The Validity of Arguments and the Validity of Argument Forms

PHIL 500

September 9th, 2019

Outline

Argument Forms

Validity of Argument Forms

Validity of Arguments and Validity of Argument Forms

An argument is *valid* iff it is impossible for its premises to all be true while its conclusion is false

An argument is *invalid* iff it is possible for its premises to all be true while its conclusion is false

Proving Invalidity

- To prove that an argument is *invalid*, it is enough to describe a possibility in which the premises are all true and in which the conclusion is false.

Proving Validity?

- To prove that an argument is *invalid*, it is enough to describe a possibility in which the premises are all true and in which the conclusion is false.

- How do we prove that an argument is valid?

Outline

Argument Forms

John went to the store unless it rained

It didn't rain

∴ John went to the store

Similar Arguments

Tabitha will be late unless she hurries

Tabitha won't hurry

∴ Tabitha will be late

The test is Friday unless I'm mistaken

I'm not mistaken

∴ The test is Friday

$\mathcal{A}$ unless $\mathcal{B}$

It is not the case that $\mathcal{B}$

$\therefore$ $\mathcal{A}$

If the gym is closed, then Aroosa will come to the party

Aroosa doesn't come to the party

∴ The gym isn't closed

If the solution is acidic, then the paper will turn red

The paper doesn't turn red

∴ The solution isn't acidic

Similar Arguments

If the Butler did it, then his blood is at the scene of the crime

The Butler's blood is not at the scene of the crime

∴ The Butler didn't do it

If $\mathscr{A}$, then $\mathscr{B}$

It is not the case that $\mathscr{B}$

∴ It is not the case that $\mathscr{A}$

- These arguments have the same *form*

Argument Forms

- These arguments have the same *form*
- Recognizing the *form* of these arguments is all that it takes to see that they are valid

Argument Forms

- These arguments have the same *form*
- Recognizing the *form* of these arguments is all that it takes to see that they are valid
- We can see that *any* arguments with this form will be valid

If $\mathscr{A}$, then $\mathscr{B}$

$\mathscr{A}$

$\therefore$ $\mathscr{B}$

Either $\mathcal{A}$ or $\mathcal{B}$

It is not the case that $\mathcal{A}$

$\therefore$ $\mathcal{B}$

Both $\mathscr{A}$ and $\mathscr{B}$

$\therefore$ $\mathscr{A}$

Variables

- A *variable* is just a place holder for which you can substitute some kind of thing.

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  - We could use 'x' as a variable for which you can substitute a *number*, as in an equation

$$f(x) = x^2$$

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    $$f(5) = 5^2$$

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$$f(24) = 24^2$$

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  $$f(Bob) = Bob^2$$

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  Bob loves Mary.

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    Sabeen loves Sabeen.

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    Spaghetti is tasty loves it is raining.

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  - We will use '$\mathscr{A}$' and '$\mathscr{B}$' as variables for which you can substitute *statements*, as in

$$\text{If } \mathscr{A}, \text{ then } \mathscr{B}.$$

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  If it is raining, then I am sad.

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  If Sam is late, then Gerald will leave him.

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If today is Tuesday, then today is Tuesday.

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    If Sabeen, then 5.

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- The things you get to replace a variable with are the things in the *range* of the variable

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- For the script letters like '$\mathcal{A}$', '$\mathcal{B}$', and '$\mathcal{C}$', the only things that can take their place is a *statement*

- The things you get to replace a variable with are the things over which the variable *ranges*

Statement Forms

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    It is both the case that $\mathcal{A}$ and that $\mathcal{B}$

    where $\mathcal{A}$ and $\mathcal{B}$ range over statements

- A *statement form* is a string of words containing variables such that, if you replace the variable with something in its range, then you get a statement.

  - E.g.,

    It is both the case that Eli is hungry and that Bob is sad

    where $\mathcal{A}$ and $\mathcal{B}$ range over statements

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    Sabeen loves Matthew

    where x and y range over names

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Argument Forms

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- For instance,

$$x \text{ loves } y$$
$$\therefore \quad y \text{ loves } x$$

$$\text{Either } \mathcal{A} \text{ or } \mathcal{B}$$
$$\therefore \quad \text{Both } \mathcal{A} \text{ and } \mathcal{B}$$

$$x \text{ is the brother of } y$$
$$\therefore \quad y \text{ is the brother of } x$$

Outline

Substitution Instances

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▷ E.g.,

if Eli is hungry, then Barcelona is in France

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▷ E.g.,

$$\text{if } \mathcal{A}, \text{ then } \mathcal{B}$$

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▷ E.g.,

>   All people are mortal.
>   Socrates is a person.
> ∴ Socrates is mortal.

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▹ E.g.,

All *P*s are *Q*.
S is a *P*.
∴ *S* is *Q*.

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Validity of Argument Forms

An argument form is *valid* if and only if every substitution instance of the argument form which has all true premises has a true conclusion as well.

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> Either $\mathcal{A}$ or $\mathcal{B}$
> It is not the case that $\mathcal{B}$
> $\therefore \mathcal{A}$

Validity of Argument Forms

An argument form is *valid* if and only if every substitution instance of the argument form which has all true premises has a true conclusion as well.

Either Trump is president or Obama is president   [T]

It is not the case that Obama is president   [T]

∴ Trump is president   [T]

An argument form is *valid* if and only if every substitution instance of the argument form which has all true premises has a true conclusion as well.

Either $\mathscr{A}$ or $\mathscr{B}$

It is not the case that $\mathscr{B}$

$\therefore \mathscr{A}$

Validity of Argument Forms

> An argument form is *valid* if and only if every substitution instance of the argument form which has all true premises has a true conclusion as well.

Either Pitt is in California or Pitt is in Pennsylvania   [T]

It is not the case that Pitt is in Pennsylvania   [F]

∴ Pitt is in California   [F]

Validity of Argument Forms

An argument form is *valid* if and only if every substitution instance of the argument form which has all true premises has a true conclusion as well.

Either $\mathscr{A}$ or $\mathscr{B}$
It is not the case that $\mathscr{B}$
$\therefore \mathscr{A}$

Validity of Argument Forms

An argument form is *valid* if and only if every substitution instance of the argument form which has all true premises has a true conclusion as well.

Either it is 2019 or I'm a monkey's uncle   [T]

It is not the case that I'm a monkey's uncle   [T]

∴ It is 2019   [T]

An argument form is *valid* if and only if every substitution instance of the argument form which has all true premises has a true conclusion as well.

Either $\mathscr{A}$ or $\mathscr{B}$

It is not the case that $\mathscr{B}$

$\therefore \mathscr{A}$

An argument form is *valid* if and only if every substitution instance of the argument form which has all true premises has a true conclusion as well.

Both $\mathscr{A}$ and it is not the case that $\mathscr{A}$.
∴ $\mathscr{B}$.

> An argument form is *valid* if and only if every substitution instance of the argument form which has all true premises has a true conclusion as well.

Both snow is white and it is not the case that snow is white.   [F]
∴ The Atlanta Braves won the Superbowl.   [F]

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Both $\mathscr{A}$ and it is not the case that $\mathscr{A}$.
∴ $\mathscr{B}$.

Validity of Argument Forms

An argument form is *valid* if and only if every substitution instance of the argument form which has all true premises has a true conclusion as well.

Both it is Tuesday and it is not the case that it is Tuesday.   [F]
∴ Grass is green.   [T]

An argument form is *valid* if and only if every substitution instance of the argument form which has all true premises has a true conclusion as well.

Both $\mathscr{A}$ and it is not the case that $\mathscr{A}$.
∴ $\mathscr{B}$.

Invalidity of Argument Forms

An argument form is *invalid* if and only if there is some substitution instance with all true premises and a false conclusion.

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Some Ps are Qs

Some Qs are Rs

∴ Some Ps are Rs

Invalidity of Argument Forms

An argument form is *invalid* if and only if there is some substitution instance with all true premises and a false conclusion.

Some *P*s are *Q*s

Some *Q*s are *R*s

∴ Some *P*s are *R*s

An argument form is *invalid* if and only if there is some substitution instance with all true premises and a false conclusion.

Some politicians are Democrats   [T]

Some Democrats are bald   [T]

∴ Some politicians are bald   [T]

An argument form is *invalid* if and only if there is some substitution instance with all true premises and a false conclusion.

Some *P*s are *Q*s

Some *Q*s are *R*s

∴ Some *P*s are *R*s

An argument form is *invalid* if and only if there is some substitution instance with all true premises and a false conclusion.

Some politicians are Democrats   [T]

Some Democrats are non-politicians   [T]

∴ Some politicians are non-politicians   [F]

An argument form is *invalid* if and only if there is some substitution instance with all true premises and a false conclusion.

Some *P*s are *Q*s
Some *Q*s are *R*s
∴ Some *P*s are *R*s

Proving Invalidity of Argument Forms

- To prove that an argument form is invalid, it is enough to provide a substitution instance which has all true premises and a false conclusion.

Proving Validity of Argument Forms

- To prove that an argument form is invalid, it is enough to provide a substitution instance which has all true premises and a false conclusion.

- How do we prove that an argument form is valid?

Outline

- What is the relationship between the validity of *arguments* and the validity of *argument forms*?

An argument is *valid* if and only if it is impossible for its premises to be true while its conclusion is false.

An argument form is *valid* if and only if every substitution instance of the argument form which has all true premises has a true conclusion as well.

Validity and Formal Validity

▹ A bold and provocative and completely non-obvious claim:

If an argument has a valid form, then it is a valid argument.

▷ A false and pernicious claim:

> **THIS IS FALSE**
> *If an argument has a invalid form, then it is a invalid argument.*
> **THIS IS FALSE**

I am in the capital of the U.S.A.

Washington D.C. is the capital of the U.S.A.

∴ I am in Washington D.C.

$$\mathscr{A}$$

$$\mathscr{B}$$

$$\therefore \mathscr{C}$$

▹ A false and pernicious claim:

> **THIS IS FALSE**
> *If an argument has a invalid form, then it is a invalid argument.*
> **THIS IS FALSE**

▷ A bold and provocative and completely non-obvious claim:

If an argument has a valid form, then it is a valid argument.

Proving Validity

- To prove that an argument is *invalid*, it is enough to describe a possibility in which the premises are all true and in which the conclusion is false.
- How do we prove that an argument is valid?

▷ A strategy for proving validity: to prove that an argument is valid, we can show that it has a valid *form*.

Proving Validity

- ▷ A strategy for proving validity: to prove that an argument is valid, we can show that it has a valid *form*.
- ▷ Since arguments with valid *forms* are valid, this will suffice to show that the original argument is itself valid.

Proving Validity of Argument Forms

- To prove that an argument form is invalid, it is enough to provide a substitution instance which has all true premises and a false conclusion.
- How do we prove that an argument form is valid?

Proving Validity

- Our plan: to prove that an argument is valid, we will:

Proving Validity

- Our plan: to prove that an argument is valid, we will:
- ▷ Uncover the *form* of that argument

Proving Validity

- Our plan: to prove that an argument is valid, we will:
- ▷ Uncover the *form* of that argument
- ▷ Prove that its form is valid

Proving Validity

- Our plan: to prove that an argument is valid, we will:
- ▷ Uncover the *form* of that argument
- ▷ Prove that its form is valid
- ▷ Conclude that the argument itself is valid

Proving Validity

- Our plan: to prove that an argument is valid, we will:

▷ Uncover the *form* of that argument

▷ Prove that its form is valid

▷ Conclude that the argument itself is valid