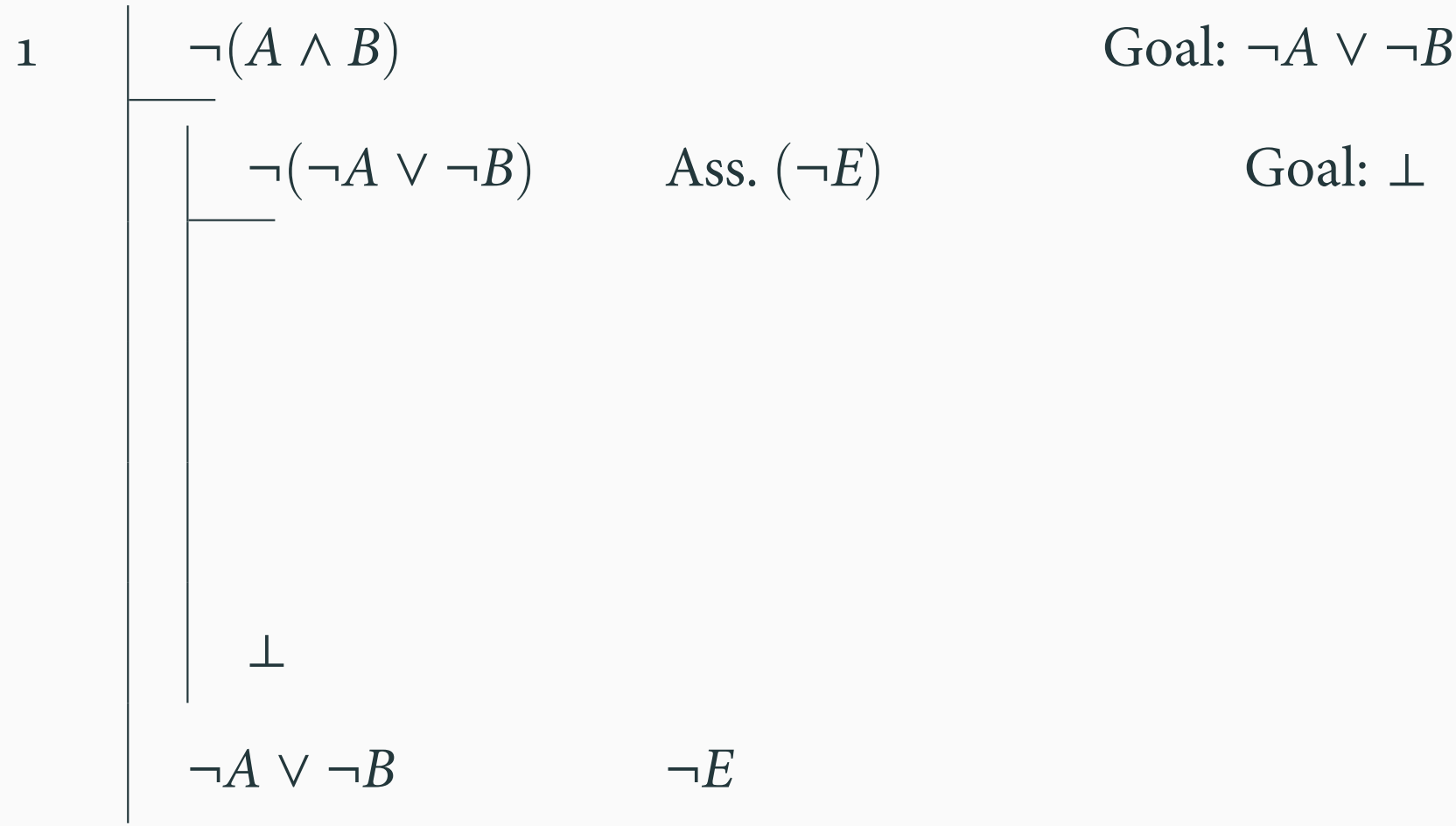
1 | $\neg(A \wedge B)$

Goal: $\neg A \vee \neg B$

$\neg(A \wedge B) \quad \vdash \quad \neg A \vee \neg B$



$\neg(A \wedge B) \quad \vdash \quad \neg A \vee \neg B$

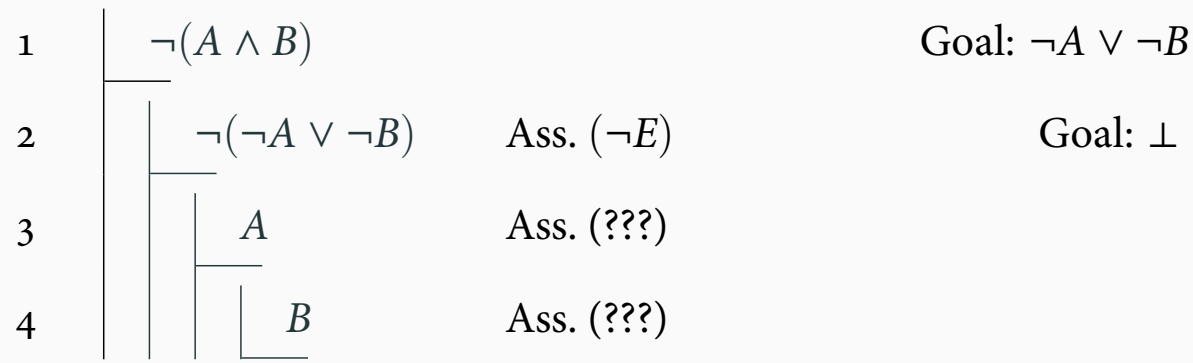


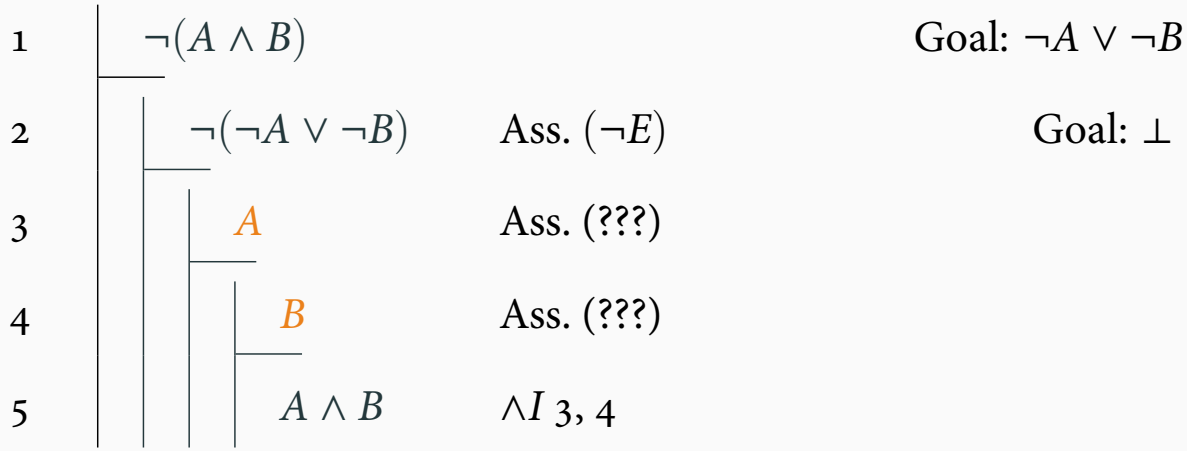
Proof Strategies

- Tip #7: If you don't have any ideas, ***just do something***

1	$\neg(A \wedge B)$		Goal: $\neg A \vee \neg B$
2	$\neg(\neg A \vee \neg B)$	Ass. $(\neg E)$	Goal: $\bot$

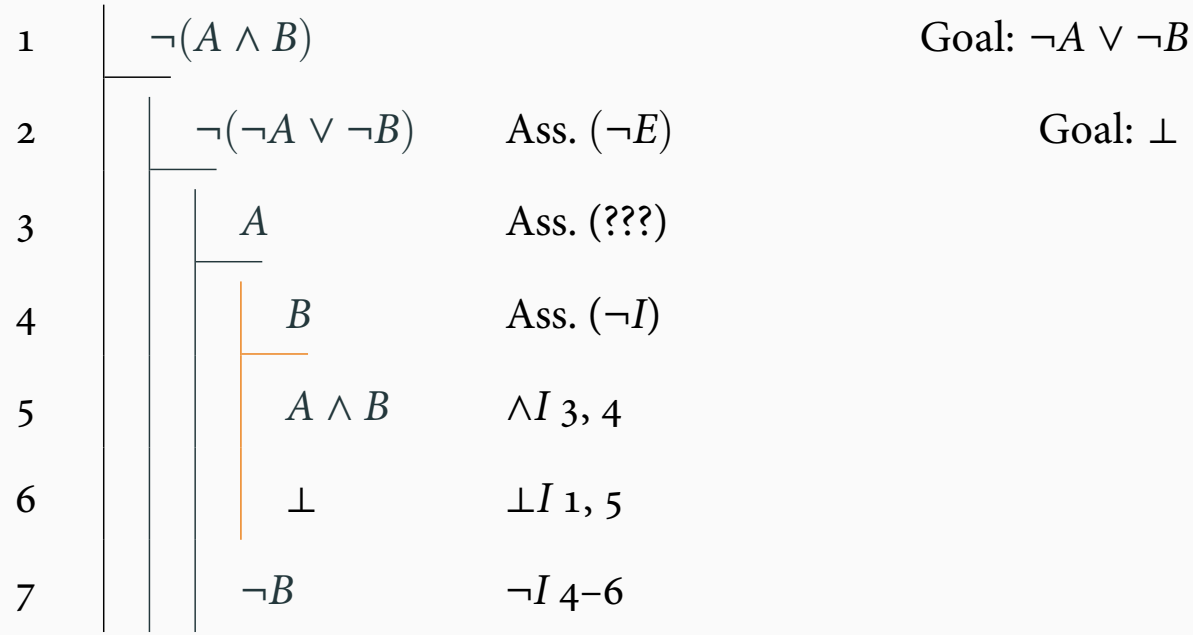
1	$\neg(A \wedge B)$		Goal: $\neg A \vee \neg B$
2	$\neg(\neg A \vee \neg B)$	Ass. $(\neg E)$	Goal: $\bot$
3	A	Ass. (???)	


1
¬(A ∧ B)
Goal: ¬A ∨ ¬B
2
¬(¬A ∨ ¬B)
Ass. (¬E)
Goal: ⊥
3
A
Ass. (???)
4
B
Ass. (???)


1
$\neg(A \wedge B)$
Goal: $\neg A \vee \neg B$
2
$\neg(\neg A \vee \neg B)$
Ass. $(\neg E)$
Goal: $\bot$
3
A
Ass. (???)
4
B
Ass. (???)
5
$A \wedge B$
$\wedge I$ 3, 4

1	$\neg(A \wedge B)$		Goal: $\neg A \vee \neg B$
2	$\neg(\neg A \vee \neg B)$	Ass. $(\neg E)$	Goal: $\bot$
3	A	Ass. (???)	
4	B	Ass. (???)	
5	$A \wedge B$	$\wedge I$ 3, 4	
6	$\bot$	$\bot I$ 1, 5	

1	$\neg(A \wedge B)$		Goal: $\neg A \vee \neg B$
2	$\neg(\neg A \vee \neg B)$	Ass. ($\neg E$)	Goal: $\bot$
3	A	Ass. (???)	
4	B	Ass. ($\neg I$)	
5	$A \wedge B$	$\wedge I$ 3, 4	
6	$\bot$	$\bot I$ 1, 5	


1 ¬(A ∧ B)
Goal: ¬A ∨ ¬B
2 ¬(¬A ∨ ¬B) Ass. (¬E)
Goal: ⊥
3 A Ass. (???)
4 B Ass. (¬I)
5 A ∧ B ∧I 3, 4
6 ⊥ ⊥I 1, 5
7 ¬B ¬I 4–6

1	$\neg(A \wedge B)$		Goal: $\neg A \vee \neg B$
2	$\neg(\neg A \vee \neg B)$	Ass. ($\neg E$)	Goal: $\bot$
3	A	Ass. (???)	
4	B	Ass. ($\neg I$)	
5	$A \wedge B$	$\wedge I$ 3, 4	
6	$\bot$	$\bot I$ 1, 5	
7	$\neg B$	$\neg I$ 4–6	
8	$\neg A \vee \neg B$	$\vee I$ 7	

1	$\neg(A \wedge B)$		Goal: $\neg A \vee \neg B$
2	$\neg(\neg A \vee \neg B)$	Ass. ($\neg E$)	Goal: $\bot$
3	A	Ass. (???)	
4	B	Ass. ($\neg I$)	
5	$A \wedge B$	$\wedge I$ 3, 4	
6	$\bot$	$\bot I$ 1, 5	
7	$\neg B$	$\neg I$ 4–6	
8	$\neg A \vee \neg B$	$\vee I$ 7	
9	$\bot$	$\bot I$ 2, 8	

1	$\neg(A \wedge B)$		Goal: $\neg A \vee \neg B$
2	$\neg(\neg A \vee \neg B)$	Ass. ($\neg E$)	Goal: $\bot$
3	A	Ass. ($\neg I$)	
4	B	Ass. ($\neg I$)	
5	$A \wedge B$	$\wedge I$ 3, 4	
6	$\bot$	$\bot I$ 1, 5	
7	$\neg B$	$\neg I$ 4–6	
8	$\neg A \vee \neg B$	$\vee I$ 7	
9	$\bot$	$\bot I$ 2, 8	

1	$\neg(A \wedge B)$		Goal: $\neg A \vee \neg B$
2	$\neg(\neg A \vee \neg B)$	Ass. $(\neg E)$	Goal: $\bot$
3	A	Ass. $(\neg I)$	
4	B	Ass. $(\neg I)$	
5	$A \wedge B$	$\wedge I$ 3, 4	
6	$\bot$	$\bot I$ 1, 5	
7	$\neg B$	$\neg I$ 4–6	
8	$\neg A \vee \neg B$	$\vee I$ 7	
9	$\bot$	$\bot I$ 2, 8	
10	$\neg A$	$\neg I$ 3–9	

1	$\neg(A \wedge B)$		Goal: $\neg A \vee \neg B$
2	$\neg(\neg A \vee \neg B)$	Ass. ($\neg E$)	Goal: $\bot$
3	A	Ass. ($\neg I$)	
4	B	Ass. ($\neg I$)	
5	$A \wedge B$	$\wedge I$ 3, 4	
6	$\bot$	$\bot I$ 1, 5	
7	$\neg B$	$\neg I$ 4–6	
8	$\neg A \vee \neg B$	$\vee I$ 7	
9	$\bot$	$\bot I$ 2, 8	
10	$\neg A$	$\neg I$ 3–9	
11	$\neg A \vee \neg B$	$\vee I$ 10	

1	$\neg(A \wedge B)$		Goal: $\neg A \vee \neg B$
2	$\neg(\neg A \vee \neg B)$	Ass. ($\neg E$)	Goal: $\bot$
3	A	Ass. ($\neg I$)	
4	B	Ass. ($\neg I$)	
5	$A \wedge B$	$\wedge I$ 3, 4	
6	$\bot$	$\bot I$ 1, 5	
7	$\neg B$	$\neg I$ 4–6	
8	$\neg A \vee \neg B$	$\vee I$ 7	
9	$\bot$	$\bot I$ 2, 8	
10	$\neg A$	$\neg I$ 3–9	
11	$\neg A \vee \neg B$	$\vee I$ 10	
12	$\bot$	$\bot I$ 2, 11	

1	$\neg(A \wedge B)$		Goal: $\neg A \vee \neg B$
2	$\neg(\neg A \vee \neg B)$	Ass. ($\neg E$)	Goal: $\bot$
3	A	Ass. ($\neg I$)	
4	B	Ass. ($\neg I$)	
5	$A \wedge B$	$\wedge I$ 3, 4	
6	$\bot$	$\bot I$ 1, 5	
7	$\neg B$	$\neg I$ 4–6	
8	$\neg A \vee \neg B$	$\vee I$ 7	
9	$\bot$	$\bot I$ 2, 8	
10	$\neg A$	$\neg I$ 3–9	
11	$\neg A \vee \neg B$	$\vee I$ 10	
12	$\bot$	$\bot I$ 2, 11	
13	$\neg A \vee \neg B$	$\neg E$ 2–12	