Predicate Logic

Natural Deduction

PHIL 500

Derived Rules

An Argument

Not everyone is fun. So, someone is not fun.

An Argument

Not everyone is fun. So, someone is not fun.

$$\neg \forall x Fx \therefore \exists x \neg Fx$$

$\neg \forall x\, Fx \vdash \exists x \neg Fx$

1	$\neg \forall x\, Fx$

$\neg \forall x\, Fx \;\vdash\; \exists x \neg Fx$

1	$\neg\forall x\, Fx$	
2	$\neg\exists x \neg Fx$	Ass ($\neg E$)

$\neg \forall x\, Fx \vdash \exists x \neg Fx$

1	$\neg \forall x\, Fx$	
2	$\neg \exists x \neg Fx$	Ass ($\neg E$)
3	$\neg Fa$	Ass ($\neg E$)

$\neg \forall x\, Fx \quad \vdash \quad \exists x \neg Fx$

1	$\neg \forall x\, Fx$	
2	$\neg \exists x \neg Fx$	Ass ($\neg E$)
3	$\neg Fa$	Ass ($\neg E$)
4	$\exists x \neg Fx$	$\exists I$ 3

$\neg \forall x\, Fx \vdash \exists x \neg Fx$

1	$\neg \forall x\, Fx$	
2	$\quad \neg \exists x \neg Fx$	Ass ($\neg E$)
3	$\quad\quad \neg Fa$	Ass ($\neg E$)
4	$\quad\quad \exists x \neg Fx$	$\exists I$ 3
5	$\quad\quad \bot$	$\bot I$ 2, 4

$\neg \forall x\, Fx \vdash \exists x \neg Fx$

1	$\neg\forall x\, Fx$	
2	$\quad \neg\exists x \neg Fx$	Ass ($\neg E$)
3	$\quad\quad \neg Fa$	Ass ($\neg E$)
4	$\quad\quad \exists x \neg Fx$	$\exists I$ 3
5	$\quad\quad \bot$	$\bot I$ 2, 4
6	$\quad Fa$	$\neg E$ 3–5

$\neg \forall x\, Fx \vdash \exists x \neg Fx$

1	$\neg\forall x\, Fx$	
2	$\quad \neg\exists x \neg Fx$	Ass $(\neg E)$
3	$\quad\quad \neg Fa$	Ass $(\neg E)$
4	$\quad\quad \exists x \neg Fx$	$\exists I$ 3
5	$\quad\quad \bot$	$\bot I$ 2, 4
6	$\quad Fa$	$\neg E$ 3–5
7	$\quad \forall x Fx$	$\forall I$ 6

$\neg \forall x\, Fx \vdash \exists x \neg Fx$

1	$\neg \forall x\, Fx$	
2	$\neg \exists x \neg Fx$	Ass $(\neg E)$
3	$\neg Fa$	Ass $(\neg E)$
4	$\exists x \neg Fx$	$\exists I$ 3
5	$\bot$	$\bot I$ 2, 4
6	Fa	$\neg E$ 3–5
7	$\forall x Fx$	$\forall I$ 6
8	$\bot$	$\bot I$ 1, 7

$\neg\forall x\, Fx \vdash \exists x \neg Fx$

1	$\neg\forall x\, Fx$	
2	$\quad \neg\exists x \neg Fx$	Ass ($\neg E$)
3	$\quad\quad \neg Fa$	Ass ($\neg E$)
4	$\quad\quad \exists x \neg Fx$	$\exists I$ 3
5	$\quad\quad \bot$	$\bot I$ 2, 4
6	$\quad Fa$	$\neg E$ 3–5
7	$\quad \forall x Fx$	$\forall I$ 6
8	$\quad \bot$	$\bot I$ 1, 7
9	$\exists x \neg Fx$	$\neg E$ 2–8

$\neg\forall x\, Fx \vdash \exists x \neg Fx$

1	$\neg\forall x\, Fx$	
2	$\quad \neg\exists x \neg Fx$	Ass ($\neg E$)
3	$\quad\quad \neg Fa$	Ass ($\neg E$)
4	$\quad\quad \exists x \neg Fx$	$\exists I$ 3
5	$\quad\quad \bot$	$\bot I$ 2, 4
6	$\quad Fa$	$\neg E$ 3–5
7	$\quad \forall x Fx$	$\forall I$ 6
8	$\quad \bot$	$\bot I$ 1, 7
9	$\exists x \neg Fx$	$\neg E$ 2–8

Change of Quantifiers (CQ)

1 $\neg\forall x \mathscr{A}(\ldots x \ldots x \ldots)$

Change of Quantifiers (CQ)

1	$\neg\forall x \mathscr{A}(\ldots x \ldots x \ldots)$	
2	$\quad \neg\exists x \neg\mathscr{A}(\ldots x \ldots x \ldots)$	Ass $(\neg E)$

Change of Quantifiers (CQ)

1	$\neg\forall x \mathscr{A}(\ldots x \ldots x \ldots)$	
2	$\neg\exists x \neg\mathscr{A}(\ldots x \ldots x \ldots)$	Ass $(\neg E)$
3	$\neg\mathscr{A}(\ldots n \ldots n \ldots)$	Ass $(\neg E)$

Change of Quantifiers (CQ)

1	$\neg\forall x \mathscr{A}(\ldots x \ldots x \ldots)$	
2	$\neg\exists x \neg\mathscr{A}(\ldots x \ldots x \ldots)$	Ass ($\neg E$)
3	$\neg\mathscr{A}(\ldots n \ldots n \ldots)$	Ass ($\neg E$)
4	$\exists x \neg\mathscr{A}(\ldots x \ldots x \ldots)$	$\exists I$ 3

Change of Quantifiers (CQ)

1	$\neg\forall x \mathscr{A}(\ldots x \ldots x \ldots)$	
2	$\quad \neg\exists x \neg\mathscr{A}(\ldots x \ldots x \ldots)$	Ass ($\neg E$)
3	$\quad\quad \neg\mathscr{A}(\ldots n \ldots n \ldots)$	Ass ($\neg E$)
4	$\quad\quad \exists x \neg\mathscr{A}(\ldots x \ldots x \ldots)$	$\exists I$ 3
5	$\quad\quad \bot$	$\bot I$ 2, 4

Change of Quantifiers (CQ)

1	$\neg\forall x \mathscr{A}(\ldots x \ldots x \ldots)$	
2	$\neg\exists x \neg\mathscr{A}(\ldots x \ldots x \ldots)$	Ass ($\neg E$)
3	$\neg\mathscr{A}(\ldots n \ldots n \ldots)$	Ass ($\neg E$)
4	$\exists x \neg\mathscr{A}(\ldots x \ldots x \ldots)$	$\exists I$ 3
5	$\bot$	$\bot I$ 2, 4
6	$\mathscr{A}(\ldots n \ldots n \ldots)$	$\neg E$ 3–5

Change of Quantifiers (CQ)

1	$\neg\forall x \mathscr{A}(\ldots x \ldots x \ldots)$	
2	$\quad \neg\exists x \neg\mathscr{A}(\ldots x \ldots x \ldots)$	Ass ($\neg E$)
3	$\quad\quad \neg\mathscr{A}(\ldots n \ldots n \ldots)$	Ass ($\neg E$)
4	$\quad\quad \exists x \neg\mathscr{A}(\ldots x \ldots x \ldots)$	$\exists I$ 3
5	$\quad\quad \bot$	$\bot I$ 2, 4
6	$\quad \mathscr{A}(\ldots n \ldots n \ldots)$	$\neg E$ 3–5
7	$\quad \forall x \mathscr{A}(\ldots x \ldots x \ldots)$	$\forall I$ 6

Change of Quantifiers (CQ)

1	$\neg\forall x \mathscr{A}(\ldots x \ldots x \ldots)$	
2	$\quad \neg\exists x \neg\mathscr{A}(\ldots x \ldots x \ldots)$	Ass ($\neg E$)
3	$\quad\quad \neg\mathscr{A}(\ldots n \ldots n \ldots)$	Ass ($\neg E$)
4	$\quad\quad \exists x \neg\mathscr{A}(\ldots x \ldots x \ldots)$	$\exists I$ 3
5	$\quad\quad \bot$	$\bot I$ 2, 4
6	$\quad \mathscr{A}(\ldots n \ldots n \ldots)$	$\neg E$ 3–5
7	$\quad \forall x \mathscr{A}(\ldots x \ldots x \ldots)$	$\forall I$ 6
8	$\quad \bot$	$\bot I$ 1, 7

Change of Quantifiers (CQ)

1	$\neg\forall x \mathscr{A}(\ldots x \ldots x \ldots)$	
2	$\neg\exists x \neg\mathscr{A}(\ldots x \ldots x \ldots)$	Ass $(\neg E)$
3	$\neg\mathscr{A}(\ldots n \ldots n \ldots)$	Ass $(\neg E)$
4	$\exists x \neg\mathscr{A}(\ldots x \ldots x \ldots)$	$\exists I$ 3
5	$\bot$	$\bot I$ 2, 4
6	$\mathscr{A}(\ldots n \ldots n \ldots)$	$\neg E$ 3–5
7	$\forall x \mathscr{A}(\ldots x \ldots x \ldots)$	$\forall I$ 6
8	$\bot$	$\bot I$ 1, 7
9	$\exists x \neg\mathscr{A}(\ldots x \ldots x \ldots)$	$\neg E$ 2-8

Change of Quantifiers (CQ)

1	$\neg\forall x \mathscr{A}(\ldots x \ldots x \ldots)$	
2	$\quad\vert\ \neg\exists x \neg\mathscr{A}(\ldots x \ldots x \ldots)$	Ass $(\neg E)$
3	$\quad\vert\quad\vert\ \neg\mathscr{A}(\ldots n \ldots n \ldots)$	Ass $(\neg E)$
4	$\quad\vert\quad\vert\ \exists x \neg\mathscr{A}(\ldots x \ldots x \ldots)$	$\exists I$ 3
5	$\quad\vert\quad\vert\ \bot$	$\bot I$ 2, 4
6	$\quad\vert\ \mathscr{A}(\ldots n \ldots n \ldots)$	$\neg E$ 3–5
7	$\quad\vert\ \forall x \mathscr{A}(\ldots x \ldots x \ldots)$	$\forall I$ 6
8	$\quad\vert\ \bot$	$\bot I$ 1, 7
9	$\exists x \neg\mathscr{A}(\ldots x \ldots x \ldots)$	$\neg E$ 2-8

Change of Quantifiers (CQ)

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$$\neg\forall x \mathscr{A} \quad \triangleright \quad \exists x \neg \mathscr{A}$$

Change of Quantifiers (CQ)

1 $\exists x \neg \mathscr{A}(\ldots x \ldots x \ldots)$

Change of Quantifiers (CQ)

1	$\exists x \neg \mathscr{A}(\ldots x \ldots x \ldots)$	
2	$\neg \mathscr{A}(\ldots n \ldots n \ldots)$	Ass ($\exists E$)

Change of Quantifiers (CQ)

1	$\exists x \neg \mathscr{A}(\ldots x \ldots x \ldots)$	
2	$\neg \mathscr{A}(\ldots n \ldots n \ldots)$	Ass ($\exists E$)
3	$\forall x \mathscr{A}(\ldots x \ldots x \ldots)$	Ass ($\neg I$)

Change of Quantifiers (CQ)

1	$\exists x \neg \mathscr{A}(\ldots x \ldots x \ldots)$	
2	$\quad \neg \mathscr{A}(\ldots n \ldots n \ldots)$	Ass ($\exists E$)
3	$\quad\quad \forall x \mathscr{A}(\ldots x \ldots x \ldots)$	Ass ($\neg I$)
4	$\quad\quad \mathscr{A}(\ldots n \ldots n \ldots)$	$\forall E$ 3

Change of Quantifiers (CQ)

1	$\exists x \neg \mathscr{A}(\ldots x \ldots x \ldots)$	
2	$\neg \mathscr{A}(\ldots n \ldots n \ldots)$	Ass ($\exists E$)
3	$\forall x \mathscr{A}(\ldots x \ldots x \ldots)$	Ass ($\neg I$)
4	$\mathscr{A}(\ldots n \ldots n \ldots)$	$\forall E$ 3
5	$\bot$	$\bot I$ 2, 4

Change of Quantifiers (CQ)

1	$\exists x \neg \mathscr{A}(\ldots x \ldots x \ldots)$	
2	$\neg \mathscr{A}(\ldots n \ldots n \ldots)$	Ass ($\exists E$)
3	$\forall x \mathscr{A}(\ldots x \ldots x \ldots)$	Ass ($\neg I$)
4	$\mathscr{A}(\ldots n \ldots n \ldots)$	$\forall E$ 3
5	$\bot$	$\bot I$ 2, 4
6	$\neg \forall x \mathscr{A}(\ldots x \ldots x \ldots)$	$\neg I$ 3–5

Change of Quantifiers (CQ)

1	$\exists x \neg \mathscr{A}(\ldots x \ldots x \ldots)$	
2	$\quad \neg \mathscr{A}(\ldots n \ldots n \ldots)$	Ass ($\exists E$)
3	$\quad\quad \forall x \mathscr{A}(\ldots x \ldots x \ldots)$	Ass ($\neg I$)
4	$\quad\quad \mathscr{A}(\ldots n \ldots n \ldots)$	$\forall E$ 3
5	$\quad\quad \bot$	$\bot I$ 2, 4
6	$\quad \neg \forall x \mathscr{A}(\ldots x \ldots x \ldots)$	$\neg I$ 3–5
7	$\neg \forall x \mathscr{A}(\ldots x \ldots x \ldots)$	$\exists E$ 1, 2–6

Change of Quantifiers (CQ)

1	$\exists x \neg \mathscr{A}(\ldots x \ldots x \ldots)$	
2	$\quad \neg \mathscr{A}(\ldots n \ldots n \ldots)$	Ass ($\exists E$)
3	$\quad\quad \forall x \mathscr{A}(\ldots x \ldots x \ldots)$	Ass ($\neg I$)
4	$\quad\quad \mathscr{A}(\ldots n \ldots n \ldots)$	$\forall E$ 3
5	$\quad\quad \bot$	$\bot I$ 2, 4
6	$\quad \neg \forall x \mathscr{A}(\ldots x \ldots x \ldots)$	$\neg I$ 3–5
7	$\neg \forall x \mathscr{A}(\ldots x \ldots x \ldots)$	$\exists E$ 1, 2–6

Change of Quantifiers (CQ)

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$\neg\forall x\mathscr{A}$ ◁ ▷ $\exists x\neg\mathscr{A}$

Change of Quantifiers (CQ)

1	$\neg\exists x \mathscr{A}(\ldots x \ldots x \ldots)$

Change of Quantifiers (CQ)

1	$\neg\exists x \mathscr{A}(\ldots x \ldots x \ldots)$	
2	$\mathscr{A}(\ldots n \ldots n \ldots)$	Ass ($\neg I$)

Change of Quantifiers (CQ)

1	$\neg\exists x \mathscr{A}(\ldots x \ldots x \ldots)$	
2	$\quad \mathscr{A}(\ldots n \ldots n \ldots)$	Ass $(\neg I)$
3	$\quad \exists x \mathscr{A}(\ldots x \ldots x \ldots)$	$\exists I$ 2

Change of Quantifiers (CQ)

1	$\neg\exists x \mathscr{A}(\ldots x \ldots x \ldots)$	
2	$\quad \mathscr{A}(\ldots n \ldots n \ldots)$	Ass ($\neg I$)
3	$\quad \exists x \mathscr{A}(\ldots x \ldots x \ldots)$	$\exists I$ 2
4	$\quad \bot$	$\bot I$ 1, 3

Change of Quantifiers (CQ)

1	$\neg\exists x \mathscr{A}(\ldots x \ldots x \ldots)$	
2	$\mathscr{A}(\ldots n \ldots n \ldots)$	Ass ($\neg I$)
3	$\exists x \mathscr{A}(\ldots x \ldots x \ldots)$	$\exists I$ 2
4	$\bot$	$\bot I$ 1, 3
5	$\neg\mathscr{A}(\ldots n \ldots n \ldots)$	$\neg I$ 2–4

Change of Quantifiers (CQ)

1	$\neg \exists x \mathscr{A}(\ldots x \ldots x \ldots)$	
2	$\mathscr{A}(\ldots n \ldots n \ldots)$	Ass ($\neg I$)
3	$\exists x \mathscr{A}(\ldots x \ldots x \ldots)$	$\exists I$ 2
4	$\bot$	$\bot I$ 1, 3
5	$\neg \mathscr{A}(\ldots n \ldots n \ldots)$	$\neg I$ 2–4
6	$\forall x \neg \mathscr{A}(\ldots x \ldots x \ldots)$	$\forall I$ 5

Change of Quantifiers (CQ)

Change of Quantifiers (CQ)

$\neg\forall x \mathscr{A}$	◃ ▹	$\exists x \neg \mathscr{A}$
$\neg\exists x \mathscr{A}$	▹	$\forall x \neg \mathscr{A}$

Change of Quantifiers (CQ)

1 $\quad \forall x \neg \mathscr{A}(\ldots x \ldots x \ldots)$

Change of Quantifiers (CQ)

1	$\forall x \neg \mathscr{A}(\ldots x \ldots x \ldots)$	
2	$\exists x \mathscr{A}(\ldots x \ldots x \ldots)$	Ass $(\neg I)$

Change of Quantifiers (CQ)

1	$\forall x \neg \mathscr{A}(\ldots x \ldots x \ldots)$	
2	$\quad \exists x \mathscr{A}(\ldots x \ldots x \ldots)$	Ass $(\neg I)$
3	$\quad\quad \mathscr{A}(\ldots n \ldots n \ldots)$	Ass $(\exists E)$

Change of Quantifiers (CQ)

1	$\forall x \neg \mathscr{A}(\ldots x \ldots x \ldots)$	
2	$\quad \exists x \mathscr{A}(\ldots x \ldots x \ldots)$	Ass $(\neg I)$
3	$\quad\quad \mathscr{A}(\ldots n \ldots n \ldots)$	Ass $(\exists E)$
4	$\quad\quad \neg \mathscr{A}(\ldots n \ldots n \ldots)$	$\forall E$ 1

Change of Quantifiers (CQ)

1	$\forall x \neg \mathscr{A}(\ldots x \ldots x \ldots)$	
2	$\exists x \mathscr{A}(\ldots x \ldots x \ldots)$	Ass ($\neg I$)
3	$\mathscr{A}(\ldots n \ldots n \ldots)$	Ass ($\exists E$)
4	$\neg \mathscr{A}(\ldots n \ldots n \ldots)$	$\forall E$ 1
5	$\bot$	$\bot I$ 3, 4

Change of Quantifiers (CQ)

1	$\forall x \neg \mathscr{A}(\ldots x \ldots x \ldots)$	
2	$\exists x \mathscr{A}(\ldots x \ldots x \ldots)$	Ass ($\neg I$)
3	$\mathscr{A}(\ldots n \ldots n \ldots)$	Ass ($\exists E$)
4	$\neg \mathscr{A}(\ldots n \ldots n \ldots)$	$\forall E$ 1
5	$\bot$	$\bot I$ 3, 4
6	$\bot$	$\exists E$ 2, 3–5

Change of Quantifiers (CQ)

1	$\forall x \neg \mathscr{A}(\ldots x \ldots x \ldots)$	
2	$\quad \exists x \mathscr{A}(\ldots x \ldots x \ldots)$	Ass $(\neg I)$
3	$\quad\quad \mathscr{A}(\ldots n \ldots n \ldots)$	Ass $(\exists E)$
4	$\quad\quad \neg \mathscr{A}(\ldots n \ldots n \ldots)$	$\forall E$ 1
5	$\quad\quad \bot$	$\bot I$ 3, 4
6	$\quad \bot$	$\exists E$ 2, 3–5
7	$\neg \exists x \mathscr{A}(\ldots x \ldots x \ldots)$	$\neg I$ 2–6

Change of Quantifiers (CQ)

Change of Quantifiers (CQ)

$\neg\forall x \mathscr{A}$	◁ ▷	$\exists x \neg \mathscr{A}$
$\neg\exists x \mathscr{A}$	◁ ▷	$\forall x \neg \mathscr{A}$

Strategies

Strategies from SL

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- Try to use the introduction rule for the main operator of the sentence you want to write down
- Try to use the elimination rule for the main operator of a sentence you have accessible

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- Come up with a 'big picture' strategy (form a *main* goal, and then about how to achieve it)
  - When forming your strategy, *think about meaning*
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- Try to use the elimination rule for the main operator of a sentence you have accessible
- If all else fails, try negative elimination

Strategies from SL

- Come up with a 'big picture' strategy (form a *main* goal, and then about how to achieve it)
  - When forming your strategy, *think about meaning*
- Form 'sub-goals' which will help you achieve your main goal, given your 'big-picture' strategy
- Try to use the introduction rule for the main operator of the sentence you want to write down
- Try to use the elimination rule for the main operator of a sentence you have accessible
- If all else fails, try negative elimination
- If you don't have any ideas, *just do something*

Strategies for PL

- When using $\forall E$, use your (sub-)goal as a guide for which name to instantiate.

Strategies for PL

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1	$\forall y(Py \to Qy)$	
2	Ps	Goal: Qs

Strategies for PL

- When using $\forall E$, use your (sub-)goal as a guide for which name to instantiate.

1	$\forall y(Py \rightarrow Qy)$	
2	Ps	Goal: Qs
3	$Pa \rightarrow Qa$	$\forall E$ 1

Strategies for PL

- When using $\forall E$, use your (sub-)goal as a guide for which name to instantiate.

1	$\forall y(Py \rightarrow Qy)$	
2	Ps	Goal: Qs
3	$Ps \rightarrow Qs$	$\forall E$ 1

Strategies for PL

- When using $\forall E$, use your (sub-)goal as a guide for which name to instantiate.

1	$\forall y(Py \rightarrow Qy)$	
2	Ps	Goal: Qs
3	$Ps \rightarrow Qs$	$\forall E$ 1
4	Qs	$\rightarrow E$ 2, 3

Strategies for PL

- If your goal is an existentially quantified sentence, '$\exists x \mathscr{A}(\ldots x \ldots x \ldots)$', set yourself the sub-goal of deriving '$\mathscr{A}(\ldots n \ldots n \ldots)$', for some name '$n$'.

Strategies for PL

- If your goal is an existentially quantified sentence, '$\exists x \mathscr{A}(\ldots x \ldots x \ldots)$', set yourself the sub-goal of deriving '$\mathscr{A}(\ldots n \ldots n \ldots)$', for some name '$n$'.

1	$\forall x Rxa$	Goal: $\exists y Ryy$

Strategies for PL

- If your goal is an existentially quantified sentence, '$\exists x \mathscr{A}(\ldots x \ldots x \ldots)$', set yourself the sub-goal of deriving '$\mathscr{A}(\ldots n \ldots n \ldots)$', for some name '$n$'.

1	$\forall x Rxa$	Goal: $\exists y Ryy$
2	Raa	$\forall E$ 1

Strategies for PL

- If your goal is an existentially quantified sentence, '$\exists x \mathscr{A}(\ldots x \ldots x \ldots)$', set yourself the sub-goal of deriving '$\mathscr{A}(\ldots n \ldots n \ldots)$', for some name '$n$'.

1	$\forall x Rxa$	Goal: $\exists y Ryy$
2	Raa	$\forall E$ 1
3	$\exists y Ryy$	$\exists I$ 2

Strategies for PL

- If your goal is a universally quantified sentence, '$\forall x \mathscr{A}(\ldots x \ldots x ..)$', then set yourself the sub-goal of deriving '$\mathscr{A}(\ldots n \ldots n \ldots)$', for some name n which doesn't appear in any open assumptions.

Strategies for PL

Strategies for PL

1	$\forall x(Px \rightarrow Qx)$	
2	$\forall y(Qy \rightarrow \neg Ry)$	Goal: $\forall z(Pz \rightarrow \neg Rz)$

Strategies for PL

1	$\forall x(Px \to Qx)$	
2	$\forall y(Qy \to \neg Ry)$	Goal: $\forall z(Pz \to \neg Rz)$
3	Pk	sub-goal: $\neg Rk$
4		
5		
6		
7	$\neg Rk$	
8	$Pk \to \neg Rk$	

Strategies for PL

1	$\forall x(Px \rightarrow Qx)$	
2	$\forall y(Qy \rightarrow \neg Ry)$	Goal: $\forall z(Pz \rightarrow \neg Rz)$
3	$\quad Pk$	sub-goal: $\neg Rk$
4	$\quad Pk \rightarrow Qk$	$\forall E$ 1
5		
6		
7	$\quad \neg Rk$	
8	$Pk \rightarrow \neg Rk$	

Strategies for PL

1	$\forall x(Px \rightarrow Qx)$	
2	$\forall y(Qy \rightarrow \neg Ry)$	Goal: $\forall z(Pz \rightarrow \neg Rz)$
3	Pk	sub-goal: $\neg Rk$
4	$Pk \rightarrow Qk$	$\forall E$ 1
5	Qk	$\rightarrow E$ 3, 4
6		
7	$\neg Rk$	
8	$Pk \rightarrow \neg Rk$	

Strategies for PL

1	$\forall x(Px \rightarrow Qx)$	
2	$\forall y(Qy \rightarrow \neg Ry)$	Goal: $\forall z(Pz \rightarrow \neg Rz)$
3	$\quad Pk$	sub-goal: $\neg Rk$
4	$\quad Pk \rightarrow Qk$	$\forall E$ 1
5	$\quad Qk$	$\rightarrow E$ 3, 4
6	$\quad Qk \rightarrow \neg Rk$	$\forall E$ 2
7	$\quad \neg Rk$	
8	$Pk \rightarrow \neg Rk$	

Strategies for PL

1	$\forall x(Px \rightarrow Qx)$	
2	$\forall y(Qy \rightarrow \neg Ry)$	Goal: $\forall z(Pz \rightarrow \neg Rz)$
3	$\quad Pk$	sub-goal: $\neg Rk$
4	$\quad Pk \rightarrow Qk$	$\forall E$ 1
5	$\quad Qk$	$\rightarrow E$ 3, 4
6	$\quad Qk \rightarrow \neg Rk$	$\forall E$ 2
7	$\quad \neg Rk$	$\rightarrow E$ 5, 6
8	$Pk \rightarrow \neg Rk$	

Strategies for PL

1	$\forall x(Px \rightarrow Qx)$	
2	$\forall y(Qy \rightarrow \neg Ry)$	Goal: $\forall z(Pz \rightarrow \neg Rz)$
3	$\quad Pk$	sub-goal: $\neg Rk$
4	$\quad Pk \rightarrow Qk$	$\forall E$ 1
5	$\quad Qk$	$\rightarrow E$ 3, 4
6	$\quad Qk \rightarrow \neg Rk$	$\forall E$ 2
7	$\quad \neg Rk$	$\rightarrow E$ 5, 6
8	$Pk \rightarrow \neg Rk$	$\rightarrow I$ 3–7

Strategies for PL

1	$\forall x(Px \rightarrow Qx)$	
2	$\forall y(Qy \rightarrow \neg Ry)$	Goal: $\forall z(Pz \rightarrow \neg Rz)$
3	$\quad Pk$	sub-goal: $\neg Rk$
4	$\quad Pk \rightarrow Qk$	$\forall E$ 1
5	$\quad Qk$	$\rightarrow E$ 3, 4
6	$\quad Qk \rightarrow \neg Rk$	$\forall E$ 2
7	$\quad \neg Rk$	$\rightarrow E$ 5, 6
8	$Pk \rightarrow \neg Rk$	$\rightarrow I$ 3–7
9	$\forall z(Pz \rightarrow \neg Rz)$	$\forall I$ 8

Strategies for PL

1	$\forall x(Px \rightarrow Qx)$	
2	$\forall y(Qy \rightarrow \neg Ry)$	Goal: $\forall z(Pz \rightarrow \neg Rz)$
3	$\quad Pk$	sub-goal: $\neg Rk$
4	$\quad Pk \rightarrow Qk$	$\forall E$ 1
5	$\quad Qk$	$\rightarrow E$ 3, 4
6	$\quad Qk \rightarrow \neg Rk$	$\forall E$ 2
7	$\quad \neg Rk$	$\rightarrow E$ 5, 6
8	$Pk \rightarrow \neg Rk$	$\rightarrow I$ 3–7
9	$\forall z(Pz \rightarrow \neg Rz)$	$\forall I$ 8

Strategies for PL

- If your goal is a universally quantified sentence, '$\forall x \mathscr{A}(\ldots x \ldots x ..)$', then set yourself the sub-goal of deriving '$\mathscr{A}(\ldots n \ldots n \ldots)$', for some name n which doesn't appear in any open assumptions.

Strategies for PL

1	$\forall x\, Fx$	Goal: $\forall z Fz$

Strategies for PL

1	$\forall x\, Fx$	Goal: $\forall z Fz$
2	Fa	$\forall E$ 1

Strategies for PL

1	$\forall x\, Fx$	Goal: $\forall z Fz$
2	Fa	$\forall E$ 1
3	$\forall z\, Fz$	$\forall I$ 2

Strategies for PL

- When you have an existentially quantified sentence '$\exists x \mathscr{A}(\ldots x \ldots x \ldots)$', start a sub-proof with '$\mathscr{A}(\ldots n \ldots n \ldots)$' (for some *new* name '$n$') before using the rule $\forall E$.

Strategies for PL

- When you have an existentially quantified sentence '$\exists x \mathscr{A}(\ldots x \ldots x \ldots)$', start a sub-proof with '$\mathscr{A}(\ldots n \ldots n \ldots)$' (for some *new* name '$n$') before using the rule $\forall E$.
- That is: eliminate existential quantifiers *before* you eliminate universal quantifiers.

Strategies for PL

1	$\forall x(Px \rightarrow Qx)$	
2	$\exists w\, Pw$	Goal: $\exists z\, Qz$

Strategies for PL

1	$\forall x(Px \to Qx)$	
2	$\exists w\, Pw$	Goal: $\exists z\, Qz$
3	$Pa \to Qa$	$\forall E$ 1

Strategies for PL

1	$\forall x(Px \rightarrow Qx)$	
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3	$Pa \rightarrow Qa$	$\forall E$ 1
4	Pa	Ass ($\exists E$)

Strategies for PL

1	$\forall x(Px \rightarrow Qx)$	
2	$\exists w\, Pw$	Goal: $\exists z\, Qz$
3	$Pa \rightarrow Qa$	$\forall E$ 1
4	$\quad Pa$	Ass ($\exists E$)
5	$\quad Qa$	$\rightarrow E$ 3, 4

Strategies for PL

1	$\forall x(Px \to Qx)$	
2	$\exists w\, Pw$	Goal: $\exists z\, Qz$
3	$Pa \to Qa$	$\forall E$ 1
4	Pa	Ass ($\exists E$)
5	Qa	$\to E$ 3, 4
6	$\exists z\, Qz$	$\exists I$ 5

Strategies for PL

1	$\forall x(Px \rightarrow Qx)$	
2	$\exists w\, Pw$	Goal: $\exists z\, Qz$
3	$Pa \rightarrow Qa$	$\forall E$ 1
4	$\quad Pa$	Ass ($\exists E$)
5	$\quad Qa$	$\rightarrow E$ 3, 4
6	$\quad \exists z\, Qz$	$\exists I$ 5
7	$\exists z\, Qz$	$\exists E$ 2, 3–6

Strategies for PL

1	$\forall x(Px \rightarrow Qx)$		
2	$\exists w\, Pw$	Goal: $\exists z\, Qz$	
3	$Pa \rightarrow Qa$	$\forall E$ 1	
4	Pa	Ass ($\exists E$)	
5	Qa	$\rightarrow E$ 3, 4	
6	$\exists z\, Qz$	$\exists I$ 5	
7	$\exists z\, Qz$	$\exists E$ 2, 3–6	$\longleftarrow$ **MISTAKE!**

Strategies for PL

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Strategies for PL

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Strategies for PL

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Strategies for PL

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2	$\exists w\, Pw$	Goal: $\exists z\, Qz$
3	$\quad Pa$	Ass ($\exists E$)
4	$\quad Pa \rightarrow Qa$	$\forall E$ 1
5	$\quad Qa$	$\rightarrow E$ 3, 4
6	$\quad \exists z\, Qz$	$\exists I$ 5

Strategies for PL

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4	$\quad Pa \rightarrow Qa$	$\forall E$ 1
5	$\quad Qa$	$\rightarrow E$ 3, 4
6	$\quad \exists z\, Qz$	$\exists I$ 5
7	$\exists z\, Qz$	$\exists E$ 2, 3–6

Strategies for PL

1	$\forall x(Px \to Qx)$	
2	$\exists w\, Pw$	Goal: $\exists z\, Qz$
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5	Qa	$\to E$ 3, 4
6	$\exists z\, Qz$	$\exists I$ 5
7	$\exists z\, Qz$	$\exists E$ 2, 3–6

Syllogisms

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- A *syllogism* is a two-premise argument whose premises are conclusions are all sentences of the form:

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(A) All $\mathscr{F}$s are $\mathscr{G}$s

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- A *syllogism* is a two-premise argument whose premises are conclusions are all sentences of the form:

  (A) All $\mathscr{F}$s are $\mathscr{G}$s

  (E) No $\mathscr{F}$s are $\mathscr{G}$s

Syllogisms

- A *syllogism* is a two-premise argument whose premises are conclusions are all sentences of the form:

  (A) All $\mathscr{F}$s are $\mathscr{G}$s

  (E) No $\mathscr{F}$s are $\mathscr{G}$s

  (I) Some $\mathscr{F}$s are $\mathscr{G}$s

Syllogisms

- A *syllogism* is a two-premise argument whose premises are conclusions are all sentences of the form:
  - (A) All $\mathscr{F}$s are $\mathscr{G}$s
  - (E) No $\mathscr{F}$s are $\mathscr{G}$s
  - (I) Some $\mathscr{F}$s are $\mathscr{G}$s
  - (O) Some $\mathscr{F}$s are not $\mathscr{G}$s

Syllogisms

All desserts are sweet. Nothing sweet is healthy. So no desserts are healthy.

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$$\begin{aligned} \text{domain} &: \text{all foods} \\ Dx & \ \_\_\__x \text{ is a dessert} \\ Sx & \ \_\_\__x \text{ is sweet} \\ Hx & \ \_\_\__x \text{ is healthy} \end{aligned}$$

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All desserts are sweet. Nothing sweet is healthy. So no desserts are healthy.

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$$\forall x(Dx \to Sx), \forall y(Sy \to \neg Hy) \ \vdash \ \forall w(Dw \to \neg Hw)$$

$\forall x(Dx \rightarrow Sx), \forall y(Sy \rightarrow \neg Hy) \vdash \forall w(Dw \rightarrow \neg Hw)$

1	$\forall x(Dx \rightarrow Sx)$	
2	$\forall y(Sy \rightarrow \neg Hy)$	Goal: $\forall w(Dw \rightarrow \neg Hw)$

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1	$\forall x(Dx \rightarrow Sx)$	
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1	$\forall x(Dx \rightarrow Sx)$	
2	$\forall y(Sy \rightarrow \neg Hy)$	Goal: $\forall w(Dw \rightarrow \neg Hw)$
3	$\quad Dm$	sub-goal: $\neg Hm$
4		
5		
6		
7	$\quad \neg Hm$	
8	$Dm \rightarrow \neg Hm$	

$\forall x(Dx \to Sx), \forall y(Sy \to \neg Hy) \vdash \forall w(Dw \to \neg Hw)$

1	$\forall x(Dx \to Sx)$	
2	$\forall y(Sy \to \neg Hy)$	Goal: $\forall w(Dw \to \neg Hw)$
3	Dm	sub-goal: $\neg Hm$
4	$Dm \to Sm$	$\forall E$ 1
5		
6		
7	$\neg Hm$	
8	$Dm \to \neg Hm$	

$\forall x(Dx \to Sx), \forall y(Sy \to \neg Hy) \vdash \forall w(Dw \to \neg Hw)$

1	$\forall x(Dx \to Sx)$	
2	$\forall y(Sy \to \neg Hy)$	Goal: $\forall w(Dw \to \neg Hw)$
3	$\quad Dm$	sub-goal: $\neg Hm$
4	$\quad Dm \to Sm$	$\forall E$ 1
5	$\quad Sm$	$\to E$ 3, 4
6		
7	$\quad \neg Hm$	
8	$Dm \to \neg Hm$	

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4	$\quad Dm \to Sm$	$\forall E$ 1
5	$\quad Sm$	$\to E$ 3, 4
6	$\quad Sm \to \neg Hm$	$\forall E$ 2
7	$\quad \neg Hm$	
8	$Dm \to \neg Hm$	

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2	$\forall y(Sy \rightarrow \neg Hy)$		Goal: $\forall w(Dw \rightarrow \neg Hw)$
3		Dm	sub-goal: $\neg Hm$
4		$Dm \rightarrow Sm$	$\forall E$ 1
5		Sm	$\rightarrow E$ 3, 4
6		$Sm \rightarrow \neg Hm$	$\forall E$ 2
7		$\neg Hm$	$\rightarrow E$ 5, 6
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2	$\forall y(Sy \rightarrow \neg Hy)$	Goal: $\forall w(Dw \rightarrow \neg Hw)$
3	$\quad Dm$	sub-goal: $\neg Hm$
4	$\quad Dm \rightarrow Sm$	$\forall E$ 1
5	$\quad Sm$	$\rightarrow E$ 3, 4
6	$\quad Sm \rightarrow \neg Hm$	$\forall E$ 2
7	$\quad \neg Hm$	$\rightarrow E$ 5, 6
8	$Dm \rightarrow \neg Hm$	$\rightarrow I$ 3–7

$\forall x(Dx \to Sx), \forall y(Sy \to \neg Hy) \vdash \forall w(Dw \to \neg Hw)$

1	$\forall x(Dx \to Sx)$	
2	$\forall y(Sy \to \neg Hy)$	Goal: $\forall w(Dw \to \neg Hw)$
3	$\quad Dm$	sub-goal: $\neg Hm$
4	$\quad Dm \to Sm$	$\forall E$ 1
5	$\quad Sm$	$\to E$ 3, 4
6	$\quad Sm \to \neg Hm$	$\forall E$ 2
7	$\quad \neg Hm$	$\to E$ 5, 6
8	$Dm \to \neg Hm$	$\to I$ 3–7
9	$\forall z(Dz \to \neg Hz)$	$\forall I$ 8

$\forall x(Dx \rightarrow Sx), \forall y(Sy \rightarrow \neg Hy) \vdash \forall w(Dw \rightarrow \neg Hw)$

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2	$\forall y(Sy \rightarrow \neg Hy)$	Goal: $\forall w(Dw \rightarrow \neg Hw)$
3	$\quad Dm$	sub-goal: $\neg Hm$
4	$\quad Dm \rightarrow Sm$	$\forall E$ 1
5	$\quad Sm$	$\rightarrow E$ 3, 4
6	$\quad Sm \rightarrow \neg Hm$	$\forall E$ 2
7	$\quad \neg Hm$	$\rightarrow E$ 5, 6
8	$Dm \rightarrow \neg Hm$	$\rightarrow I$ 3–7
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Syllogisms

All tasks worth pursuing are hard. Some tasks worth pursuing are achievable. So: some hard tasks are achievable.

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$$\begin{aligned} \text{domain} &: \text{tasks} \\ Hx &: \_\_\__x \text{ is hard} \\ Ax &: \_\_\__x \text{ is achievable} \\ Wx &: \_\_\__x \text{ is worth pursuing} \end{aligned}$$

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$$\forall x(Wx \rightarrow Hx),\ \exists y(Wy \wedge Ay) \ \vdash\ \exists z(Hz \wedge Az)$$

1 $\forall x(Wx \rightarrow Hx)$

2 $\exists y(Wy \wedge Ay)$ Goal: $\exists z(Hz \wedge Az)$

1 $\forall x(Wx \rightarrow Hx)$

2 $\exists y(Wy \wedge Ay)$ Goal: $\exists z(Hz \wedge Az)$

3 $Wi \wedge Ai$ Ass ($\exists E$)

1	$\forall x(Wx \rightarrow Hx)$	
2	$\exists y(Wy \wedge Ay)$	Goal: $\exists z(Hz \wedge Az)$
3	$Wi \wedge Ai$	Ass ($\exists E$)
4	$Wi \rightarrow Hi$	$\forall E$ 1

1	$\forall x(Wx \rightarrow Hx)$	
2	$\exists y(Wy \wedge Ay)$	Goal: $\exists z(Hz \wedge Az)$
3	$\quad Wi \wedge Ai$	Ass ($\exists E$)
4	$\quad Wi \rightarrow Hi$	$\forall E$ 1
5	$\quad Wi$	$\wedge E$ 3

1	$\forall x(Wx \rightarrow Hx)$	
2	$\exists y(Wy \wedge Ay)$	Goal: $\exists z(Hz \wedge Az)$
3	$Wi \wedge Ai$	Ass ($\exists E$)
4	$Wi \rightarrow Hi$	$\forall E$ 1
5	Wi	$\wedge E$ 3
6	Hi	$\rightarrow E$ 4, 5

1	$\forall x(Wx \to Hx)$	
2	$\exists y(Wy \wedge Ay)$	Goal: $\exists z(Hz \wedge Az)$
3	$Wi \wedge Ai$	Ass ($\exists E$)
4	$Wi \to Hi$	$\forall E$ 1
5	Wi	$\wedge E$ 3
6	Hi	$\to E$ 4, 5
7	Ai	$\wedge E$ 3

1	$\forall x(Wx \rightarrow Hx)$	
2	$\exists y(Wy \land Ay)$	Goal: $\exists z(Hz \land Az)$
3	$Wi \land Ai$	Ass ($\exists E$)
4	$Wi \rightarrow Hi$	$\forall E$ 1
5	Wi	$\land E$ 3
6	Hi	$\rightarrow E$ 4, 5
7	Ai	$\land E$ 3
8	$Hi \land Ai$	$\land I$ 6, 7

1	$\forall x(Wx \rightarrow Hx)$	
2	$\exists y(Wy \wedge Ay)$	Goal: $\exists z(Hz \wedge Az)$
3	$Wi \wedge Ai$	Ass ($\exists E$)
4	$Wi \rightarrow Hi$	$\forall E$ 1
5	Wi	$\wedge E$ 3
6	Hi	$\rightarrow E$ 4, 5
7	Ai	$\wedge E$ 3
8	$Hi \wedge Ai$	$\wedge I$ 6, 7
9	$\exists z(Hz \wedge Az)$	$\exists I$ 8

1	$\forall x(Wx \rightarrow Hx)$	
2	$\exists y(Wy \wedge Ay)$	Goal: $\exists z(Hz \wedge Az)$
3	$Wi \wedge Ai$	Ass ($\exists E$)
4	$Wi \rightarrow Hi$	$\forall E$ 1
5	Wi	$\wedge E$ 3
6	Hi	$\rightarrow E$ 4, 5
7	Ai	$\wedge E$ 3
8	$Hi \wedge Ai$	$\wedge I$ 6, 7
9	$\exists z(Hz \wedge Az)$	$\exists I$ 8
10	$\exists z(Hz \wedge Az)$	$\exists E$ 2, 3–9

1	$\forall x(Wx \rightarrow Hx)$	
2	$\exists y(Wy \wedge Ay)$	Goal: $\exists z(Hz \wedge Az)$
3	$\quad Wi \wedge Ai$	Ass ($\exists E$)
4	$\quad Wi \rightarrow Hi$	$\forall E$ 1
5	$\quad Wi$	$\wedge E$ 3
6	$\quad Hi$	$\rightarrow E$ 4, 5
7	$\quad Ai$	$\wedge E$ 3
8	$\quad Hi \wedge Ai$	$\wedge I$ 6, 7
9	$\quad \exists z(Hz \wedge Az)$	$\exists I$ 8
10	$\exists z(Hz \wedge Az)$	$\exists E$ 2, 3–9

Syllogisms

Some truths are commonly believed. No lie is a truth. So: some commonly believed claims are not lies.

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$$\begin{aligned} \text{domain} &: \text{all claims} \\ Tx &: \underline{\quad}_x \text{ is a truth} \\ Cx &: \underline{\quad}_x \text{ is commonly believed} \\ Lx &: \underline{\quad}_x \text{ is a lie} \end{aligned}$$

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$$\exists x(Tx \land Cx), \forall y(Ly \rightarrow \neg Ty) \vdash \exists z(Cz \land \neg Lz)$$

1 $\exists x(Tx \land Cx)$

2 $\forall y(Ly \rightarrow \neg Ty)$ Goal: $\exists z(Cz \land \neg Lz)$

1	$\exists x(Tx \land Cx)$	
2	$\forall y(Ly \rightarrow \neg Ty)$	Goal: $\exists z(Cz \land \neg Lz)$
3	$Tc \land Cc$	Ass ($\exists E$)

1	$\exists x(Tx \land Cx)$	
2	$\forall y(Ly \rightarrow \neg Ty)$	Goal: $\exists z(Cz \land \neg Lz)$
3	$Tc \land Cc$	Ass ($\exists E$)
4	$Lc \rightarrow \neg Tc$	$\forall E$ 2

1	$\exists x(Tx \land Cx)$	
2	$\forall y(Ly \rightarrow \neg Ty)$	Goal: $\exists z(Cz \land \neg Lz)$
3	$Tc \land Cc$	Ass ($\exists E$)
4	$Lc \rightarrow \neg Tc$	$\forall E$ 2
5	Lc	Ass ($\neg I$)

1	$\exists x(Tx \land Cx)$	
2	$\forall y(Ly \rightarrow \neg Ty)$	Goal: $\exists z(Cz \land \neg Lz)$
3	$Tc \land Cc$	Ass ($\exists E$)
4	$Lc \rightarrow \neg Tc$	$\forall E$ 2
5	Lc	Ass ($\neg I$)
6	Tc	$\land E$ 3

1	$\exists x(Tx \land Cx)$	
2	$\forall y(Ly \to \neg Ty)$	Goal: $\exists z(Cz \land \neg Lz)$
3	$\quad Tc \land Cc$	Ass ($\exists E$)
4	$\quad Lc \to \neg Tc$	$\forall E$ 2
5	$\quad\quad Lc$	Ass ($\neg I$)
6	$\quad\quad Tc$	$\land E$ 3
7	$\quad\quad \neg Tc$	$\to E$ 4, 5

1	$\exists x(Tx \land Cx)$	
2	$\forall y(Ly \rightarrow \neg Ty)$	Goal: $\exists z(Cz \land \neg Lz)$
3	$Tc \land Cc$	Ass ($\exists E$)
4	$Lc \rightarrow \neg Tc$	$\forall E$ 2
5	Lc	Ass ($\neg I$)
6	Tc	$\land E$ 3
7	$\neg Tc$	$\rightarrow E$ 4, 5
8	$\bot$	$\bot I$ 6, 7

1	$\exists x(Tx \wedge Cx)$	
2	$\forall y(Ly \rightarrow \neg Ty)$	Goal: $\exists z(Cz \wedge \neg Lz)$
3	$Tc \wedge Cc$	Ass ($\exists E$)
4	$Lc \rightarrow \neg Tc$	$\forall E$ 2
5	Lc	Ass ($\neg I$)
6	Tc	$\wedge E$ 3
7	$\neg Tc$	$\rightarrow E$ 4, 5
8	$\bot$	$\bot I$ 6, 7
9	$\neg Lc$	$\neg I$ 5–8

1	$\exists x(Tx \land Cx)$	
2	$\forall y(Ly \rightarrow \neg Ty)$	Goal: $\exists z(Cz \land \neg Lz)$
3	$\quad Tc \land Cc$	Ass ($\exists E$)
4	$\quad Lc \rightarrow \neg Tc$	$\forall E$ 2
	$\quad \vdots$	
9	$\quad \neg Lc$	$\neg I$ 5–8

1	$\exists x(Tx \land Cx)$	
2	$\forall y(Ly \rightarrow \neg Ty)$	Goal: $\exists z(Cz \land \neg Lz)$
3	$Tc \land Cc$	Ass ($\exists E$)
4	$Lc \rightarrow \neg Tc$	$\forall E$ 2
	$\vdots$	
9	$\neg Lc$	$\neg I$ 5–8
10	Cc	$\land E$ 3

1	$\exists x(Tx \wedge Cx)$	
2	$\forall y(Ly \rightarrow \neg Ty)$	Goal: $\exists z(Cz \wedge \neg Lz)$
3	$Tc \wedge Cc$	Ass ($\exists E$)
4	$Lc \rightarrow \neg Tc$	$\forall E$ 2
	$\vdots$	
9	$\neg Lc$	$\neg I$ 5–8
10	Cc	$\wedge E$ 3
11	$Cc \wedge \neg Lc$	$\wedge I$ 9, 10

1	$\exists x(Tx \land Cx)$	
2	$\forall y(Ly \rightarrow \neg Ty)$	Goal: $\exists z(Cz \land \neg Lz)$
3	$Tc \land Cc$	Ass ($\exists E$)
4	$Lc \rightarrow \neg Tc$	$\forall E$ 2
	$\vdots$	
9	$\neg Lc$	$\neg I$ 5–8
10	Cc	$\land E$ 3
11	$Cc \land \neg Lc$	$\land I$ 9, 10
12	$\exists z(Cz \land \neg Lz)$	$\exists I$ 11

1	$\exists x(Tx \wedge Cx)$	
2	$\forall y(Ly \rightarrow \neg Ty)$	Goal: $\exists z(Cz \wedge \neg Lz)$
3	$\quad Tc \wedge Cc$	Ass ($\exists E$)
4	$\quad Lc \rightarrow \neg Tc$	$\forall E$ 2
	$\quad \vdots$	
9	$\quad \neg Lc$	$\neg I$ 5–8
10	$\quad Cc$	$\wedge E$ 3
11	$\quad Cc \wedge \neg Lc$	$\wedge I$ 9, 10
12	$\quad \exists z(Cz \wedge \neg Lz)$	$\exists I$ 11
13	$\exists z(Cz \wedge \neg Lz)$	$\exists E$ 1, 3–12

1	$\exists x(Tx \wedge Cx)$	
2	$\forall y(Ly \rightarrow \neg Ty)$	Goal: $\exists z(Cz \wedge \neg Lz)$
3	$\quad Tc \wedge Cc$	Ass ($\exists E$)
4	$\quad Lc \rightarrow \neg Tc$	$\forall E$ 2
	$\quad \vdots$	
9	$\quad \neg Lc$	$\neg I$ 5–8
10	$\quad Cc$	$\wedge E$ 3
11	$\quad Cc \wedge \neg Lc$	$\wedge I$ 9, 10
12	$\quad \exists z(Cz \wedge \neg Lz)$	$\exists I$ 11
13	$\exists z(Cz \wedge \neg Lz)$	$\exists E$ 1, 3–12

Predicate Logic

Natural Deduction

PHIL 500

Tautologies

Tautologies

- To show that '$\mathscr{A}$' is a tautology, we may prove it from *no* assumptions,

$$\vdash \mathscr{A}$$

$\vdash \forall x\, Qx \to \neg\exists x \neg Qx$

1

$\vdash \forall x\, Qx \rightarrow \neg\exists x\, \neg Qx$

1	$\forall x\, Qx$	Ass ($\rightarrow I$) sub-goal: $\neg\exists x \neg Qx$

$\vdash\ \forall x\, Qx \rightarrow \neg\exists x\, \neg Qx$

1	$\forall x\, Qx$	Ass ($\rightarrow I$) sub-goal: $\neg\exists x \neg Qx$
2	$\exists x\, \neg Qx$	Ass ($\neg I$) sub -goal: $\bot$

$\vdash \forall x\, Qx \to \neg \exists x \neg Qx$

1	$\forall x\, Qx$	Ass ($\to I$) sub-goal: $\neg \exists x \neg Qx$
2	$\exists x \neg Qx$	Ass ($\neg I$) sub -goal: $\bot$
3	$\neg \forall x\, Qx$	CQ 2

$\vdash \forall x\, Qx \rightarrow \neg\exists x\, \neg Qx$

1	$\forall x\, Qx$	Ass ($\rightarrow I$) sub-goal: $\neg\exists x \neg Qx$
2	$\quad \exists x\, \neg Qx$	Ass ($\neg I$) sub -goal: $\bot$
3	$\quad \neg\forall x\, Qx$	CQ 2
4	$\quad \bot$	$\bot I$ 1, 3

$\vdash \forall x\, Qx \to \neg\exists x\, \neg Qx$

1	$\forall x\, Qx$	Ass ($\to I$) sub-goal: $\neg\exists x \neg Qx$
2	$\quad \exists x\, \neg Qx$	Ass ($\neg I$) sub -goal: $\bot$
3	$\quad \neg\forall x\, Qx$	CQ 2
4	$\quad \bot$	$\bot I$ 1, 3
5	$\neg\exists x\, \neg Qx$	$\neg I$ 2–4

$\vdash \forall x\, Qx \rightarrow \neg\exists x\, \neg Qx$

1	$\forall x\, Qx$	Ass ($\rightarrow I$) sub-goal: $\neg\exists x \neg Qx$
2	$\quad \exists x\, \neg Qx$	Ass ($\neg I$) sub -goal: $\bot$
3	$\quad \neg\forall x\, Qx$	CQ 2
4	$\quad \bot$	$\bot I$ 1, 3
5	$\neg\exists x\, \neg Qx$	$\neg I$ 2–4
6	$\forall x\, Qx \rightarrow \neg\exists x\, \neg Qx$	$\rightarrow I$ 1–5

$\vdash \forall x\, Qx \to \neg\exists x\, \neg Qx$

1	$\forall x\, Qx$	Ass ($\to I$) sub-goal: $\neg\exists x \neg Qx$
2	$\exists x\, \neg Qx$	Ass ($\neg I$) sub -goal: $\bot$
3	$\neg\forall x\, Qx$	CQ 2
4	$\bot$	$\bot I$ 1, 3
5	$\neg\exists x\, \neg Qx$	$\neg I$ 2–4
6	$\forall x\, Qx \to \neg\exists x\, \neg Qx$	$\to I$ 1–5

$\vdash \forall x \forall y\, Rxy \to \exists x\, Rxx$

1

$\vdash \forall x \forall y\, Rxy \to \exists x\, Rxx$

1	$\forall x \forall y\, Rxy$	Ass ($\to I$) sub-goal: $\exists x\, Rxx$

$\vdash \forall x \forall y\, Rxy \to \exists x\, Rxx$

1	$\forall x \forall y\, Rxy$	Ass ($\to I$) sub-goal: $\exists x\, Rxx$
2	$\forall y\, Ray$	$\forall E$ 1

$\vdash \forall x \forall y\, Rxy \rightarrow \exists x\, Rxx$

1	$\forall x \forall y\, Rxy$	Ass ($\rightarrow I$) sub-goal: $\exists x\, Rxx$
2	$\forall y\, Ray$	$\forall E$ 1
3	Raa	$\forall E$ 2

$\vdash \forall x \forall y\, Rxy \rightarrow \exists x\, Rxx$

1	$\forall x \forall y\, Rxy$	Ass ($\rightarrow I$) sub-goal: $\exists x\, Rxx$
2	$\forall y\, Ray$	$\forall E$ 1
3	Raa	$\forall E$ 2
4	$\exists x\, Rxx$	$\exists I$ 3

$\vdash \forall x \forall y\, Rxy \rightarrow \exists x\, Rxx$

1	$\forall x \forall y\, Rxy$	Ass ($\rightarrow I$) sub-goal: $\exists x\, Rxx$
2	$\forall y\, Ray$	$\forall E$ 1
3	Raa	$\forall E$ 2
4	$\exists x\, Rxx$	$\exists I$ 3
5	$\forall x \forall y\, Rxy \rightarrow \exists x\, Rxx$	$\rightarrow I$ 1–5

$\vdash \forall x \forall y\, Rxy \to \exists x\, Rxx$

1	$\forall x \forall y\, Rxy$	Ass ($\to I$) sub-goal: $\exists x\, Rxx$
2	$\forall y\, Ray$	$\forall E$ 1
3	Raa	$\forall E$ 2
4	$\exists x\, Rxx$	$\exists I$ 3
5	$\forall x \forall y\, Rxy \to \exists x\, Rxx$	$\to I$ 1–5

$\vdash \neg\exists x Bx \rightarrow \forall y \neg By$

1

$\vdash \neg\exists x Bx \rightarrow \forall y \neg By$

1	$\neg\exists x\, Bx$	Ass ($\rightarrow I$)

$\vdash \neg\exists x Bx \rightarrow \forall y \neg By$

1	$\neg\exists x\, Bx$	Ass ($\rightarrow I$)
2	$\forall x\, \neg Bx$	CQ 1

$\vdash \neg\exists x Bx \to \forall y \neg By$

1	$\neg\exists x\, Bx$	Ass ($\to I$)
2	$\forall x\, \neg Bx$	CQ 1
3	$\neg Bd$	$\forall E$ 2

$\vdash \neg\exists x Bx \rightarrow \forall y \neg By$

1	$\neg\exists x\, Bx$	Ass ($\rightarrow I$)
2	$\forall x\, \neg Bx$	CQ 1
3	$\neg Bd$	$\forall E$ 2
4	$\forall y\, \neg By$	$\forall I$ 3

$\vdash \neg\exists x Bx \rightarrow \forall y \neg By$

1	$\neg\exists x\, Bx$	Ass ($\rightarrow I$)
2	$\forall x \neg Bx$	CQ 1
3	$\neg Bd$	$\forall E$ 2
4	$\forall y \neg By$	$\forall I$ 3
5	$\neg\exists x\, Bx \rightarrow \forall y \neg By$	$\rightarrow I$ 1–4

$\vdash \forall x(Px \vee \neg Px)$

1

$\vdash \forall x(Px \vee \neg Px)$

1	Pa	Ass (LEM)

$\vdash \forall x(Px \vee \neg Px)$

1	Pa	Ass (LEM)
2	$Pa \vee \neg Pa$	$\vee I$ 1

$\vdash \forall x(Px \vee \neg Px)$

1	Pa	Ass (LEM)
2	$Pa \vee \neg Pa$	$\vee I$ 1
3	$\neg Pa$	Ass (LEM)

$\vdash \forall x(Px \lor \neg Px)$

1	Pa	Ass (LEM)
2	$Pa \lor \neg Pa$	$\lor I$ 1
3	$\neg Pa$	Ass (LEM)
4	$Pa \lor \neg Pa$	$\lor I$ 3

$\vdash \forall x(Px \lor \neg Px)$

1	Pa	Ass (LEM)
2	$Pa \lor \neg Pa$	$\lor I$ 1
3	$\neg Pa$	Ass (LEM)
4	$Pa \lor \neg Pa$	$\lor I$ 3
5	$Pa \lor \neg Pa$	LEM 1–2, 3–4

$\vdash \forall x(Px \vee \neg Px)$

1	$\quad Pa$	Ass (LEM)
2	$\quad Pa \vee \neg Pa$	$\vee I$ 1
3	$\quad \neg Pa$	Ass (LEM)
4	$\quad Pa \vee \neg Pa$	$\vee I$ 3
5	$Pa \vee \neg Pa$	LEM 1–2, 3–4
6	$\forall x(Px \vee \neg Px)$	$\forall I$ 5

$\vdash \forall x(Px \lor \neg Px)$

1

$\vdash \forall x(Px \vee \neg Px)$

1	$\neg(Pa \vee \neg Pa)$	Ass ($\neg E$)

$\vdash \forall x(Px \lor \neg Px)$

1	$\neg(Pa \lor \neg Pa)$	Ass $(\neg E)$
2	$\neg Pa \land \neg\neg Pa$	DeM 1

$\vdash \forall x(Px \lor \neg Px)$

1		$\neg(Pa \lor \neg Pa)$	Ass ($\neg E$)
2		$\neg Pa \land \neg\neg Pa$	DeM 1
3		$\neg Pa$	$\land E$ 2

$\vdash \forall x(Px \vee \neg Px)$

1	$\neg(Pa \vee \neg Pa)$	Ass ($\neg E$)
2	$\neg Pa \wedge \neg\neg Pa$	DeM 1
3	$\neg Pa$	$\wedge E$ 2
4	$\neg\neg Pa$	$\wedge E$ 2

$\vdash \forall x(Px \vee \neg Px)$

1	$\neg(Pa \vee \neg Pa)$	Ass ($\neg E$)
2	$\neg Pa \wedge \neg\neg Pa$	DeM 1
3	$\neg Pa$	$\wedge E$ 2
4	$\neg\neg Pa$	$\wedge E$ 2
5	$\bot$	$\bot I$ 3, 4

$\vdash \forall x(Px \vee \neg Px)$

1	$\quad \neg(Pa \vee \neg Pa)$	Ass ($\neg E$)
2	$\quad \neg Pa \wedge \neg\neg Pa$	DeM 1
3	$\quad \neg Pa$	$\wedge E$ 2
4	$\quad \neg\neg Pa$	$\wedge E$ 2
5	$\quad \bot$	$\bot I$ 3, 4
6	$Pa \vee \neg Pa$	$\neg E$

$\vdash \forall x(Px \vee \neg Px)$

1	$\neg(Pa \vee \neg Pa)$	Ass ($\neg E$)
2	$\neg Pa \wedge \neg\neg Pa$	DeM 1
3	$\neg Pa$	$\wedge E$ 2
4	$\neg\neg Pa$	$\wedge E$ 2
5	$\bot$	$\bot I$ 3, 4
6	$Pa \vee \neg Pa$	$\neg E$
7	$\forall x\,(Px \vee \neg Px)$	$\forall I$ 5

$\vdash \forall x(Px \lor \neg Px)$

1

$\vdash \forall x(Px \lor \neg Px)$

1	$\neg\forall x\,(Px \lor \neg Px)$	Ass ($\neg E$)

$\vdash \forall x(Px \vee \neg Px)$

1	$\neg\forall x\,(Px \vee \neg Px)$	Ass ($\neg E$)
2	$\exists x\,\neg(Px \vee \neg Px)$	CQ 1

$\vdash \forall x(Px \lor \neg Px)$

1	$\neg\forall x\,(Px \lor \neg Px)$	Ass ($\neg E$)
2	$\exists x\,\neg(Px \lor \neg Px)$	CQ 1
3	$\neg(Pa \lor \neg Pa)$	Ass ($\exists E$)

$\vdash \forall x(Px \lor \neg Px)$

1	$\neg\forall x\,(Px \lor \neg Px)$	Ass ($\neg E$)
2	$\exists x\,\neg(Px \lor \neg Px)$	CQ 1
3	$\quad \neg(Pa \lor \neg Pa)$	Ass ($\exists E$)
4	$\quad \neg Pa \land \neg\neg Pa$	DeM 3

$\vdash \forall x(Px \lor \neg Px)$

1	$\neg\forall x\,(Px \lor \neg Px)$	Ass ($\neg E$)
2	$\exists x\,\neg(Px \lor \neg Px)$	CQ 1
3	$\quad \neg(Pa \lor \neg Pa)$	Ass ($\exists E$)
4	$\quad \neg Pa \land \neg\neg Pa$	DeM 3
5	$\quad \neg Pa$	$\land E$ 4

$\vdash \forall x(Px \lor \neg Px)$

1	$\neg\forall x\,(Px \lor \neg Px)$	Ass ($\neg E$)
2	$\exists x\,\neg(Px \lor \neg Px)$	CQ 1
3	$\quad\neg(Pa \lor \neg Pa)$	Ass ($\exists E$)
4	$\quad\neg Pa \land \neg\neg Pa$	DeM 3
5	$\quad\neg Pa$	$\land E$ 4
6	$\quad\neg\neg Pa$	$\land E$ 4

$\vdash \forall x(Px \vee \neg Px)$

1	$\neg\forall x\,(Px \vee \neg Px)$	Ass ($\neg E$)
2	$\exists x\,\neg(Px \vee \neg Px)$	CQ 1
3	$\quad \neg(Pa \vee \neg Pa)$	Ass ($\exists E$)
4	$\quad \neg Pa \wedge \neg\neg Pa$	DeM 3
5	$\quad \neg Pa$	$\wedge E$ 4
6	$\quad \neg\neg Pa$	$\wedge E$ 4
7	$\quad \bot$	$\bot I$ 5, 6

$\vdash \forall x(Px \lor \neg Px)$

1	$\neg\forall x\,(Px \lor \neg Px)$	Ass ($\neg E$)
2	$\exists x\,\neg(Px \lor \neg Px)$	CQ 1
3	$\quad \neg(Pa \lor \neg Pa)$	Ass ($\exists E$)
4	$\quad \neg Pa \land \neg\neg Pa$	DeM 3
5	$\quad \neg Pa$	$\land E$ 4
6	$\quad \neg\neg Pa$	$\land E$ 4
7	$\quad \bot$	$\bot I$ 5, 6
8	$\bot$	$\exists E$ 2, 3–7

$\vdash \forall x(Px \lor \neg Px)$

1	$\neg\forall x\,(Px \lor \neg Px)$	Ass ($\neg E$)
2	$\exists x\,\neg(Px \lor \neg Px)$	CQ 1
3	$\neg(Pa \lor \neg Pa)$	Ass ($\exists E$)
4	$\neg Pa \land \neg\neg Pa$	DeM 3
5	$\neg Pa$	$\land E$ 4
6	$\neg\neg Pa$	$\land E$ 4
7	$\bot$	$\bot I$ 5, 6
8	$\bot$	$\exists E$ 2, 3–7
9	$\forall x\,(Px \lor \neg Px)$	$\neg E$ 1–8

$\vdash \exists x Rxx \lor \neg \exists y Ryy$

1

$\vdash \exists x Rxx \lor \neg \exists y Ryy$

1	$\exists x\, Rxx$	Ass (LEM)

$\vdash \exists x Rxx \lor \neg \exists y Ryy$

1	$\exists x\, Rxx$	Ass (LEM)
2	$\exists x\, Rxx \lor \neg \exists y\, Ryy$	$\lor I$ 1

$\vdash \exists x Rxx \lor \neg \exists y Ryy$

1	$\exists x\, Rxx$	Ass (LEM)
2	$\exists x\, Rxx \lor \neg \exists y\, Ryy$	$\lor I$ 1
3	$\neg \exists x\, Rxx$	Ass (LEM)

$\vdash \exists x Rxx \vee \neg \exists y Ryy$

1	$\exists x\, Rxx$	Ass (LEM)
2	$\exists x\, Rxx \vee \neg \exists y\, Ryy$	$\vee I$ 1
3	$\neg \exists x\, Rxx$	Ass (LEM)
4	$\forall x\, \neg Rxx$	CQ 3

$\vdash \exists x Rxx \lor \neg \exists y Ryy$

1	$\exists x\, Rxx$	Ass (LEM)
2	$\exists x\, Rxx \lor \neg\exists y\, Ryy$	$\lor I$ 1
3	$\neg\exists x\, Rxx$	Ass (LEM)
4	$\forall x\, \neg Rxx$	CQ 3
5	$\neg Raa$	$\forall E$ 4

$\vdash \exists x Rxx \vee \neg \exists y Ryy$

1	$\exists x\, Rxx$	Ass (LEM)
2	$\exists x\, Rxx \vee \neg \exists y\, Ryy$	$\vee I$ 1
3	$\neg \exists x\, Rxx$	Ass (LEM)
4	$\forall x\, \neg Rxx$	CQ 3
5	$\neg Raa$	$\forall E$ 4
6	$\forall y\, \neg Ryy$	$\forall I$ 5

$\vdash \exists x Rxx \lor \neg \exists y Ryy$

1	$\exists x\, Rxx$	Ass (LEM)
2	$\exists x\, Rxx \lor \neg \exists y\, Ryy$	$\lor I$ 1
3	$\neg \exists x\, Rxx$	Ass (LEM)
4	$\forall x\, \neg Rxx$	CQ 3
5	$\neg Raa$	$\forall E$ 4
6	$\forall y\, \neg Ryy$	$\forall I$ 5
7	$\neg \exists y\, Ryy$	CQ 6

$\vdash \exists x Rxx \lor \neg\exists y Ryy$

1	$\exists x\, Rxx$	Ass (LEM)
2	$\exists x\, Rxx \lor \neg\exists y\, Ryy$	$\lor I$ 1
3	$\neg\exists x\, Rxx$	Ass (LEM)
4	$\forall x\, \neg Rxx$	CQ 3
5	$\neg Raa$	$\forall E$ 4
6	$\forall y\, \neg Ryy$	$\forall I$ 5
7	$\neg\exists y\, Ryy$	CQ 6
8	$\exists x\, Rxx \lor \neg\exists y\, Ryy$	$\lor I$ 7

$\vdash \exists x Rxx \vee \neg \exists y Ryy$

1	$\exists x\, Rxx$	Ass (LEM)
2	$\exists x\, Rxx \vee \neg \exists y\, Ryy$	$\vee I$ 1
3	$\neg \exists x\, Rxx$	Ass (LEM)
4	$\forall x\, \neg Rxx$	CQ 3
5	$\neg Raa$	$\forall E$ 4
6	$\forall y\, \neg Ryy$	$\forall I$ 5
7	$\neg \exists y\, Ryy$	CQ 6
8	$\exists x\, Rxx \vee \neg \exists y\, Ryy$	$\vee I$ 7
9	$\exists x\, Rxx \vee \neg \exists y\, Ryy$	LEM 1–2, 3–8

$\vdash \exists x(Fx \vee \neg\exists y\, Fy)$

1

$\vdash\ \exists x(Fx \lor \neg\exists y\, Fy)$

1	$\neg\exists x\,(Fx \lor \neg\exists y\, Fy)$	Ass ($\neg E$)

$\vdash \exists x(Fx \lor \neg\exists y\, Fy)$

1	$\neg\exists x\,(Fx \lor \neg\exists y\, Fy)$	Ass ($\neg E$)
2	$\forall x\,\neg(Fx \lor \neg\exists y\, Fy)$	CQ 1

$\vdash \exists x(Fx \lor \neg\exists y\, Fy)$

1	$\neg\exists x\,(Fx \lor \neg\exists y\, Fy)$	Ass ($\neg E$)
2	$\forall x\,\neg(Fx \lor \neg\exists y\, Fy)$	CQ 1
3	$\neg(Fa \lor \forall y \neg Fy)$	$\forall E$ 2

$\vdash \exists x(Fx \lor \neg\exists y\, Fy)$

1	$\neg\exists x\,(Fx \lor \neg\exists y\, Fy)$	Ass ($\neg E$)
2	$\forall x\,\neg(Fx \lor \neg\exists y\, Fy)$	CQ 1
3	$\neg(Fa \lor \forall y \neg Fy)$	$\forall E$ 2
4	$\neg Fa \land \neg\forall y \neg Fy$	DeM 3

$\vdash \exists x(Fx \vee \neg\exists y\, Fy)$

1	$\neg\exists x\,(Fx \vee \neg\exists y\, Fy)$	Ass ($\neg E$)
2	$\forall x\,\neg(Fx \vee \neg\exists y\, Fy)$	CQ 1
3	$\neg(Fa \vee \forall y \neg Fy)$	$\forall E$ 2
4	$\neg Fa \wedge \neg\forall y \neg Fy$	DeM 3
5	$\neg Fa$	$\wedge E$ 4

$\vdash \exists x(Fx \vee \neg\exists y\, Fy)$

1	$\neg\exists x\,(Fx \vee \neg\exists y\, Fy)$	Ass ($\neg E$)
2	$\forall x\,\neg(Fx \vee \neg\exists y\, Fy)$	CQ 1
3	$\neg(Fa \vee \forall y \neg Fy)$	$\forall E$ 2
4	$\neg Fa \wedge \neg\forall y \neg Fy$	DeM 3
5	$\neg Fa$	$\wedge E$ 4
6	$\forall y \neg Fy$	$\forall I$ 5

$\vdash \exists x(Fx \lor \neg\exists y\, Fy)$

1	$\neg\exists x\,(Fx \lor \neg\exists y\, Fy)$	Ass ($\neg E$)
2	$\forall x\,\neg(Fx \lor \neg\exists y\, Fy)$	CQ 1
3	$\neg(Fa \lor \forall y \neg Fy)$	$\forall E$ 2
4	$\neg Fa \land \neg\forall y \neg Fy$	DeM 3
5	$\neg Fa$	$\land E$ 4
6	$\forall y \neg Fy$	$\forall I$ 5
7	$\neg\forall y \neg Fy$	$\land E$ 4

$\vdash\ \exists x(Fx \vee \neg\exists y\, Fy)$

1	$\neg\exists x\,(Fx \vee \neg\exists y\, Fy)$	Ass ($\neg E$)
2	$\forall x\,\neg(Fx \vee \neg\exists y\, Fy)$	CQ 1
3	$\neg(Fa \vee \forall y \neg Fy)$	$\forall E$ 2
4	$\neg Fa \wedge \neg\forall y \neg Fy$	DeM 3
5	$\neg Fa$	$\wedge E$ 4
6	$\forall y \neg Fy$	$\forall I$ 5
7	$\neg\forall y \neg Fy$	$\wedge E$ 4
8	$\bot$	$\bot I$ 6, 7

$\vdash \exists x(Fx \lor \neg\exists y\, Fy)$

1	$\neg\exists x\,(Fx \lor \neg\exists y\, Fy)$	Ass ($\neg E$)
2	$\forall x\,\neg(Fx \lor \neg\exists y\, Fy)$	CQ 1
3	$\neg(Fa \lor \forall y \neg Fy)$	$\forall E$ 2
4	$\neg Fa \land \neg\forall y \neg Fy$	DeM 3
5	$\neg Fa$	$\land E$ 4
6	$\forall y \neg Fy$	$\forall I$ 5
7	$\neg\forall y \neg Fy$	$\land E$ 4
8	$\bot$	$\bot I$ 6, 7
9	$\exists x\,(Fx \lor \neg\exists y Fy)$	$\neg E$ 1–8

Evaluating Arguments

Evaluating Arguments

Nobody funny is a bore. All of my friends are bores. So: some friends of mine aren't funny.

Evaluating Arguments

Nobody funny is a bore. All of my friends are bores. So: some friends of mine aren't funny.

$$\begin{aligned} \text{domain} &: \text{all people} \\ Fx &: \underline{\quad}_x \text{ is funny} \\ Bx &: \underline{\quad}_x \text{ is a bore} \\ Mx &: \underline{\quad}_x \text{ is a friend of mine} \end{aligned}$$

Evaluating Arguments

Nobody funny is a bore. All of my friends are bores. So: some friends of mine aren't funny.

$$\begin{aligned} \text{domain} &: \text{all people} \\ Fx &: \underline{\quad}_x \text{ is funny} \\ Bx &: \underline{\quad}_x \text{ is a bore} \\ Mx &: \underline{\quad}_x \text{ is a friend of mine} \end{aligned}$$

$$\forall x\,(Fx \rightarrow \neg Bx),\ \forall y\,(My \rightarrow By)\ \therefore\ \exists z(Mz \wedge \neg Fz)$$

Evaluating Arguments

$$\forall x\,(Fx \rightarrow \neg Bx),\ \forall y\,(My \rightarrow By)\ \therefore\ \exists z(Mz \wedge \neg Fz)$$

$$\begin{aligned} \text{domain} &: 1 \\ F &: \\ B &: \\ M &: \end{aligned}$$

Evaluating Arguments

All Democrats hate freedom. No one who hates freedom is a conservative. So no Democrats are conservatives.

Evaluating Arguments

All Democrats hate freedom. No one who hates freedom is a conservative. So no Democrats are conservatives.

$$\begin{aligned} \text{domain} &: \text{all people} \\ Dx &: \underline{\quad}_x \ \text{is a Democrat} \\ Hx &: \underline{\quad}_x \ \text{hates freedom} \\ Cx &: \underline{\quad}_x \ \text{is a conservative} \end{aligned}$$

Evaluating Arguments

All Democrats hate freedom. No one who hates freedom is a conservative. So no Democrats are conservatives.

$$\begin{aligned} \text{domain} &: \text{all people} \\ Dx &: \underline{\quad}_x \ \text{is a Democrat} \\ Hx &: \underline{\quad}_x \ \text{hates freedom} \\ Cx &: \underline{\quad}_x \ \text{is a conservative} \end{aligned}$$

$$\forall x\,(Dx \rightarrow Hx),\ \forall y\,(Hy \rightarrow \neg Cy)\ \therefore\ \forall z\,(Dz \rightarrow \neg Cz)$$

Evaluating Arguments

1 $\forall x\,(Dx \rightarrow Hx)$

2 $\forall y\,(Hy \rightarrow \neg Cy)$ Goal: $\forall z\,(Dz \rightarrow \neg Cz)$

Evaluating Arguments

1	$\forall x\,(Dx \rightarrow Hx)$	
2	$\forall y\,(Hy \rightarrow \neg Cy)$	Goal: $\forall z\,(Dz \rightarrow \neg Cz)$
3	$\quad De$	Ass ($\rightarrow I$)

Evaluating Arguments

1	$\forall x\,(Dx \rightarrow Hx)$	
2	$\forall y\,(Hy \rightarrow \neg Cy)$	Goal: $\forall z\,(Dz \rightarrow \neg Cz)$
3	$\quad De$	Ass ($\rightarrow I$)
4	$\quad De \rightarrow He$	$\forall E$ 1

Evaluating Arguments

1	$\forall x\,(Dx \rightarrow Hx)$	
2	$\forall y\,(Hy \rightarrow \neg Cy)$	Goal: $\forall z\,(Dz \rightarrow \neg Cz)$
3	De	Ass ($\rightarrow I$)
4	$De \rightarrow He$	$\forall E$ 1
5	He	$\rightarrow E$ 3, 4

Evaluating Arguments

1	$\forall x\,(Dx \rightarrow Hx)$	
2	$\forall y\,(Hy \rightarrow \neg Cy)$	Goal: $\forall z\,(Dz \rightarrow \neg Cz)$
3	$\quad De$	Ass ($\rightarrow I$)
4	$\quad De \rightarrow He$	$\forall E$ 1
5	$\quad He$	$\rightarrow E$ 3, 4
6	$\quad He \rightarrow \neg Ce$	$\forall E$ 2

Evaluating Arguments

1	$\forall x\,(Dx \to Hx)$	
2	$\forall y\,(Hy \to \neg Cy)$	Goal: $\forall z\,(Dz \to \neg Cz)$
3	De	Ass ($\to I$)
4	$De \to He$	$\forall E$ 1
5	He	$\to E$ 3, 4
6	$He \to \neg Ce$	$\forall E$ 2
7	$\neg Ce$	$\to E$ 5, 6

Evaluating Arguments

1	$\forall x\,(Dx \rightarrow Hx)$	
2	$\forall y\,(Hy \rightarrow \neg Cy)$	Goal: $\forall z\,(Dz \rightarrow \neg Cz)$
3	$\quad De$	Ass ($\rightarrow I$)
4	$\quad De \rightarrow He$	$\forall E$ 1
5	$\quad He$	$\rightarrow E$ 3, 4
6	$\quad He \rightarrow \neg Ce$	$\forall E$ 2
7	$\quad \neg Ce$	$\rightarrow E$ 5, 6
8	$De \rightarrow \neg Ce$	$\rightarrow I$ 3–7

Evaluating Arguments

1	$\forall x\,(Dx \to Hx)$	
2	$\forall y\,(Hy \to \neg Cy)$	Goal: $\forall z\,(Dz \to \neg Cz)$
3	De	Ass ($\to I$)
4	$De \to He$	$\forall E$ 1
5	He	$\to E$ 3, 4
6	$He \to \neg Ce$	$\forall E$ 2
7	$\neg Ce$	$\to E$ 5, 6
8	$De \to \neg Ce$	$\to I$ 3–7
9	$\forall z\,(Dz \to \neg Cz)$	$\forall I$ 8

Evaluating Arguments

All dogs are pets. Some pets are furry. So some dogs are furry.

Evaluating Arguments

All dogs are pets. Some pets are furry. So some dogs are furry.

$$\begin{aligned} \text{domain} &: \text{all animals} \\ Dx &: \underline{\quad}_x \text{ is a dog} \\ Px &: \underline{\quad}_x \text{ is a pet} \\ Fx &: \underline{\quad}_x \text{ is furry} \end{aligned}$$

Evaluating Arguments

All dogs are pets. Some pets are furry. So some dogs are furry.

$$\begin{aligned} \text{domain} &: \text{all animals} \\ Dx &: \underline{\quad}_x \text{ is a dog} \\ Px &: \underline{\quad}_x \text{ is a pet} \\ Fx &: \underline{\quad}_x \text{ is furry} \end{aligned}$$

$$\forall x\,(Dx \to Px),\ \exists y\,(Py \land Fy)\ \therefore\ \exists z\,(Dz \land Fz)$$

Evaluating Arguments

$$\forall x\,(Dx \longrightarrow Px),\; \exists y\,(Py \wedge Fy) \;\;\therefore\;\; \exists z\,(Dz \wedge Fz)$$

Evaluating Arguments

$$\forall x\,(Dx \rightarrow Px),\ \exists y\,(Py \wedge Fy)\ \therefore\ \exists z\,(Dz \wedge Fz)$$

$$\begin{aligned} \text{domain} &: 1 \\ D &: \\ P &: 1 \\ F &: 1 \end{aligned}$$

Evaluating Arguments

Bill is friends with everybody unless he's not friends with Ted. Bill is friends with Ted. So Bill is friends with everyone.

Evaluating Arguments

Bill is friends with everybody unless he's not friends with Ted. Bill is friends with Ted. So Bill is friends with everyone.

$$\begin{aligned} \text{domain} &: \text{all people} \\ Fxy &: \_\_\__x \text{ is friends with } \_\_\__y \\ b &: \text{Bill} \\ t &: \text{Ted} \end{aligned}$$

Evaluating Arguments

Bill is friends with everybody unless he's not friends with Ted. Bill is friends with Ted. So Bill is friends with everyone.

$$\begin{aligned} \text{domain} &: \text{all people} \\ Fxy &: \underline{\quad}_x \text{ is friends with } \underline{\quad}_y \\ b &: \text{Bill} \\ t &: \text{Ted} \end{aligned}$$

$$\forall x\, Fbx \lor \neg Fbt,\ Fbt \ \therefore\ \forall x\, Fbx$$

Evaluating Arguments

1 | $\forall x\, Fbx \lor \neg Fbt$

2 | Fbt

Goal: $\forall x\, Fbx$

Evaluating Arguments

1	$\forall x\, Fbx \lor \neg Fbt$	
2	Fbt	Goal: $\forall x\, Fbx$
3	$\quad \neg Fbt$	Ass ($\neg I$)

Evaluating Arguments

1	$\forall x\, Fbx \lor \neg Fbt$	
2	Fbt	Goal: $\forall x\, Fbx$
3	$\neg Fbt$	Ass ($\neg I$)
4	$\bot$	$\bot I$ 2, 3

Evaluating Arguments

1	$\forall x\, Fbx \vee \neg Fbt$	
2	Fbt	Goal: $\forall x\, Fbx$
3	$\quad \neg Fbt$	Ass ($\neg I$)
4	$\quad \bot$	$\bot I$ 2, 3
5	$\neg\neg Fbt$	$\neg I$ 3–4

Evaluating Arguments

1	$\forall x\, Fbx \lor \neg Fbt$	
2	Fbt	Goal: $\forall x\, Fbx$
3	$\quad \neg Fbt$	Ass ($\neg I$)
4	$\quad \bot$	$\bot I$ 2, 3
5	$\neg\neg Fbt$	$\neg I$ 3–4
6	$\forall x\, Fbx$	DS 1, 5

Evaluating Arguments

Bill is friends with everybody unless he's not friends with Ted. Bill is not friends with Ted. So Bill is not friends with everyone.

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Fxy &: \_\_\__x \text{ is friends with } \_\_\__y \\
b &: \text{Bill} \\
t &: \text{Ted}
\end{aligned}
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$$\forall x\, Fbx \lor \neg Fbt,\ \neg Fbt \ \therefore\ \neg\forall x\, Fbx$$

Evaluating Arguments

1	$\forall x\, Fbx \lor \neg Fbt$	
2	$\neg Fbt$	Goal: $\neg\forall x\, Fbx$

Evaluating Arguments

1	$\forall x\, Fbx \vee \neg Fbt$	
2	$\neg Fbt$	Goal: $\neg\forall x\, Fbx$
3	$\exists x \neg Fbx$	$\exists I$ 2

Evaluating Arguments

1	$\forall x\, Fbx \vee \neg Fbt$	
2	$\neg Fbt$	Goal: $\neg\forall x\, Fbx$
3	$\exists x \neg Fbx$	$\exists I$ 2
4	$\neg\forall x Fbx$	CQ 3