

Sentential Logic

Entailment, Satisfiability, Tautologies, and
Contradictions

PHIL 500

Outline

Semantics for SL

Semantic Notions

Entailment

Satisfiability

Tautologies and Contradictions

Semantics for SL

Semantic Notions

Entailment

Satisfiability

Tautologies and Contradictions

The Meaning of '¬'

\mathcal{A}	$\neg\mathcal{A}$
T	F
F	T

The Meaning of ' \wedge '

A	B	$A \wedge B$
T	T	T
T	F	F
F	T	F
F	F	F

The Meaning of '∨'

A	B	$A \vee B$
T	T	T
T	F	T
F	T	T
F	F	F

The Meaning of '→'

A	B	$A \rightarrow B$
T	T	T
T	F	F
F	T	T
F	F	T

The Meaning of ' \leftrightarrow '

A	B	$A \rightarrow B$
T	T	T
T	F	F
F	T	F
F	F	T

Determining the truth-value of a sentence of SL

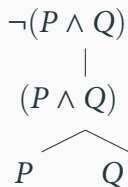
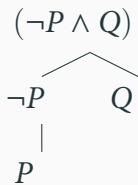
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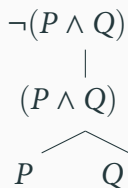
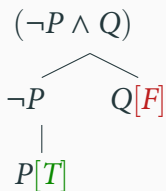
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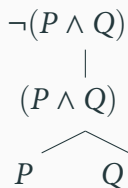
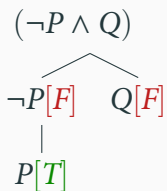
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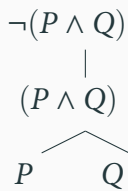
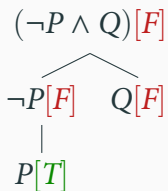
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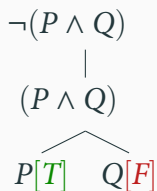
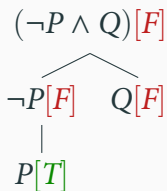
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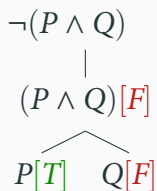
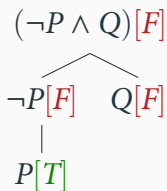
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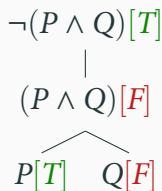
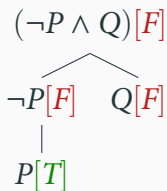
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Truth Tables

- Truth-table for ' $\neg P \wedge Q$ ':

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P	Q	$\neg P \wedge Q$
T	T	
T	F	
F	T	
F	F	

Truth Tables

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P	Q	$\neg P$	$P \wedge Q$
T	T	F	T
T	F	F	F
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F	F	T	F

Truth Tables

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P	Q	\neg	P	\wedge	Q
T	T	F	T		T
T	F	F	T		F
F	T	T	F		T
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Truth Tables

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P	Q	\neg	P	\wedge	Q
T	T	F	T	F	T
T	F	F	T	F	F
F	T	T	F	T	T
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Truth Tables

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P	Q	$\neg (P \wedge Q)$
T	T	T T T
T	F	T F F
F	T	F F T
F	F	F F F

Truth Tables

- Truth-table for $\neg(P \wedge Q)$:

P	Q	\neg	$(P$	\wedge	$Q)$
T	T	F	T	T	T
T	F	T	T	F	F
F	T	T	F	F	T
F	F	T	F	F	F

Truth-functionality

- In SL, the truth-value of non-atomic sentences is a function of the truth-values of the atomic sentences appearing therein.

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- ▶ In order to know the truth-value of ' $\neg A$ ', you only need to know the *truth-value* of ' A '. You don't need to know what ' A ' means.

Truth-functionality

- In SL, the truth-value of non-atomic sentences is a function of the truth-values of the atomic sentences appearing therein.
- That's because the operators of SL are *truth-functional*.
- ▶ In order to know the truth-value of ' $\neg\mathcal{A}$ ', you only need to know the *truth-value* of ' \mathcal{A} '. You don't need to know what ' \mathcal{A} ' means.
- Not all sentential operators are truth-functional like this.

Truth-functionality

- Suppose both ' \mathcal{A} ' and ' \mathcal{B} ' are true.

Truth-functionality

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- Then, “Both \mathcal{A} and \mathcal{B} ”

Truth-functionality

- Suppose both ' \mathcal{A} ' and ' \mathcal{B} ' are true.
- Then, “Both \mathcal{A} and \mathcal{B} ” must be true.

Truth-functionality

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- But: " \mathcal{A} because \mathcal{B} "...

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 - ▷ Snow is white because grass is green

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Truth-functionality

- Suppose both ‘ A ’ and ‘ B ’ are true.
- Then, “Both A and B ” must be true.
- But: “ A because B ” could be true and could be false.
- ▶ The tides come in because the moon exerts a gravitational force on the ocean
- ▶ Snow is white because grass is green

Valuations

- Because the operators of SL are truth-functional, all we need in order to say whether the sentences of SL are true or false is a *valuation*.

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A **VALUATION** is an assignment of truth-values (true or false) to the statement letters of SL.

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Validity and Entailment

Validity

An argument $\mathcal{A}_1, \mathcal{A}_2, \dots, \mathcal{A}_N \therefore \mathcal{C}$ is *valid* if and only if there is no **possibility** in which $\mathcal{A}_1, \mathcal{A}_2, \dots, \mathcal{A}_N$ are all true while \mathcal{C} is false.

Validity and Entailment

Validity

An argument $\mathcal{A}_1, \mathcal{A}_2, \dots, \mathcal{A}_N \therefore \mathcal{C}$ is *valid* if and only if there is no **possibility** in which $\mathcal{A}_1, \mathcal{A}_2, \dots, \mathcal{A}_N$ are all true while \mathcal{C} is false.

Entailment

$\mathcal{A}_1, \mathcal{A}_2, \dots, \mathcal{A}_N$ *entail* \mathcal{C} if and only if there is no **valuation** on which $\mathcal{A}_1, \mathcal{A}_2, \dots, \mathcal{A}_N$ are all true while \mathcal{C} is false.

Notation

- We will write:

$$\mathcal{A}_1, \mathcal{A}_2, \dots, \mathcal{A}_N \models \mathcal{C}$$

to mean that $\mathcal{A}_1, \mathcal{A}_2, \dots, \mathcal{A}_N$ entail \mathcal{C} .

Entailment

$$P \rightarrow Q, \neg Q \stackrel{?}{\models} \neg P$$

P	Q	$P \rightarrow Q$	$\neg Q$	$\neg P$
T	T			
T	F			
F	T			
F	F			

Entailment

$$P \rightarrow Q, \neg Q \stackrel{?}{\models} \neg P$$

P	Q	$P \rightarrow Q$	$\neg Q$	$\neg P$
T	T	T		
T	F	T		
F	T	F		
F	F	F		

Entailment

$$P \rightarrow Q, \neg Q \stackrel{?}{\models} \neg P$$

P	Q	$P \rightarrow Q$	$\neg Q$	$\neg P$
T	T	T	F	F
T	F	F	T	F
F	T	T	F	T
F	F	T	T	T

Entailment

$$P \rightarrow Q, \neg Q \stackrel{?}{\models} \neg P$$

P	Q	$P \rightarrow Q$	$\neg Q$	$\neg P$
T	T	T	F	F
T	F	F	T	F
F	T	T	F	T
F	F	T	T	T

Entailment

$$P \rightarrow Q, \neg Q \stackrel{?}{\models} \neg P$$

P	Q	$P \rightarrow Q$	$\neg Q$	$\neg P$
T	T	T	F	F
T	F	F	T	F
F	T	T	F	T
F	F	T	T	T

Entailment

$$P \rightarrow Q, \neg Q \stackrel{?}{\models} \neg P$$

P	Q	$P \rightarrow Q$	$\neg Q$	$\neg P$
T	T	T	F	F
T	F	F	T	F
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Entailment

$$P \rightarrow Q, \neg Q \stackrel{?}{\models} \neg P$$

P	Q	$P \rightarrow Q$	$\neg Q$	$\neg P$
T	T	T	F	F
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Entailment

$$P \rightarrow Q, \neg Q \stackrel{?}{\models} \neg P$$

P	Q	$P \rightarrow Q$	$\neg Q$	$\neg P$
T	T	T		
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Entailment

$$P \rightarrow Q, \neg Q \stackrel{?}{\models} \neg P$$

P	Q	$P \rightarrow Q$	$\neg Q$	$\neg P$
T	T	T	F	F
T	F	F	T	F
F	T	T	F	T
F	F	T	T	T

Entailment

$$P \rightarrow Q, \neg Q \stackrel{?}{\models} \neg P$$

P	Q	$P \rightarrow Q$	$\neg Q$	$\neg P$
T	T	T	F	
T	F	F	T	
F	T	T	F	
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Entailment

$$P \rightarrow Q, \neg Q \stackrel{?}{\models} \neg P$$

P	Q	$P \rightarrow Q$	$\neg Q$	$\neg P$
T	T	T	F	
T	F	F	T	
F	T	T		T
F	F	T		T

Entailment

$$P \rightarrow Q, \neg Q \stackrel{?}{\models} \neg P$$

P	Q	$P \rightarrow Q$	$\neg Q$	$\neg P$
T	T	T	F	
T	F	F	T	
F	T	T	F	
F	F	T	T	

Entailment

$$P \rightarrow Q, \neg Q \stackrel{?}{\models} \neg P$$

P	Q	$P \rightarrow Q$	$\neg Q$	$\neg P$
T	T	T	F	F
T	F	F	T	F
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Entailment

$$P \rightarrow Q, \neg Q \stackrel{?}{\models} \neg P$$

P	Q	$P \rightarrow Q$	$\neg Q$	$\neg P$
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T	F	F	T	
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$$P \rightarrow Q, \neg Q \stackrel{?}{\models} \neg P$$

P	Q	$P \rightarrow Q$	$\neg Q$	$\neg P$
T	T	T	F	T
T	F	F	T	T
F	T	T	F	F
F	F	T	T	F

Entailment

$$P \rightarrow Q, \neg Q \stackrel{?}{\models} \neg P$$

P	Q	$P \rightarrow Q$	$\neg Q$	$\neg P$
T	T	T	F	F
T	F	F	T	F
F	T	T	F	T
F	F	T	T	T

Entailment

$$P \rightarrow Q, \neg Q \stackrel{?}{\models} \neg P$$

P	Q	$P \rightarrow Q$	$\neg Q$	$\neg P$
T	T	T	F	F
T	F	F	T	F
F	T	T	F	T
F	F	T	T	T

Entailment

$$P \rightarrow Q, \neg Q \stackrel{?}{\models} \neg P$$

P	Q	$P \rightarrow Q$	$\neg Q$	$\neg P$
T	T	T	F	F
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Entailment

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P	Q	$P \rightarrow Q$	$\neg Q$	$\neg P$
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T	F	F	T	F
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Entailment

$$P \rightarrow Q, \neg Q \stackrel{?}{\models} \neg P$$

P	Q	$P \rightarrow Q$	$\neg Q$	$\neg P$
T	T	T	F	F
T	F	F	T	F
F	T	T	F	T
F	F	T	T	T

Entailment

$$P \rightarrow Q, \neg Q \stackrel{?}{\models} \neg P$$

P	Q	$P \rightarrow Q$	$\neg Q$	$\neg P$
T	T	T	F	F
T	F	F	T	F
F	T	T	F	T
F	F	T	T	T

Entailment

$$P \rightarrow Q, \neg Q \stackrel{\checkmark}{\models} \neg P$$

P	Q	$P \rightarrow Q$	$\neg Q$	$\neg P$
T	T	T	F	F
T	F	F	T	F
F	T	T	F	T
F	F	T	T	T

Entailment

$$P \rightarrow Q, \neg Q \stackrel{\checkmark}{\models} \neg P$$

P	Q	$P \rightarrow Q$	$\neg Q$	$\neg P$
T	T	T	F	F
T	F	F	T	F
F	T	T	F	T
F	F	T	T	T

Entailment

$$P \vee Q, \neg P \stackrel{?}{\models} Q$$

Entailment

$$P \vee Q, \neg P \stackrel{?}{\models} Q$$

P	Q	$P \vee Q$	$\neg P$	Q
T	T			
T	F			
F	T			
F	F			

Entailment

$$P \vee Q, \neg P \stackrel{?}{\models} Q$$

P	Q	$P \vee Q$	$\neg P$	Q
T	T	T		
T	F	T		
F	T	F		
F	F	F		

Entailment

$$P \vee Q, \neg P \stackrel{?}{\models} Q$$

P	Q	$P \vee Q$	$\neg P$	Q
T	T	T		
T	F	T		
F	T	F		
F	F	F		

Entailment

$$P \vee Q, \neg P \stackrel{?}{\models} Q$$

P	Q	$P \vee Q$	$\neg P$	Q
T	T	T	F	T
T	F	T	F	F
F	T	T	T	T
F	F	F	T	F

Entailment

$$P \vee Q, \neg P \stackrel{?}{\models} Q$$

P	Q	$P \vee Q$	$\neg P$	Q
T	T	T		
T	F	T		
F	T	F		
F	F	F		

Entailment

$$P \vee Q, \neg P \stackrel{?}{\models} Q$$

P	Q	$P \vee Q$	$\neg P$	Q
T	T	T	F	T
T	F	T	F	F
F	T	T	T	T
F	F	F	T	F

Entailment

$$P \vee Q, \neg P \stackrel{?}{\models} Q$$

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T	T	T	F	T
T	F	T	F	F
F	T	T	T	T
F	F	F	T	F

Entailment

$$P \vee Q, \neg P \stackrel{?}{\models} Q$$

P	Q	$P \vee Q$	$\neg P$	Q
T	T	T		
T	F	T		
F	T	T		
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Entailment

$$P \vee Q, \neg P \stackrel{?}{\models} Q$$

P	Q	$P \vee Q$	$\neg P$	Q
T	T	T	F	T
T	F	T	F	F
F	T	T	T	T
F	F	F	T	F

Entailment

$$P \vee Q, \neg P \stackrel{?}{\models} Q$$

P	Q	$P \vee Q$	$\neg P$	P	Q
T	T	T	F	T	
T	F	T		T	
F	T	T		F	
F	F	F		F	

Entailment

$$P \vee Q, \neg P \stackrel{?}{\models} Q$$

P	Q	$P \vee Q$	$\neg P$	P	Q
T	T	T	F	T	
T	F	T	F	T	
F	T	T		F	
F	F	F		F	

Entailment

$$P \vee Q, \neg P \stackrel{?}{\models} Q$$

P	Q	$P \vee Q$	$\neg P$	P	Q
T	T	T	F	T	
T	F	T	F	T	
F	T	T	T	F	
F	F	F		F	

Entailment

$$P \vee Q, \neg P \stackrel{?}{\models} Q$$

P	Q	$P \vee Q$	$\neg P$	P	Q
T	T	T	F	T	
T	F	T	F	T	
F	T	T	T	F	
F	F	F	T	F	

Entailment

$$P \vee Q, \neg P \stackrel{?}{\models} Q$$

P	Q	$P \vee Q$	$\neg P$	Q
T	T	T	F	
T	F	T	F	
F	T	T	T	
F	F	F	T	

Entailment

$$P \vee Q, \neg P \stackrel{?}{\models} Q$$

P	Q	$P \vee Q$	$\neg P$	Q
T	T	T	F	T
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Entailment

$$P \vee Q, \neg P \stackrel{?}{\models} Q$$

P	Q	$P \vee Q$	$\neg P$	Q
T	T	T	F	T
T	F	T	F	F
F	T	T	T	T
F	F	F	T	F

Entailment

$$P \vee Q, \neg P \stackrel{\checkmark}{\models} Q$$

P	Q	$P \vee Q$	$\neg P$	Q
T	T	T	F	T
T	F	T	F	F
F	T	T	T	T
F	F	F	T	F

Entailment

$$P \vee Q, P \rightarrow R, Q \rightarrow R \stackrel{?}{\models} R$$

Entailment

$$P \vee Q, P \rightarrow R, Q \rightarrow R \stackrel{?}{\models} R$$

P	Q	R	$P \vee Q$	$P \rightarrow R$	$Q \rightarrow R$	R
T						
T						
T						
T						

Entailment

$$P \vee Q, P \rightarrow R, Q \rightarrow R \stackrel{?}{\models} R$$

<i>P</i>	<i>Q</i>	<i>R</i>	<i>P</i> \vee <i>Q</i>	<i>P</i> \rightarrow <i>R</i>	<i>Q</i> \rightarrow <i>R</i>	<i>R</i>
<i>T</i>						
<i>T</i>						
<i>T</i>						
<i>T</i>						
<i>F</i>						
<i>F</i>						
<i>F</i>						
<i>F</i>						

Entailment

$$P \vee Q, P \rightarrow R, Q \rightarrow R \stackrel{?}{\models} R$$

P	Q	R	$P \vee Q$	$P \rightarrow R$	$Q \rightarrow R$	R
T	T					
T	T					
T						
T						
F						
F						
F						
F						

Entailment

$$P \vee Q, P \rightarrow R, Q \rightarrow R \stackrel{?}{\models} R$$

P	Q	R	$P \vee Q$	$P \rightarrow R$	$Q \rightarrow R$	R
T	T					
T	T					
T	F					
T	F					
F						
F						
F						
F						

Entailment

$$P \vee Q, P \rightarrow R, Q \rightarrow R \stackrel{?}{\models} R$$

P	Q	R	$P \vee Q$	$P \rightarrow R$	$Q \rightarrow R$	R
T	T					
T	T					
T	F					
T	F					
F	T					
F	T					
F						
F						

Entailment

$$P \vee Q, P \rightarrow R, Q \rightarrow R \stackrel{?}{\models} R$$

P	Q	R	$P \vee Q$	$P \rightarrow R$	$Q \rightarrow R$	R
T	T					
T	T					
T	F					
T	F					
F	T					
F	T					
F	F					
F	F					

Entailment

$$P \vee Q, P \rightarrow R, Q \rightarrow R \stackrel{?}{\models} R$$

P	Q	R	$P \vee Q$	$P \rightarrow R$	$Q \rightarrow R$	R
T	T	T				
T	T					
T	F					
T	F					
F	T					
F	T					
F	F					
F	F					

Entailment

$$P \vee Q, P \rightarrow R, Q \rightarrow R \stackrel{?}{\models} R$$

P	Q	R	$P \vee Q$	$P \rightarrow R$	$Q \rightarrow R$	R
T	T	T				
T	T	F				
T	F					
T	F					
F	T					
F	T					
F	F					
F	F					

Entailment

$$P \vee Q, P \rightarrow R, Q \rightarrow R \stackrel{?}{\models} R$$

P	Q	R	$P \vee Q$	$P \rightarrow R$	$Q \rightarrow R$	R
T	T	T				
T	T	F				
T	F	T				
T	F					
F	T					
F	T					
F	F					
F	F					

Entailment

$$P \vee Q, P \rightarrow R, Q \rightarrow R \stackrel{?}{\models} R$$

P	Q	R	$P \vee Q$	$P \rightarrow R$	$Q \rightarrow R$	R
T	T	T				
T	T	F				
T	F	T				
T	F	F				
F	T					
F	T					
F	F					
F	F					

Entailment

$$P \vee Q, P \rightarrow R, Q \rightarrow R \stackrel{?}{\models} R$$

P	Q	R	$P \vee Q$	$P \rightarrow R$	$Q \rightarrow R$	R
T	T	T				
T	T	F				
T	F	T				
T	F	F				
F	T	T				
F	T					
F	F					
F	F					

Entailment

$$P \vee Q, P \rightarrow R, Q \rightarrow R \stackrel{?}{\models} R$$

P	Q	R	$P \vee Q$	$P \rightarrow R$	$Q \rightarrow R$	R
T	T	T				
T	T	F				
T	F	T				
T	F	F				
F	T	T				
F	T	F				
F	F					
F	F					

Entailment

$$P \vee Q, P \rightarrow R, Q \rightarrow R \stackrel{?}{\models} R$$

P	Q	R	$P \vee Q$	$P \rightarrow R$	$Q \rightarrow R$	R
T	T	T				
T	T	F				
T	F	T				
T	F	F				
F	T	T				
F	T	F				
F	F	T				
F	F					

Entailment

$$P \vee Q, P \rightarrow R, Q \rightarrow R \stackrel{?}{\models} R$$

P	Q	R	$P \vee Q$	$P \rightarrow R$	$Q \rightarrow R$	R
T	T	T				
T	T	F				
T	F	T				
T	F	F				
F	T	T				
F	T	F				
F	F	T				
F	F	F				

Entailment

$$P \vee Q, P \rightarrow R, Q \rightarrow R \stackrel{?}{\models} R$$

P	Q	R	$P \vee Q$	$P \rightarrow R$	$Q \rightarrow R$	R
T	T	T	T			
T	T	F	T			
T	F	T	T			
T	F	F	T			
F	T	T	F			
F	T	F	F			
F	F	T	F			
F	F	F	F			

Entailment

$$P \vee Q, P \rightarrow R, Q \rightarrow R \stackrel{?}{\models} R$$

P	Q	R	$P \vee Q$	$P \rightarrow R$	$Q \rightarrow R$	R
T	T	T	T			
T	T	F	T			
T	F	T	T			
T	F	F	T			
F	T	T	F			
F	T	F	F			
F	F	T	F			
F	F	F	F			

Entailment

$$P \vee Q, P \rightarrow R, Q \rightarrow R \stackrel{?}{\models} R$$

P	Q	R	$P \vee Q$	$P \rightarrow R$	$Q \rightarrow R$	R
T	T	T	T	T	T	
T	T	F	T	F	T	
T	F	T	T	T	F	
T	F	F	T	T	F	
F	T	T	F	T	T	
F	T	F	F	T	T	
F	F	T	F	T	F	
F	F	F	F	T	F	

Entailment

$$P \vee Q, P \rightarrow R, Q \rightarrow R \stackrel{?}{\models} R$$

P	Q	R	$P \vee Q$	$P \rightarrow R$	$Q \rightarrow R$	R
T	T	T	T	T	T	T
T	T	F	T	F	T	F
T	F	T	T	T	T	T
T	F	F	T	T	T	F
F	T	T	F	T	F	T
F	T	F	F	T	T	F
F	F	T	F	T	T	T
F	F	F	F	T	T	F

Entailment

$$P \vee Q, P \rightarrow R, Q \rightarrow R \stackrel{?}{\models} R$$

P	Q	R	$P \vee Q$	$P \rightarrow R$	$Q \rightarrow R$	R
T	T	T	T	T	T	
T	T	F	T	F	T	
T	F	T	T	T	T	
T	F	F	T	T	T	
F	T	T	F	T	T	
F	T	F	F	T	T	
F	F	T	F	T	T	
F	F	F	F	T	T	

Entailment

$$P \vee Q, P \rightarrow R, Q \rightarrow R \stackrel{?}{\models} R$$

P	Q	R	$P \vee Q$	$P \rightarrow R$	$Q \rightarrow R$	R
T	T	T	T			
T	T	F	T			
T	F	T	T			
T	F	F	T			
F	T	T	T			
F	T	F	T			
F	F	T	F			
F	F	F	F			

Entailment

$$P \vee Q, P \rightarrow R, Q \rightarrow R \stackrel{?}{\models} R$$

P	Q	R	$P \vee Q$	$P \rightarrow R$	$Q \rightarrow R$	R
T	T	T	T	T	T	
T	T	F	T	F	T	
T	F	T	T	T	T	
T	F	F	T	T	T	
F	T	T	F	T	T	
F	T	F	F	T	T	
F	F	T	F	T	T	
F	F	F	F	T	T	

Entailment

$$P \vee Q, P \rightarrow R, Q \rightarrow R \stackrel{?}{\models} R$$

P	Q	R	$P \vee Q$	$P \rightarrow R$	$Q \rightarrow R$	R
T	T	T	T	T	T	
T	T	F	T	F	T	
T	F	T	T	T	T	
T	F	F	T	T	T	
F	T	T	F	T	T	
F	T	F	F	T	T	
F	F	T	F	T	T	
F	F	F	F	T	T	

Entailment

$$P \vee Q, P \rightarrow R, Q \rightarrow R \stackrel{?}{\models} R$$

P	Q	R	$P \vee Q$	$P \rightarrow R$	$Q \rightarrow R$	R
T	T	T	T			
T	T	F	T			
T	F	T	T			
T	F	F	T			
F	T	T	F			
F	T	F	F			
F	F	T	F			
F	F	F	F			

Entailment

$$P \vee Q, P \rightarrow R, Q \rightarrow R \stackrel{?}{\models} R$$

P	Q	R	$P \vee Q$	$P \rightarrow R$	$Q \rightarrow R$	R
T	T	T	T	T	T	
T	T	F	T	F	T	
T	F	T	T	T	T	
T	F	F	T	T	T	
F	T	T	T	T	T	
F	T	F	T	T	T	
F	F	T	F	T	T	
F	F	F	F	T	T	

Entailment

$$P \vee Q, P \rightarrow R, Q \rightarrow R \stackrel{?}{\models} R$$

P	Q	R	$P \vee Q$	$P \rightarrow R$	$Q \rightarrow R$	R
T	T	T	T			
T	T	F	T			
T	F	T	T			
T	F	F	T			
F	T	T	T			
F	T	F	T			
F	F	T	F			
F	F	F	F			

Entailment

$$P \vee Q, P \rightarrow R, Q \rightarrow R \stackrel{?}{\models} R$$

P	Q	R	$P \vee Q$	$P \rightarrow R$	$Q \rightarrow R$	R
T	T	T	T	T		
T	T	F	T	T		
T	F	T	T	T		
T	F	F	T	T		
F	T	T	T	F		
F	T	F	T	F		
F	F	T	F	F		
F	F	F	F	F		

Entailment

$$P \vee Q, P \rightarrow R, Q \rightarrow R \stackrel{?}{\models} R$$

P	Q	R	$P \vee Q$	$P \rightarrow R$	$Q \rightarrow R$	R
T	T	T	T	T		
T	T	F	T	F		
T	F	T	T	T		
T	F	F	T	F		
F	T	T	T	F		
F	T	F	T	F		
F	F	T	F	F		
F	F	F	F	F		

Entailment

$$P \vee Q, P \rightarrow R, Q \rightarrow R \stackrel{?}{\models} R$$

P	Q	R	$P \vee Q$	$P \rightarrow R$	$Q \rightarrow R$	R
T	T	T	T	T	T	
T	T	F	T	F		
T	F	T	T	T	T	
T	F	F	T	T	F	
F	T	T	T	F	T	
F	T	F	T	F	F	
F	F	T	F	F	T	
F	F	F	F	F	F	

Entailment

$$P \vee Q, P \rightarrow R, Q \rightarrow R \stackrel{?}{\models} R$$

P	Q	R	$P \vee Q$	$P \rightarrow R$	$Q \rightarrow R$	R
T	T	T	T	T	T	
T	T	F	T	F	F	
T	F	T	T	T	T	
T	F	F	T	T	F	
F	T	T	T	F	T	
F	T	F	T	F	F	
F	F	T	F	F	T	
F	F	F	F	F	F	

Entailment

$$P \vee Q, P \rightarrow R, Q \rightarrow R \stackrel{?}{\models} R$$

P	Q	R	$P \vee Q$	$P \rightarrow R$	$Q \rightarrow R$	R
T	T	T	T	T	T	
T	T	F	T	F	F	
T	F	T	T	T	T	
T	F	F	T	T	F	
F	T	T	T	F	T	
F	T	F	T	F	F	
F	F	T	F	F	T	
F	F	F	F	F	F	

Entailment

$$P \vee Q, P \rightarrow R, Q \rightarrow R \stackrel{?}{\models} R$$

P	Q	R	$P \vee Q$	$P \rightarrow R$	$Q \rightarrow R$	R
T	T	T	T	T	T	
T	T	F	T	F	F	
T	F	T	T	T	T	
T	F	F	T	F	F	
F	T	T	T	F	T	
F	T	F	T	F	F	
F	F	T	F	F	T	
F	F	F	F	F	F	

Entailment

$$P \vee Q, P \rightarrow R, Q \rightarrow R \stackrel{?}{\models} R$$

P	Q	R	$P \vee Q$	$P \rightarrow R$	$Q \rightarrow R$	R
T	T	T	T	T	T	
T	T	F	T	F	F	
T	F	T	T	T	T	
T	F	F	T	F	F	
F	T	T	T	F	T	
F	T	F	T	F	F	
F	F	T	F	F	T	
F	F	F	F	F	F	

Entailment

$$P \vee Q, P \rightarrow R, Q \rightarrow R \stackrel{?}{\models} R$$

P	Q	R	$P \vee Q$	$P \rightarrow R$	$Q \rightarrow R$	R
T	T	T	T	T	T	
T	T	F	T	F	F	
T	F	T	T	T	T	
T	F	F	T	F	F	
F	T	T	T	F	T	
F	T	F	T	F	F	
F	F	T	F	F	T	
F	F	F	F	F	F	

Entailment

$$P \vee Q, P \rightarrow R, Q \rightarrow R \stackrel{?}{\models} R$$

P	Q	R	$P \vee Q$	$P \rightarrow R$	$Q \rightarrow R$	R
T	T	T	T	T	T	
T	T	F	T	F	F	
T	F	T	T	T	T	
T	F	F	T	F	F	
F	T	T	T	F	T	
F	T	F	T	F	F	
F	F	T	F	F	T	
F	F	F	F	F	F	

Entailment

$$P \vee Q, P \rightarrow R, Q \rightarrow R \stackrel{?}{\models} R$$

P	Q	R	$P \vee Q$	$P \rightarrow R$	$Q \rightarrow R$	R
T	T	T	T	T	T	
T	T	F	T	F	F	
T	F	T	T	T	T	
T	F	F	T	F	F	
F	T	T	T	F	T	
F	T	F	T	F	F	
F	F	T	F	F	T	
F	F	F	F	F	F	

Entailment

$$P \vee Q, P \rightarrow R, Q \rightarrow R \stackrel{?}{\models} R$$

<i>P</i>	<i>Q</i>	<i>R</i>	<i>P</i> \vee <i>Q</i>	<i>P</i> \rightarrow <i>R</i>	<i>Q</i> \rightarrow <i>R</i>	<i>R</i>
<i>T</i>	<i>T</i>	<i>T</i>	<i>T</i>	<i>T</i>		
<i>T</i>	<i>T</i>	<i>F</i>	<i>T</i>	<i>F</i>		
<i>T</i>	<i>F</i>	<i>T</i>	<i>T</i>	<i>T</i>		
<i>T</i>	<i>F</i>	<i>F</i>	<i>T</i>	<i>F</i>		
<i>F</i>	<i>T</i>	<i>T</i>	<i>T</i>	<i>T</i>		
<i>F</i>	<i>T</i>	<i>F</i>	<i>T</i>	<i>T</i>		
<i>F</i>	<i>F</i>	<i>T</i>	<i>F</i>	<i>T</i>		
<i>F</i>	<i>F</i>	<i>F</i>	<i>F</i>	<i>T</i>		

Entailment

$$P \vee Q, P \rightarrow R, Q \rightarrow R \stackrel{?}{\models} R$$

P	Q	R	$P \vee Q$	$P \rightarrow R$	$Q \rightarrow R$	R
T	T	T	T	T	T	
T	T	F	T	F	T	
T	F	T	T	T	F	
T	F	F	T	F	F	
F	T	T	T	T	T	
F	T	F	T	T	T	
F	F	T	F	T	F	
F	F	F	F	T	F	

Entailment

$$P \vee Q, P \rightarrow R, Q \rightarrow R \stackrel{?}{\models} R$$

P	Q	R	$P \vee Q$	$P \rightarrow R$	$Q \rightarrow R$	R
T	T	T	T	T	T	T
T	T	F	T	F	T	F
T	F	T	T	T	F	T
T	F	F	T	F	F	F
F	T	T	T	T	T	T
F	T	F	T	T	T	F
F	F	T	F	T	F	T
F	F	F	F	T	F	F

Entailment

$$P \vee Q, P \rightarrow R, Q \rightarrow R \stackrel{?}{\models} R$$

<i>P</i>	<i>Q</i>	<i>R</i>	<i>P</i> \vee <i>Q</i>	<i>P</i> \rightarrow <i>R</i>	<i>Q</i> \rightarrow <i>R</i>	<i>R</i>
<i>T</i>	<i>T</i>	<i>T</i>	<i>T</i>	<i>T</i>	<i>T</i> <i>T</i> <i>T</i>	
<i>T</i>	<i>T</i>	<i>F</i>	<i>T</i>	<i>F</i>	<i>T</i> <i>F</i>	
<i>T</i>	<i>F</i>	<i>T</i>	<i>T</i>	<i>T</i>	<i>F</i> <i>T</i>	
<i>T</i>	<i>F</i>	<i>F</i>	<i>T</i>	<i>F</i>	<i>F</i> <i>F</i>	
<i>F</i>	<i>T</i>	<i>T</i>	<i>T</i>	<i>T</i>	<i>T</i> <i>T</i>	
<i>F</i>	<i>T</i>	<i>F</i>	<i>T</i>	<i>T</i>	<i>T</i> <i>F</i>	
<i>F</i>	<i>F</i>	<i>T</i>	<i>F</i>	<i>T</i>	<i>F</i> <i>T</i>	
<i>F</i>	<i>F</i>	<i>F</i>	<i>F</i>	<i>T</i>	<i>F</i> <i>F</i>	

Entailment

$$P \vee Q, P \rightarrow R, Q \rightarrow R \stackrel{?}{\models} R$$

P	Q	R	$P \vee Q$	$P \rightarrow R$	$Q \rightarrow R$	R
T	T	T	T	T	T	T
T	T	F	T	F	F	F
T	F	T	T	T	T	T
T	F	F	T	F	T	F
F	T	T	T	T	T	T
F	T	F	T	T	T	F
F	F	T	F	T	T	T
F	F	F	F	T	T	F

Entailment

$$P \vee Q, P \rightarrow R, Q \rightarrow R \stackrel{?}{\models} R$$

<i>P</i>	<i>Q</i>	<i>R</i>	<i>P</i> \vee <i>Q</i>	<i>P</i> \rightarrow <i>R</i>	<i>Q</i> \rightarrow <i>R</i>	<i>R</i>
<i>T</i>	<i>T</i>	<i>T</i>	<i>T</i>	<i>T</i>	<i>T</i>	<i>T</i>
<i>T</i>	<i>T</i>	<i>F</i>	<i>T</i>	<i>F</i>	<i>F</i>	<i>F</i>
<i>T</i>	<i>F</i>	<i>T</i>	<i>T</i>	<i>T</i>	<i>T</i>	<i>T</i>
<i>T</i>	<i>F</i>	<i>F</i>	<i>T</i>	<i>F</i>	<i>F</i>	<i>F</i>
<i>F</i>	<i>T</i>	<i>T</i>	<i>T</i>	<i>T</i>	<i>T</i>	<i>T</i>
<i>F</i>	<i>T</i>	<i>F</i>	<i>T</i>	<i>T</i>	<i>F</i>	<i>F</i>
<i>F</i>	<i>F</i>	<i>T</i>	<i>F</i>	<i>T</i>	<i>F</i>	<i>T</i>
<i>F</i>	<i>F</i>	<i>F</i>	<i>F</i>	<i>T</i>	<i>F</i>	<i>F</i>

Entailment

$$P \vee Q, P \rightarrow R, Q \rightarrow R \stackrel{?}{\models} R$$

<i>P</i>	<i>Q</i>	<i>R</i>	<i>P</i> \vee <i>Q</i>	<i>P</i> \rightarrow <i>R</i>	<i>Q</i> \rightarrow <i>R</i>	<i>R</i>
<i>T</i>	<i>T</i>	<i>T</i>	<i>T</i>	<i>T</i>	<i>T</i>	<i>T</i>
<i>T</i>	<i>T</i>	<i>F</i>	<i>T</i>	<i>F</i>	<i>T</i>	<i>F</i>
<i>T</i>	<i>F</i>	<i>T</i>	<i>T</i>	<i>T</i>	<i>F</i>	<i>T</i>
<i>T</i>	<i>F</i>	<i>F</i>	<i>T</i>	<i>F</i>	<i>F</i>	<i>F</i>
<i>F</i>	<i>T</i>	<i>T</i>	<i>T</i>	<i>T</i>	<i>T</i>	<i>T</i>
<i>F</i>	<i>T</i>	<i>F</i>	<i>T</i>	<i>T</i>	<i>T</i>	<i>F</i>
<i>F</i>	<i>F</i>	<i>T</i>	<i>F</i>	<i>T</i>	<i>F</i>	<i>T</i>
<i>F</i>	<i>F</i>	<i>F</i>	<i>F</i>	<i>T</i>	<i>F</i>	<i>F</i>

Entailment

$$P \vee Q, P \rightarrow R, Q \rightarrow R \stackrel{?}{\models} R$$

P	Q	R	$P \vee Q$	$P \rightarrow R$	$Q \rightarrow R$	R
T	T	T	T	T	T	T
T	T	F	T	F	F	F
T	F	T	T	T	T	T
T	F	F	T	F	T	F
F	T	T	T	T	T	T
F	T	F	T	T	F	F
F	F	T	F	T	T	T
F	F	F	F	T	F	F

Entailment

$$P \vee Q, P \rightarrow R, Q \rightarrow R \stackrel{?}{\models} R$$

<i>P</i>	<i>Q</i>	<i>R</i>	<i>P</i> \vee <i>Q</i>	<i>P</i> \rightarrow <i>R</i>	<i>Q</i> \rightarrow <i>R</i>	<i>R</i>
<i>T</i>	<i>T</i>	<i>T</i>	<i>T</i>	<i>T</i>	<i>T</i> <i>T</i> <i>T</i>	
<i>T</i>	<i>T</i>	<i>F</i>	<i>T</i>	<i>F</i>	<i>T</i> <i>F</i> <i>F</i>	
<i>T</i>	<i>F</i>	<i>T</i>	<i>T</i>	<i>T</i>	<i>F</i> <i>T</i> <i>T</i>	
<i>T</i>	<i>F</i>	<i>F</i>	<i>T</i>	<i>F</i>	<i>F</i> <i>T</i> <i>F</i>	
<i>F</i>	<i>T</i>	<i>T</i>	<i>T</i>	<i>T</i>	<i>T</i> <i>T</i> <i>T</i>	
<i>F</i>	<i>T</i>	<i>F</i>	<i>T</i>	<i>T</i>	<i>T</i> <i>F</i> <i>F</i>	
<i>F</i>	<i>F</i>	<i>T</i>	<i>F</i>	<i>T</i>	<i>F</i> <i>T</i>	
<i>F</i>	<i>F</i>	<i>F</i>	<i>F</i>	<i>T</i>	<i>F</i> <i>F</i>	

Entailment

$$P \vee Q, P \rightarrow R, Q \rightarrow R \stackrel{?}{\models} R$$

P	Q	R	$P \vee Q$	$P \rightarrow R$	$Q \rightarrow R$	R
T	T	T	T	T	T	T
T	T	F	T	F	F	F
T	F	T	T	T	F	T
T	F	F	T	F	F	F
F	T	T	T	T	T	T
F	T	F	T	T	F	F
F	F	T	F	T	F	T
F	F	F	F	T	F	F

Entailment

$$P \vee Q, P \rightarrow R, Q \rightarrow R \stackrel{?}{\models} R$$

P	Q	R	$P \vee Q$	$P \rightarrow R$	$Q \rightarrow R$	R
T	T	T	T	T	T	T
T	T	F	T	F	F	F
T	F	T	T	T	T	T
T	F	F	T	F	T	F
F	T	T	T	T	T	T
F	T	F	T	T	F	F
F	F	T	F	T	T	T
F	F	F	F	T	T	F

Entailment

$$P \vee Q, P \rightarrow R, Q \rightarrow R \stackrel{?}{\models} R$$

P	Q	R	$P \vee Q$	$P \rightarrow R$	$Q \rightarrow R$	R
T	T	T	T	T	T	
T	T	F	T	F	F	
T	F	T	T	T	T	
T	F	F	T	F	T	
F	T	T	T	T	T	
F	T	F	T	T	F	
F	F	T	F	T	T	
F	F	F	F	T	T	

Entailment

$$P \vee Q, P \rightarrow R, Q \rightarrow R \stackrel{?}{\models} R$$

P	Q	R	$P \vee Q$	$P \rightarrow R$	$Q \rightarrow R$	R
T	T	T	T	T	T	
T	T	F	T	F	F	
T	F	T	T	T	T	
T	F	F	T	F	T	
F	T	T	T	T	T	
F	T	F	T	T	F	
F	F	T	F	T	T	
F	F	F	F	T	T	

Entailment

$$P \vee Q, P \rightarrow R, Q \rightarrow R \stackrel{?}{\models} R$$

P	Q	R	$P \vee Q$	$P \rightarrow R$	$Q \rightarrow R$	R
T	T	T	T	T	T	T
T	T	F	T	F	F	F
T	F	T	T	T	T	T
T	F	F	T	F	T	F
F	T	T	T	T	T	T
F	T	F	T	T	F	F
F	F	T	F	T	T	T
F	F	F	F	T	T	F

Entailment

$$P \vee Q, P \rightarrow R, Q \rightarrow R \stackrel{?}{\models} R$$

P	Q	R	$P \vee Q$	$P \rightarrow R$	$Q \rightarrow R$	R
T	T	T	T	T	T	T
T	T	F	T	F	F	F
T	F	T	T	T	T	T
T	F	F	T	F	T	F
F	T	T	T	T	T	T
F	T	F	T	T	F	F
F	F	T	F	T	T	T
F	F	F	F	T	T	F

Entailment

$$P \vee Q, P \rightarrow R, Q \rightarrow R \stackrel{\checkmark}{\models} R$$

P	Q	R	$P \vee Q$	$P \rightarrow R$	$Q \rightarrow R$	R
T	T	T	T	T	T	T
T	T	F	T	F	F	F
T	F	T	T	T	T	T
T	F	F	T	F	T	F
F	T	T	T	T	T	T
F	T	F	T	T	F	F
F	F	T	F	T	T	T
F	F	F	F	T	T	F

Entailment

$$B \rightarrow A, \neg B \stackrel{?}{\models} \neg A$$

Entailment

$$B \rightarrow A, \neg B \stackrel{?}{\models} \neg A$$

A	B	$B \rightarrow A$	$\neg B$	$\neg A$
T	T			
T	F			
F	T			
F	F			

Entailment

$$B \rightarrow A, \neg B \stackrel{?}{\models} \neg A$$

A	B	$B \rightarrow A$	$\neg B$	$\neg A$
T	T	T		
T	F	F		
F	T	T		
F	F	F		

Entailment

$$B \rightarrow A, \neg B \stackrel{?}{\models} \neg A$$

A	B	$B \rightarrow A$	$\neg B$	$\neg A$
T	T	T		
T	F	F		
F	T	T		
F	F	F		

Entailment

$$B \rightarrow A, \neg B \stackrel{?}{\models} \neg A$$

A	B	$B \rightarrow A$	$\neg B$	$\neg A$
T	T	T	F	F
T	F	F	T	F
F	T	T	F	T
F	F	F	T	T

Entailment

$$B \rightarrow A, \neg B \stackrel{?}{\models} \neg A$$

A	B	$B \rightarrow A$	$\neg B$	$\neg A$
T	T	T	F	F
T	F	F	T	F
F	T	T	F	T
F	F	T	T	T

Entailment

$$B \rightarrow A, \neg B \stackrel{?}{\models} \neg A$$

A	B	$B \rightarrow A$	$\neg B$	$\neg A$
T	T	T	F	F
T	F	F	T	F
F	T	T	F	T
F	F	T	T	T

Entailment

$$B \rightarrow A, \neg B \stackrel{?}{\models} \neg A$$

A	B	$B \rightarrow A$	$\neg B$	$\neg A$
T	T	T	F	F
T	F	F	T	F
F	T	T	F	T
F	F	F	T	T

Entailment

$$B \rightarrow A, \neg B \stackrel{?}{\models} \neg A$$

A	B	$B \rightarrow A$	$\neg B$	$\neg A$
T	T	T		
T	F	T		
F	T	F		
F	F	T		

Entailment

$$B \rightarrow A, \neg B \stackrel{?}{\models} \neg A$$

A	B	$B \rightarrow A$	$\neg B$	$\neg A$
T	T	T	F	F
T	F	T	T	F
F	T	F	F	T
F	F	T	T	T

Entailment

$$B \rightarrow A, \neg B \stackrel{?}{\models} \neg A$$

A	B	$B \rightarrow A$	$\neg B$	$\neg A$
T	T	T	F	F
T	F	T	T	F
F	T	F	F	T
F	F	T	T	T

Entailment

$$B \rightarrow A, \neg B \stackrel{?}{\models} \neg A$$

A	B	$B \rightarrow A$	$\neg B$	$\neg A$
T	T	T	F	F
T	F	T	T	F
F	T	F	F	T
F	F	T	T	T

Entailment

$$B \rightarrow A, \neg B \stackrel{?}{\models} \neg A$$

A	B	$B \rightarrow A$	$\neg B$	$\neg A$
T	T	T	F	F
T	F	T	T	F
F	T	F	F	T
F	F	T	T	T

Entailment

$$B \rightarrow A, \neg B \stackrel{?}{\models} \neg A$$

A	B	$B \rightarrow A$	$\neg B$	$\neg A$
T	T	T	F	F
T	F	T	T	F
F	T	F	F	T
F	F	T	T	T

Entailment

$$B \rightarrow A, \neg B \stackrel{?}{\models} \neg A$$

A	B	$B \rightarrow A$	$\neg B$	$\neg A$
T	T	T	F	
T	F	T	T	
F	T	F	F	
F	F	T	T	

Entailment

$$B \rightarrow A, \neg B \stackrel{?}{\models} \neg A$$

A	B	$B \rightarrow A$	$\neg B$	$\neg A$
T	T	T	F	T
T	F	T	T	T
F	T	F	F	F
F	F	T	T	F

Entailment

$$B \rightarrow A, \neg B \stackrel{?}{\models} \neg A$$

A	B	$B \rightarrow A$	$\neg B$	$\neg A$
T	T	T	F	F
T	F	T	T	F
F	T	F	F	T
F	F	T	T	T

Entailment

$$B \rightarrow A, \neg B \stackrel{?}{\models} \neg A$$

A	B	$B \rightarrow A$	$\neg B$	$\neg A$
T	T	T	F	F
T	F	T	T	F
F	T	F	F	T
F	F	T	T	T

Entailment

$$B \rightarrow A, \neg B \stackrel{?}{\models} \neg A$$

A	B	$B \rightarrow A$	$\neg B$	$\neg A$
T	T	T	F	F
T	F	T	T	F
F	T	F	F	T
F	F	T	T	T

Entailment

$$B \rightarrow A, \neg B \stackrel{?}{\models} \neg A$$

A	B	$B \rightarrow A$	$\neg B$	$\neg A$
T	T	T	F	F
T	F	T	T	F
F	T	F	F	T
F	F	T	T	T

Entailment

$$B \rightarrow A, \neg B \stackrel{?}{\models} \neg A$$

A	B	$B \rightarrow A$	$\neg B$	$\neg A$
T	T	T	F	F
T	F	T	T	F
F	T	F	F	T
F	F	T	T	T

Entailment

$$B \rightarrow A, \neg B \stackrel{?}{\models} \neg A$$

A	B	$B \rightarrow A$	$\neg B$	$\neg A$
T	T	T	F	F
T	F	T	T	F
F	T	F	F	T
F	F	T	T	T

Entailment

$$B \rightarrow A, \neg B \stackrel{\times}{\models} \neg A$$

<i>A</i>	<i>B</i>	<i>B</i> → <i>A</i>	¬ <i>B</i>	¬ <i>A</i>
<i>T</i>	<i>T</i>	<i>T</i>	<i>F</i>	<i>F</i>
<i>T</i>	<i>F</i>	<i>T</i>	<i>T</i>	<i>F</i>
<i>F</i>	<i>T</i>	<i>F</i>	<i>F</i>	<i>T</i>
<i>F</i>	<i>F</i>	<i>T</i>	<i>T</i>	<i>T</i>

Entailment and Validity

- $A :=$ It is pouring

Entailment and Validity

- ▷ $A :=$ It is pouring
- ▷ $B :=$ It is raining

Entailment and Validity

- ▷ $A :=$ It is pouring
- ▷ $B :=$ It is raining

$$B \rightarrow A$$

$$\neg B$$

$$\therefore \neg A$$

Entailment and Validity

- ▶ $A :=$ It is pouring
- ▶ $B :=$ It is raining

If it's raining, then it's pouring

It's not raining

\therefore It's not pouring

Entailment and Validity

- If $\mathcal{A}_1, \mathcal{A}_2, \dots, \mathcal{A}_N$ entail \mathcal{C} , then the argument $\mathcal{A}_1, \mathcal{A}_2, \dots, \mathcal{A}_N \therefore \mathcal{C}$ is valid.

Entailment and Validity

- If $\mathcal{A}_1, \mathcal{A}_2, \dots, \mathcal{A}_N$ entail \mathcal{C} , then the argument $\mathcal{A}_1, \mathcal{A}_2, \dots, \mathcal{A}_N \therefore \mathcal{C}$ is valid.
- ▶ *But just because $\mathcal{A}_1, \mathcal{A}_2, \dots, \mathcal{A}_N$ don't entail \mathcal{C} , this doesn't mean that $\mathcal{A}_1, \mathcal{A}_2, \dots, \mathcal{A}_N \therefore \mathcal{C}$ is invalid.*

Entailment and Validity

- If $\mathcal{A}_1, \mathcal{A}_2, \dots, \mathcal{A}_N$ entail \mathcal{C} , then the argument $\mathcal{A}_1, \mathcal{A}_2, \dots, \mathcal{A}_N \therefore \mathcal{C}$ is valid.
- ▷ *But just because $\mathcal{A}_1, \mathcal{A}_2, \dots, \mathcal{A}_N$ don't entail \mathcal{C} , this doesn't mean that $\mathcal{A}_1, \mathcal{A}_2, \dots, \mathcal{A}_N \therefore \mathcal{C}$ is invalid.*
- ▷ Let $S :=$ Sally is taller than John, and $J :=$ John is taller than Sally.

Entailment and Validity

- If $\mathcal{A}_1, \mathcal{A}_2, \dots, \mathcal{A}_N$ entail \mathcal{C} , then the argument $\mathcal{A}_1, \mathcal{A}_2, \dots, \mathcal{A}_N \therefore \mathcal{C}$ is valid.
- ▷ *But just because $\mathcal{A}_1, \mathcal{A}_2, \dots, \mathcal{A}_N$ don't entail \mathcal{C} , this doesn't mean that $\mathcal{A}_1, \mathcal{A}_2, \dots, \mathcal{A}_N \therefore \mathcal{C}$ is invalid.*
- ▷ Let $S :=$ Sally is taller than John, and $J :=$ John is taller than Sally.
- ▷ S doesn't entail $\neg J$. But $S \therefore \neg J$ is a valid argument.

Entailment and Validity

- All *possibilities* are represented in some *valuation*.

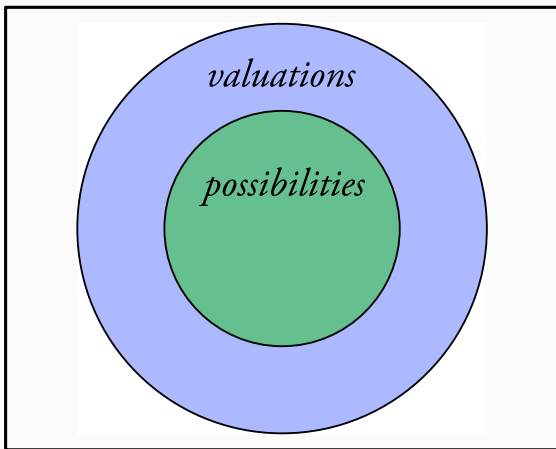
Entailment and Validity

- All *possibilities* are represented in some *valuation*.
- But not all *valuations* correspond to some *possibility*.

Entailment and Validity

- All *possibilities* are represented in some *valuation*.
- But not all *valuations* correspond to some *possibility*.
- Consider the valuation which makes *S* true and *J* true.

Entailment and Validity



Entailment and Validity

- If we know something about *every* valuation, then we know something about every possibility.

Entailment and Validity

- If we know something about *every* valuation, then we know something about every possibility.
- But, just because we know something about *some* valuation, that doesn't tell us anything about any possibility.

Entailment and Validity

- Entailment \Rightarrow Valid Argument

Entailment and Validity

- ▶ Entailment \Rightarrow Valid Argument
- ▶ Not an Entailment \nRightarrow Not a Valid Argument

Special Cases of Entailment

$$P, \neg P \models Q$$

Special Cases of Entailment

$$P, \neg P \models Q$$

P	Q	P	$\neg P$	P	Q
T	T				
T	F				
F	T				
F	F				

Special Cases of Entailment

$$P, \neg P \models Q$$

P	Q	P	$\neg P$	P	Q
T	T	T			
T	F	T			
F	T	F			
F	F	F			

Special Cases of Entailment

$$P, \neg P \models Q$$

P	Q	P	$\neg P$	P	Q
T	T	T		T	
T	F	T		T	
F	T	F		F	
F	F	F		F	

Special Cases of Entailment

$$P, \neg P \models Q$$

P	Q	P	$\neg P$	P	Q
T	T	T	F	T	
T	F	T		T	
F	T	F		F	
F	F	F		F	

Special Cases of Entailment

$$P, \neg P \models Q$$

<i>P</i>	<i>Q</i>	<i>P</i>	\neg	<i>P</i>	<i>Q</i>
<i>T</i>	<i>T</i>	<i>T</i>	<i>F</i>	<i>T</i>	
<i>T</i>	<i>F</i>	<i>T</i>	<i>F</i>	<i>T</i>	
<i>F</i>	<i>T</i>	<i>F</i>		<i>F</i>	
<i>F</i>	<i>F</i>	<i>F</i>		<i>F</i>	

Special Cases of Entailment

$$P, \neg P \models Q$$

P	Q	P	$\neg P$	P	Q
T	T	T	F	T	
T	F	T	F	T	
F	T	F	T	F	
F	F	F		F	

Special Cases of Entailment

$$P, \neg P \models Q$$

P	Q	P	$\neg P$	P	Q
T	T	T	F	T	
T	F	T	F	T	
F	T	F	T	F	
F	F	F	T	F	

Special Cases of Entailment

$$P, \neg P \models Q$$

P	Q	P	$\neg P$	P	Q
T	T	T	F		
T	F	T	F		
F	T	F	T		
F	F	F	T		

Special Cases of Entailment

$$P, \neg P \models Q$$

P	Q	P	$\neg P$	P	Q
T	T	T	F		T
T	F	T	F		F
F	T	F	T		T
F	F	F	T		F

Special Cases of Entailment

$$Q \models P \vee \neg P$$

Special Cases of Entailment

$$Q \models P \vee \neg P$$

<i>P</i>	<i>Q</i>	<i>Q</i>	<i>P</i> \vee \neg <i>P</i>
<i>T</i>	<i>T</i>		
<i>T</i>	<i>F</i>		
<i>F</i>	<i>T</i>		
<i>F</i>	<i>F</i>		

Special Cases of Entailment

$$Q \models P \vee \neg P$$

<i>P</i>	<i>Q</i>	<i>Q</i>	<i>P</i> \vee \neg <i>P</i>
<i>T</i>	<i>T</i>	<i>T</i>	
<i>T</i>	<i>F</i>	<i>F</i>	
<i>F</i>	<i>T</i>	<i>T</i>	
<i>F</i>	<i>F</i>	<i>F</i>	

Special Cases of Entailment

$$Q \models P \vee \neg P$$

<i>P</i>	<i>Q</i>	<i>Q</i>	<i>P</i> \vee \neg <i>P</i>
<i>T</i>	<i>T</i>	<i>T</i>	<i>T</i>
<i>T</i>	<i>F</i>	<i>F</i>	<i>T</i>
<i>F</i>	<i>T</i>	<i>T</i>	<i>F</i>
<i>F</i>	<i>F</i>	<i>F</i>	<i>F</i>

Special Cases of Entailment

$$Q \models P \vee \neg P$$

<i>P</i>	<i>Q</i>	<i>Q</i>	<i>P</i>	\vee	\neg	<i>P</i>
<i>T</i>	<i>T</i>	<i>T</i>			<i>F</i>	<i>T</i>
<i>T</i>	<i>F</i>	<i>F</i>				<i>T</i>
<i>F</i>	<i>T</i>	<i>T</i>				<i>F</i>
<i>F</i>	<i>F</i>	<i>F</i>				<i>F</i>

Special Cases of Entailment

$$Q \models P \vee \neg P$$

<i>P</i>	<i>Q</i>	<i>Q</i>	<i>P</i>	\vee	\neg	<i>P</i>
<i>T</i>	<i>T</i>	<i>T</i>			<i>F</i>	<i>T</i>
<i>T</i>	<i>F</i>	<i>F</i>			<i>F</i>	<i>T</i>
<i>F</i>	<i>T</i>	<i>T</i>				<i>F</i>
<i>F</i>	<i>F</i>	<i>F</i>				<i>F</i>

Special Cases of Entailment

$$Q \models P \vee \neg P$$

<i>P</i>	<i>Q</i>	<i>Q</i>	<i>P</i>	\vee	\neg	<i>P</i>
<i>T</i>	<i>T</i>	<i>T</i>			<i>F</i>	<i>T</i>
<i>T</i>	<i>F</i>	<i>F</i>			<i>F</i>	<i>T</i>
<i>F</i>	<i>T</i>	<i>T</i>			<i>T</i>	<i>F</i>
<i>F</i>	<i>F</i>	<i>F</i>				<i>F</i>

Special Cases of Entailment

$$Q \models P \vee \neg P$$

<i>P</i>	<i>Q</i>	<i>Q</i>	<i>P</i>	\vee	\neg	<i>P</i>
<i>T</i>	<i>T</i>	<i>T</i>			<i>F</i>	<i>T</i>
<i>T</i>	<i>F</i>	<i>F</i>			<i>F</i>	<i>T</i>
<i>F</i>	<i>T</i>	<i>T</i>			<i>T</i>	<i>F</i>
<i>F</i>	<i>F</i>	<i>F</i>			<i>T</i>	<i>F</i>

Special Cases of Entailment

$$Q \models P \vee \neg P$$

<i>P</i>	<i>Q</i>	<i>Q</i>	<i>P</i> \vee \neg <i>P</i>
<i>T</i>	<i>T</i>	<i>T</i>	<i>F</i>
<i>T</i>	<i>F</i>	<i>F</i>	<i>F</i>
<i>F</i>	<i>T</i>	<i>T</i>	<i>T</i>
<i>F</i>	<i>F</i>	<i>F</i>	<i>T</i>

Special Cases of Entailment

$$Q \models P \vee \neg P$$

<i>P</i>	<i>Q</i>	<i>Q</i>	<i>P</i>	\vee	\neg	<i>P</i>
<i>T</i>	<i>T</i>	<i>T</i>	<i>T</i>			<i>F</i>
<i>T</i>	<i>F</i>	<i>F</i>	<i>T</i>			<i>F</i>
<i>F</i>	<i>T</i>	<i>T</i>	<i>F</i>			<i>T</i>
<i>F</i>	<i>F</i>	<i>F</i>	<i>F</i>			<i>T</i>

Special Cases of Entailment

$$Q \models P \vee \neg P$$

<i>P</i>	<i>Q</i>	<i>Q</i>	<i>P</i> \vee \neg <i>P</i>		
<i>T</i>	<i>T</i>	<i>T</i>	<i>T</i>	<i>T</i>	<i>F</i>
<i>T</i>	<i>F</i>	<i>F</i>	<i>T</i>		<i>F</i>
<i>F</i>	<i>T</i>	<i>T</i>	<i>F</i>		<i>T</i>
<i>F</i>	<i>F</i>	<i>F</i>	<i>F</i>		<i>T</i>

Special Cases of Entailment

$$Q \models P \vee \neg P$$

<i>P</i>	<i>Q</i>	<i>Q</i>	<i>P</i> \vee \neg <i>P</i>		
<i>T</i>	<i>T</i>	<i>T</i>	<i>T</i>	<i>T</i>	<i>F</i>
<i>T</i>	<i>F</i>	<i>F</i>	<i>T</i>	<i>T</i>	<i>F</i>
<i>F</i>	<i>T</i>	<i>T</i>	<i>F</i>		<i>T</i>
<i>F</i>	<i>F</i>	<i>F</i>	<i>F</i>		<i>T</i>

Special Cases of Entailment

$$Q \models P \vee \neg P$$

<i>P</i>	<i>Q</i>	<i>Q</i>	<i>P</i> \vee \neg <i>P</i>		
<i>T</i>	<i>T</i>	<i>T</i>	<i>T</i>	<i>T</i>	<i>F</i>
<i>T</i>	<i>F</i>	<i>F</i>	<i>T</i>	<i>T</i>	<i>F</i>
<i>F</i>	<i>T</i>	<i>T</i>	<i>F</i>	<i>T</i>	<i>T</i>
<i>F</i>	<i>F</i>	<i>F</i>	<i>F</i>		<i>T</i>

Special Cases of Entailment

$$Q \models P \vee \neg P$$

<i>P</i>	<i>Q</i>	<i>Q</i>	<i>P</i> \vee \neg <i>P</i>		
<i>T</i>	<i>T</i>	<i>T</i>	<i>T</i>	<i>T</i>	<i>F</i>
<i>T</i>	<i>F</i>	<i>F</i>	<i>T</i>	<i>T</i>	<i>F</i>
<i>F</i>	<i>T</i>	<i>T</i>	<i>F</i>	<i>T</i>	<i>T</i>
<i>F</i>	<i>F</i>	<i>F</i>	<i>F</i>	<i>T</i>	<i>T</i>

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<i>P</i>	<i>Q</i>	<i>Q</i>	<i>P</i> \vee \neg <i>P</i>
<i>T</i>	<i>T</i>	<i>T</i>	<i>T</i>
<i>T</i>	<i>F</i>	<i>F</i>	<i>T</i>
<i>F</i>	<i>T</i>	<i>T</i>	<i>T</i>
<i>F</i>	<i>F</i>	<i>F</i>	<i>T</i>

Special Cases of Entailment

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P	Q	Q	$P \vee \neg P$
T	T	T	T
T	F	F	T
F	T	T	T
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Semantics for SL

Semantic Notions

Entailment

Satisfiability

Tautologies and Contradictions

Joint Possibility and Satisfiability

Joint Possibility

$\mathcal{A}_1, \mathcal{A}_2, \dots, \mathcal{A}_N$ are *jointly possible* if and only if there is some **possibility** in which $\mathcal{A}_1, \mathcal{A}_2, \dots, \mathcal{A}_N$ are all true.

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Satisfiability

$\mathcal{A}_1, \mathcal{A}_2, \dots, \mathcal{A}_N$ are *satisfiable* if and only if there is some **valuation** on which $\mathcal{A}_1, \mathcal{A}_2, \dots, \mathcal{A}_N$ are all true.

Are $P \leftrightarrow \neg Q$, $\neg P$, and $Q \vee P$ satisfiable ?

Satisfiability

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P	Q	$P \leftrightarrow \neg Q$	$\neg P$	$Q \vee P$
T	T			
T	F			
F	T			
F	F			

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T	F	F	F	T
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Joint Impossibility and Unsatisfiability

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$\mathcal{A}_1, \mathcal{A}_2, \dots, \mathcal{A}_N$ are *jointly impossible* if and only if there is no **possibility** in which $\mathcal{A}_1, \mathcal{A}_2, \dots, \mathcal{A}_N$ are all true.

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Satisfiability

$\mathcal{A}_1, \mathcal{A}_2, \dots, \mathcal{A}_N$ are *unsatisfiable* if and only if there is no **valuation** on which $\mathcal{A}_1, \mathcal{A}_2, \dots, \mathcal{A}_N$ are all true.

Unsatisfiability

Are $\neg(P \vee Q)$, $P \rightarrow Q$, and P satisfiable ?

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P	Q	$\neg (P \vee Q)$	$P \rightarrow Q$	P
T	T			
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T	T		T	T	T		
T	F		T		F		
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T	F		T	T	F		
F	T		F		T		
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T	F		T	T	F		
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T	T	F		
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T	T	F	T	T
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P	Q	$\neg (P \vee Q)$	$P \rightarrow Q$	P
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T	F	F	F	T
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T	F	F	F	T
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P	Q	$\neg (P \vee Q)$	$P \rightarrow Q$	P
T	T	F	T	T
T	F	F	F	T
F	T	F	T	F
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P	Q	$\neg (P \vee Q)$	$P \rightarrow Q$	P
T	T	F	T	T
T	F	F	F	T
F	T	F	T	F
F	F	T	T	F

Unsatisfiability

Are $\neg(P \vee Q)$, $P \rightarrow Q$, and P satisfiable \times

P	Q	$\neg (P \vee Q)$	$P \rightarrow Q$	P
T	T	F	T	T
T	F	F	F	T
F	T	F	T	F
F	F	T	T	F

Unsatisfiability and Joint Impossibility

- If a collection of sentences of SL are unsatisfiable, then they are jointly impossible.

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- ▷ Consider: $S :=$ Sally is taller than John, and $J :=$ John is taller than Sally.

Unsatisfiability and Joint Impossibility

- If a collection of sentences of SL are unsatisfiable, then they are jointly impossible.
 - ▷ *But just because a collection of sentences of SL are satisfiable, this doesn't mean that they are jointly possible.*
 - ▷ Consider: $S :=$ Sally is taller than John, and $J :=$ John is taller than Sally.
 - ▷ S and J are satisfiable, but not jointly possible.

Unsatisfiability and Joint Impossibility

- All *possibilities* are represented in some *valuation*.

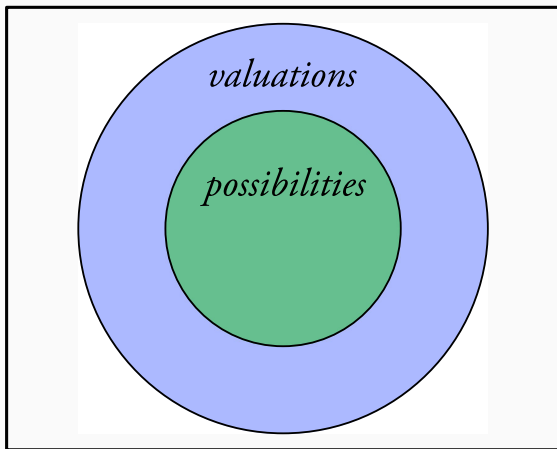
Unsatisfiability and Joint Impossibility

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Unsatisfiability and Joint Impossibility

- All *possibilities* are represented in some *valuation*.
- But not all *valuations* correspond to some *possibility*.
- ▶ Consider the valuation which makes both S true and J true.

Unsatisfiability and Joint Impossibility



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Unsatisfiability and Joint Impossibility

- If we know something about *every* valuation, then we know something about every possibility.
- But, just because we know something about *some* valuation, that doesn't tell us anything about any possibility.

Entailment and Validity

- ▷ Unsatisfiable \Rightarrow Jointly Impossible

Entailment and Validity

- ▶ Unsatisfiable \Rightarrow Jointly Impossible
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Necessary Truth and Tautology

Necessary Truth

A sentence \mathcal{A} is a *necessary truth* if and only if it is true in every possibility.

Necessary Truth and Tautology

Necessary Truth

A sentence \mathcal{A} is a *necessary truth* if and only if it is true in every possibility.

Tautology

A sentence \mathcal{A} is a *tautology* if and only if it is true on every valuation.

Necessary Falsehood and Contradiction

Necessary Falsehood

A sentence \mathcal{A} is a *necessary falsehood* if and only if it is false in every **possibility**.

Necessary Falsehood and Contradiction

Necessary Falsehood

A sentence \mathcal{A} is a *necessary falsehood* if and only if it is false in every **possibility**.

Contradiction

A sentence \mathcal{A} is a *contradiction* if and only if it is false on every **valuation**.

Tautologies and Contradictions

Is $P \rightarrow (P \rightarrow P)$ a tautology, a contradiction, or neither?

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P	$P \rightarrow (P \rightarrow P)$
T	
F	

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P	$P \rightarrow (P \rightarrow P)$
T	T
F	F

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T	T
F	F

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P	$P \rightarrow (P \rightarrow P)$
T	$T \quad T \quad T$
F	$F \quad T \quad F$

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P	$P \leftrightarrow \neg P$
T	
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- If a sentence is a tautology, then it is a necessary truth.

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- If a sentence is a tautology, then it is a necessary truth.
- ▷ *But just because a sentence is not a tautology, this doesn't mean that it isn't a necessary truth.*
- ▷ Consider: $T :=$ There's no one who is taller than themselves.
- ▷ T is not a tautology, but is a necessary truth.

Necessary Truths and Tautologies

- All *possibilities* are represented in some *valuation*.

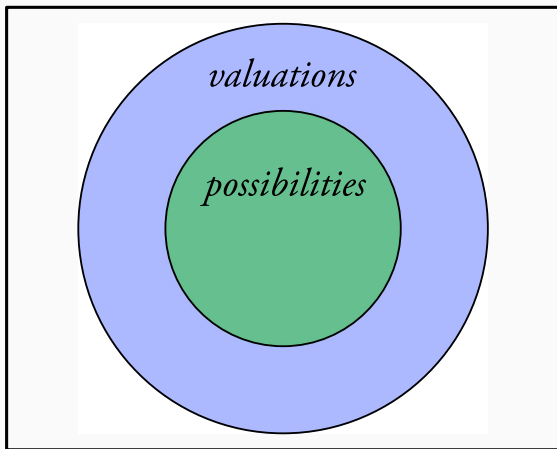
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Necessary Truths and Tautologies

- All *possibilities* are represented in some *valuation*.
- But not all *valuations* correspond to some *possibility*.
- ▶ Consider the valuation which makes T true.

Necessary Truths and Tautologies



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- If we know something about *every* valuation, then we know something about every possibility.
- But, just because we know something about *some* valuation, that doesn't tell us anything about any possibility.

Necessary Truths and Tautologies

▷ Tautology \Rightarrow Necessary Truth

Necessary Truths and Tautologies

- ▶ Tautology \Rightarrow Necessary Truth
- ▶ Not a tautology \nRightarrow Not a necessary truth

Necessary Falsehoods and Contradictions

- Contradiction \Rightarrow Necessary Falsehood

Necessary Falsehoods and Contradictions

- ▶ Contradiction \Rightarrow Necessary Falsehood
- ▶ Not a contradiction $\not\Rightarrow$ Not a necessary falsehood

Special Cases of Entailment

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 - *ex falso quodlibet*
 - *explosion*