Predicate Logic

Entailment

PHIL 500

Outline

PL Entailment

Satisfiability in PL

Tautologies and Contradictions in PL

PL Entailment

Validity and Entailment

Validity

An argument $\mathscr{A}_1, \mathscr{A}_2, \ldots \mathscr{A}_N \therefore \mathscr{C}$ is *valid* iff there is no possibility in which $\mathscr{A}_1, \mathscr{A}_2, \ldots \mathscr{A}_N$ are all true while $\mathscr{C}$ is false.

Validity and Entailment

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SL Entailment

$\mathscr{A}_1, \mathscr{A}_2, \ldots \mathscr{A}_N$ *entail* $\mathscr{C}$ in SL,

$$\mathscr{A}_1, \mathscr{A}_2, \ldots \mathscr{A}_N \models \mathscr{C}$$

iff there is no valuation on which $\mathscr{A}_1, \mathscr{A}_2, \ldots \mathscr{A}_N$ are all true while $\mathscr{C}$ is false.

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PL Entailment

$\mathscr{A}_1, \mathscr{A}_2, \ldots \mathscr{A}_N$ *entail* $\mathscr{C}$ in PL,

$$\mathscr{A}_1, \mathscr{A}_2, \ldots \mathscr{A}_N \models \mathscr{C}$$

iff there is no interpretation on which $\mathscr{A}_1, \mathscr{A}_2, \ldots \mathscr{A}_N$ are all true while $\mathscr{C}$ is false.

Notation and Terminology

- For the remainder of the course, whenever we say 'entail', we will mean 'entail *in PL*'.

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- For the remainder of the course, whenever we write '$\mathscr{A}_1, \mathscr{A}_2, \ldots \mathscr{A}_N \models \mathscr{C}$', we will mean that $\mathscr{A}_1, \mathscr{A}_2, \ldots \mathscr{A}_N$ entail $\mathscr{C}$ *in PL*.

Entailment and Validity

- If $\mathscr{A}_1, \mathscr{A}_2, \ldots \mathscr{A}_N$ entail $\mathscr{C}$, then the argument $\mathscr{A}_1, \mathscr{A}_2, \ldots \mathscr{A}_N \therefore \mathscr{C}$ is valid.

Entailment and Validity

- If $\mathscr{A}_1, \mathscr{A}_2, \ldots \mathscr{A}_N$ entail $\mathscr{C}$, then the argument $\mathscr{A}_1, \mathscr{A}_2, \ldots \mathscr{A}_N \therefore \mathscr{C}$ is valid.
- However, just because the argument $\mathscr{A}_1, \mathscr{A}_2, \ldots \mathscr{A}_N \therefore \mathscr{C}$ is valid, this doesn't necessarily mean that $\mathscr{A}_1, \mathscr{A}_2, \ldots \mathscr{A}_N$ entail $\mathscr{C}$.

Entailment and Validity

Everything in my house is red

$\therefore$ Everything in my house is colored

Entailment and Validity

$$\forall x(Hx \rightarrow Rx)$$

$\therefore$ Everything in my house is colored

Entailment and Validity

$$\begin{array}{rl} & \forall x(Hx \rightarrow Rx) \\ \therefore & \forall y(Hy \rightarrow Cy) \end{array}$$

Entailment and Validity

$$\forall x(Hx \rightarrow Rx)$$
$$\therefore \ \forall y(Hy \rightarrow Cy)$$

$$\begin{aligned} \text{domain} &: 1 \\ H &: 1 \\ R &: 1 \\ C &: \end{aligned}$$

Entailment and Validity

$$\forall x(Hx \rightarrow Rx) \text{ [T]}$$

$$\therefore \forall y(Hy \rightarrow Cy)$$

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Entailment and Validity

$$\forall x(Hx \rightarrow Rx) \text{ [T]}$$
$$\therefore\ \forall y(Hy \rightarrow Cy) \text{ [F]}$$

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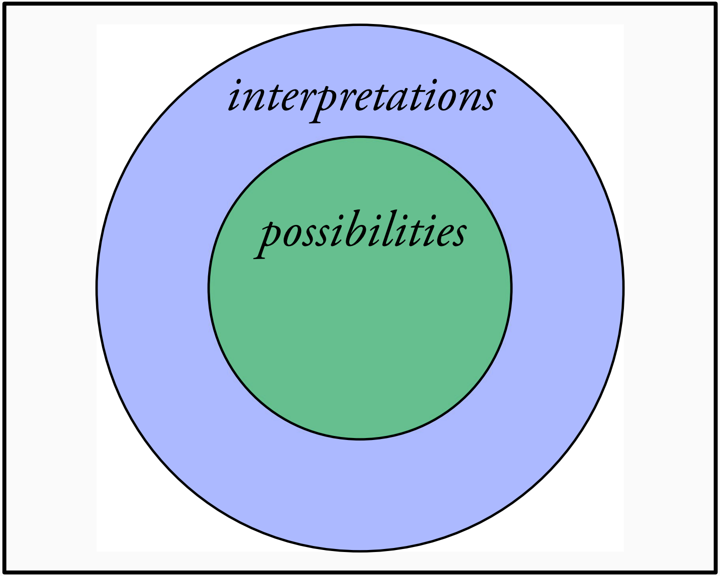
Entailment and Validity

- All *possibilities* are represented in some *interpretation*

Entailment and Validity

- All *possibilities* are represented in some *interpretation*
- But not all *interpretations* correspond to some *possibility*

Entailment and Validity



Entailment and Validity

- If we know something about *every* interpretation, then we know something about every possibility.

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- If we know something about *every* interpretation, then we know something about every possibility.
- But, just because we know something about *some* interpretation, that doesn't tell us that anything about any possibility.

Entailment and Validity

- Entailment $\Rightarrow$ Valid Argument

Entailment and Validity

- Entailment $\Longrightarrow$ Valid Argument
- Not an Entailment $\not\Longrightarrow$ Invalid Argument

Entailment and Validity

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▹ We can attempt to reason about these interpretations to prove that an argument is an entailment.

Entailment and Validity

- In SL, we could check every valuation (every row of the truth-table) to establish that an argument was an entailment.
- In PL, there's no way to check every possible interpretation (there's infinitely many)
  - ▹ We can attempt to reason about these interpretations to prove that an argument is an entailment.
  - ▹ Or: we can use natural deduction (that's what we'll do in this class)

Proving an Argument isn't an Entailment

- So, for now, we'll consider how to use interpretations to prove that an argument *isn't* an entailment in PL.

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  - Remember: *this doesn't show that the argument is invalid*
  - The way to show that an argument is invalid is to point at a possibility in which the premises are true and the conclusion is false.

Proving an Argument isn't an Entailment

- So, for now, we'll consider how to use interpretations to prove that an argument *isn't* an entailment in PL.
  - Remember: *this doesn't show that the argument is invalid*
  - The way to show that an argument is invalid is to point at a possibility in which the premises are true and the conclusion is false.
  - Still, knowing that an argument isn't an entailment will prevent you from wasting time trying to prove it with a natural deduction proof.

Proving an Argument isn't an Entailment

- To prove that an argument isn't an entailment in PL:

Proving an Argument isn't an Entailment

- To prove that an argument isn't an entailment in PL:
  - Provide an interpretation (*any* interpretation) which makes its premises true and its conclusion false.

Proving an Argument isn't an Entailment

Someone is fast, and someone is tall. So someone is fast and tall.

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$$\begin{aligned} \text{domain} &: \text{all people} \\ Fx &: \underline{\quad}_x \text{ is fast} \\ Tx &: \underline{\quad}_x \text{ is tall} \end{aligned}$$

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$$\exists x \quad Fx \quad \wedge \quad \exists y \quad Ty \quad \therefore \quad \exists z \quad (Fz \wedge Tz)$$

Proving an Argument isn't an Entailment

Someone is fast, and someone is tall. So someone is fast and tall.

$$\begin{aligned} \text{domain} &: \text{Amy, Bill} \\ F &: \text{Amy} \\ T &: \text{Bill} \end{aligned}$$

$$\exists x \quad Fx \quad \wedge \quad \exists y \quad Ty \quad \therefore \quad \exists z \quad (Fz \wedge Tz)$$

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$$
\begin{aligned}
\text{domain} &: \text{Amy, Bill} \\
F &: \text{Amy} \\
T &: \text{Bill} \\
x &: \text{Amy}
\end{aligned}
$$

$$\exists x \quad Fx \quad \wedge \quad \exists y \quad Ty \quad \therefore \quad \exists z \quad (Fz \wedge Tz)$$

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$$\quad\quad T$$

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$$\begin{array}{ccccccccc} \exists x & Fx & \wedge & \exists y & Ty & \therefore & \exists z & (Fz \wedge Tz) \\ T & T & & & & & & \\ & F & & & & & & \end{array}$$

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T	T						
	F						

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	F						

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$$\underset{T}{}$$

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$$\begin{array}{cc} T & F \\ & F \end{array}$$

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		T					F
							F

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		T				F	F
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Proving an Argument isn't an Entailment

$$\exists x\, Fx \land \exists y\, Ty \nvDash \exists z\, (Fz \land Tz)$$

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  - $\exists x \forall y(\neg Lyy \rightarrow Lxy)$

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  - $\exists x \forall y(\neg Lyy \rightarrow Lxy)$
  - "Someone loves everyone who doesn't love themselves."

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- A word of warning: think through the semantics when deciding whether a sentence is true or false on an interpretation.
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  - ▹ $\exists x \forall y(\neg Lyy \rightarrow Lxy)$
  - ▹ "Someone loves everyone who doesn't love themselves."

$$\begin{aligned} \text{domain} &: \text{Abelard, Heloise} \\ L &: \langle \text{ A, A } \rangle, \langle \text{ A, H } \rangle \end{aligned}$$

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Some lawyers are politicians. Some politicians are rich. So, some lawyers are rich.

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$$\begin{aligned} \text{domain} &: \text{all people} \\ Lx &: \underline{\quad}_x \text{ is a lawyer} \\ Px &: \underline{\quad}_x \text{ is a politician} \\ Rx &: \underline{\quad}_x \text{ is rich} \end{aligned}$$

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$$\exists x \quad (Lx \land Px) \quad , \quad \exists y \quad (Py \land Ry) \quad \therefore \quad \exists z \quad (Lz \land Rz)$$

Proving an Argument isn't an Entailment

Some lawyers are politicians. Some politicians are rich. So, some lawyers are rich.

$$\begin{aligned} \text{domain} &: 1, 2 \\ L &: 1 \\ P &: 1, 2 \\ R &: 2 \end{aligned}$$

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$$
\begin{aligned}
\text{domain} &: 1, 2 \\
L &: 1 \\
P &: 1, 2 \\
R &: 2 \\
y &: 2
\end{aligned}
$$

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T $\qquad$ T

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$$\underset{T}{\exists x} \quad (Lx \wedge Px) \quad , \quad \underset{T}{\exists y} \quad (\underset{T}{Py} \wedge Ry) \quad \therefore \quad \exists z \quad (Lz \wedge Rz)$$

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$$\exists x \quad (Lx \wedge Px) \quad , \quad \exists y \quad (Py \wedge Ry) \quad \therefore \quad \exists z \quad (Lz \wedge Rz)$$

T (under $\exists x$), T (under $\exists y$), F (under $\wedge$ in $Lz \wedge Rz$)

F

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Some lawyers are politicians. Some politicians are rich. So, some lawyers are rich.

$$\begin{aligned} \text{domain} &: 1, 2 \\ L &: 1 \\ P &: 1, 2 \\ R &: 2 \end{aligned}$$

$\exists x$	$(Lx \wedge Px)$	,	$\exists y$	$(Py \wedge Ry)$	$\therefore$	$\exists z$	$(Lz \wedge Rz)$
T			T				F
							F

Proving an Argument isn't an Entailment

Some lawyers are politicians. Some politicians are rich. So, some lawyers are rich.

$$\begin{aligned} \text{domain} &: 1, 2 \\ L &: 1 \\ P &: 1, 2 \\ R &: 2 \end{aligned}$$

$$\begin{array}{ccccccccc} \exists x & (Lx \wedge Px) & , & \exists y & (Py \wedge Ry) & \therefore & \exists z & (Lz \wedge Rz) \\ T & & & T & & & F & F \\ & & & & & & & F \end{array}$$

Proving an Argument isn't an Entailment

Some lawyers are politicians. Some politicians are rich. So, some lawyers are rich.

$$\begin{aligned} \text{domain} &: 1, 2 \\ L &: 1 \\ P &: 1, 2 \\ R &: 2 \end{aligned}$$

$$\underset{T}{\exists x} \quad (Lx \wedge Px) \quad , \quad \underset{T}{\exists y} \quad (Py \wedge Ry) \quad \therefore \quad \underset{F}{\exists z} \quad (Lz \wedge Rz)$$

Proving an Argument isn't an Entailment

$$\exists x\,(Lx \wedge Px),\ \exists y\,(Py \wedge Ry) \not\models \exists z\,(Lz \wedge Rz)$$

Proving an Argument isn't an Entailment

All wealthy people are republicans. So some republicans are wealthy.

Proving an Argument isn't an Entailment

All wealthy people are republicans. So some republicans are wealthy.

$$\begin{aligned} \text{domain} &: \text{all people} \\ Wx &: \underline{\quad}_x \text{ is wealthy} \\ Rx &: \underline{\quad}_x \text{ is a republican} \end{aligned}$$

Proving an Argument isn't an Entailment

All wealthy people are republicans. So some republicans are wealthy.

$$\begin{aligned} \text{domain} &: \text{all people} \\ Wx &: \underline{\quad}_x \text{ is wealthy} \\ Rx &: \underline{\quad}_x \text{ is a republican} \end{aligned}$$

$$\forall x\,(Wx \rightarrow Rx) \quad \therefore \quad \exists y(Ry \wedge Wy)$$

Proving an Argument isn't an Entailment

All wealthy people are republicans. So some republicans are wealthy.

$$\begin{aligned} \text{domain} &: 1 \\ W &: \\ R &: \end{aligned}$$

$$\forall x\,(Wx \rightarrow Rx) \quad \therefore \quad \exists y(Ry \wedge Wy)$$

Proving an Argument isn't an Entailment

All wealthy people are republicans. So some republicans are wealthy.

$$\begin{aligned} \text{domain} &: 1 \\ W &: \\ R &: \end{aligned}$$

$$\forall x\,(Wx \rightarrow Rx) \quad \therefore \quad \exists y(Ry \wedge Wy)$$

T

Proving an Argument isn't an Entailment

All wealthy people are republicans. So some republicans are wealthy.

$$
\begin{aligned}
\text{domain} &: 1 \\
W &: \\
R &:
\end{aligned}
$$

$$
\begin{array}{ccc}
\forall x\,(Wx \rightarrow Rx) & \therefore & \exists y(Ry \wedge Wy) \\
T & & F
\end{array}
$$

Proving an Argument isn't an Entailment

$$\forall x\,(Wx \rightarrow Rx) \not\models \exists y\,(Ry \land Wy)$$

Proving an Argument isn't an Entailment

$$\forall x\,(Wx \rightarrow Rx),\ \exists y Wy \models \exists y\,(Ry \land Wy)$$

Proving an Argument isn't an Entailment

Luella is taller than Sabeen. So Sabeen is shorter than someone.

Proving an Argument isn't an Entailment

Luella is taller than Sabeen. So Sabeen is shorter than someone.

$$
\begin{aligned}
\text{domain} &: \text{all people} \\
Txy &: \_\_\__x \text{ is taller than } \_\_\__y \\
Sxy &: \_\_\__x \text{ is shorter than } \_\_\__y \\
l &: \text{Luella} \\
s &: \text{Sabeen}
\end{aligned}
$$

Proving an Argument isn't an Entailment

Luella is taller than Sabeen. So Sabeen is shorter than someone.

$$\begin{aligned} \text{domain} &: \text{all people} \\ Txy &: \_\_\__x \text{ is taller than } \_\_\__y \\ Sxy &: \_\_\__x \text{ is shorter than } \_\_\__y \\ l &: \text{Luella} \\ s &: \text{Sabeen} \end{aligned}$$

$$Tls \quad \therefore \quad \exists x\, Ssx$$

Proving an Argument isn't an Entailment

Luella is taller than Sabeen. So Sabeen is shorter than someone.

$$\begin{aligned} \text{domain} &: 1 \\ T &: \langle\, 1, 1 \,\rangle \\ S &: \\ s &: 1 \\ l &: 1 \end{aligned}$$

$$Tls \quad \therefore \quad \exists x\, Ssx$$

Proving an Argument isn't an Entailment

Luella is taller than Sabeen. So Sabeen is shorter than someone.

$$\begin{aligned} \text{domain} &: 1 \\ T &: \langle\, 1, 1 \,\rangle \\ S &: \\ s &: 1 \\ l &: 1 \end{aligned}$$

$$Tls \quad \therefore \quad \exists x\, Ssx$$

$$T$$

Proving an Argument isn't an Entailment

Luella is taller than Sabeen. So Sabeen is shorter than someone.

$$\begin{aligned} \text{domain} &: 1 \\ T &: \langle\, 1, 1 \,\rangle \\ S &: \\ s &: 1 \\ l &: 1 \end{aligned}$$

Tls	$\therefore$	$\exists x\, Ssx$
T		F

Proving an Argument isn't an Entailment

$$Tls \not\models \exists x\, Ssx$$

Proving an Argument isn’t an Entailment

Abelard loves everyone who loves him. Heloise doesn’t love Abelard. So Abelard doesn’t love Heloise.

Proving an Argument isn't an Entailment

Abelard loves everyone who loves him. Heloise doesn't love Abelard. So Abelard doesn't love Heloise.

$$
\begin{aligned}
\text{domain} &: \text{all people} \\
Lxy &: \underline{\quad}_x \text{ loves } \underline{\quad}_y \\
a &: \text{Abelard} \\
h &: \text{Heloise}
\end{aligned}
$$

Proving an Argument isn't an Entailment

Abelard loves everyone who loves him. Heloise doesn't love Abelard. So Abelard doesn't love Heloise.

$$
\begin{aligned}
\text{domain} &: \text{all people} \\
Lxy &: \_\_\__x \text{ loves } \_\_\__y \\
a &: \text{Abelard} \\
h &: \text{Heloise}
\end{aligned}
$$

$$\forall x(Lxa \rightarrow Lax) \quad , \quad \neg Lha \quad \therefore \quad \neg Lah$$

Proving an Argument isn't an Entailment

Abelard loves everyone who loves him. Heloise doesn't love Abelard. So Abelard doesn't love Heloise.

$$\begin{aligned} \text{domain} &: \text{Abelard, Heloise, Simone} \\ L &: \langle \text{ A, H } \rangle, \langle \text{ S, A } \rangle, \langle \text{ A, S } \rangle \\ a &: \text{Abelard} \\ h &: \text{Heloise} \end{aligned}$$

$$\forall x(Lxa \rightarrow Lax) \quad , \quad \neg Lha \quad \therefore \quad \neg Lah$$

Proving an Argument isn't an Entailment

Abelard loves everyone who loves him. Heloise doesn't love Abelard. So Abelard doesn't love Heloise.

$$\begin{aligned} \text{domain} &: \text{Abelard, Heloise, Simone} \\ L &: \langle \text{A, H} \rangle, \langle \text{S, A} \rangle, \langle \text{A, S} \rangle \\ a &: \text{Abelard} \\ h &: \text{Heloise} \end{aligned}$$

$$\forall x(Lxa \rightarrow Lax) \quad , \quad \neg Lha \quad \therefore \quad \neg Lah$$

$$F$$

Proving an Argument isn't an Entailment

Abelard loves everyone who loves him. Heloise doesn't love Abelard. So Abelard doesn't love Heloise.

$$
\begin{aligned}
\text{domain} &: \text{Abelard, Heloise, Simone} \\
L &: \langle \text{ A, H } \rangle, \langle \text{ S, A } \rangle, \langle \text{ A, S } \rangle \\
a &: \text{Abelard} \\
h &: \text{Heloise}
\end{aligned}
$$

$$\forall x(Lxa \rightarrow Lax) \quad , \quad \neg Lha \quad \therefore \quad \neg Lah$$

$$\qquad\qquad\qquad\qquad T \qquad\qquad F$$

Proving an Argument isn't an Entailment

Abelard loves everyone who loves him. Heloise doesn't love Abelard. So Abelard doesn't love Heloise.

$$\begin{aligned} \text{domain} &: \text{Abelard, Heloise, Simone} \\ L &: \langle \text{A, H} \rangle, \langle \text{S, A} \rangle, \langle \text{A, S} \rangle \\ a &: \text{Abelard} \\ h &: \text{Heloise} \end{aligned}$$

$$\forall x(Lxa \rightarrow Lax) \quad , \quad \neg Lha \quad \therefore \quad \neg Lah$$

$$T \qquad\qquad\qquad T \qquad\qquad F$$

Proving an Argument isn't an Entailment

Abelard loves everyone who loves him. Heloise doesn't love Abelard. So Abelard doesn't love Heloise.

$$\begin{aligned} \text{domain} &: \text{Abelard, Heloise, Simone} \\ L &: \langle \text{A, H} \rangle, \langle \text{S, A} \rangle, \langle \text{A, S} \rangle \\ a &: \text{Abelard} \\ h &: \text{Heloise} \end{aligned}$$

$$\underset{T}{\forall x(Lxa \to Lax)} \quad , \quad \underset{T}{\neg Lha} \quad \therefore \quad \underset{F}{\neg Lah}$$

Proving an Argument isn't an Entailment

Abelard loves everyone who loves him. Heloise doesn't love Abelard. So Abelard doesn't love Heloise.

$$
\begin{aligned}
\text{domain} &: \text{Abelard, Heloise} \\
L &: \langle \text{ Abelard, Heloise } \rangle \\
a &: \text{Abelard} \\
h &: \text{Heloise}
\end{aligned}
$$

$$
\begin{array}{ccccc}
\forall x(Lxa \rightarrow Lax) & , & \neg Lha & \therefore & \neg Lah \\
T & & T & & F
\end{array}
$$

Proving an Argument isn't an Entailment

$$\forall x(Lxa \to Lax), \neg Lha \not\models \neg Lah$$

Proving an Argument isn't an Entailment

Everyone loves anyone who loves themselves. Someone is loved. So someone loves themselves.

Proving an Argument isn't an Entailment

Everyone loves anyone who loves themselves. Someone is loved. So someone loves themselves.

$$\begin{aligned} \text{domain} &: \text{all people} \\ Lxy &: \_\_\__x \text{ loves } \_\_\__y \end{aligned}$$

Proving an Argument isn't an Entailment

Everyone loves anyone who loves themselves. Someone is loved. So someone loves themselves.

$$\begin{aligned} \text{domain} &: \text{all people} \\ Lxy &: \_\_\__x \text{ loves } \_\_\__y \end{aligned}$$

$$\forall x \quad (Lxx \to \forall y\, Lyx) \qquad\qquad\qquad \therefore$$

Proving an Argument isn't an Entailment

Everyone loves anyone who loves themselves. Someone is loved. So someone loves themselves.

$$\text{domain} : \text{all people}$$
$$Lxy : \_\_\__x \text{ loves } \_\_\__y$$

$$\forall x \quad (Lxx \to \forall y\, Lyx) \quad \exists z\, \exists w\, Lzw \quad \therefore$$

Proving an Argument isn't an Entailment

Everyone loves anyone who loves themselves. Someone is loved. So someone loves themselves.

$$\begin{aligned} \text{domain} &: \text{all people} \\ Lxy &: \underline{\quad}_x \text{ loves } \underline{\quad}_y \end{aligned}$$

$$\forall x \quad (Lxx \to \forall y\, Lyx) \quad \exists z\, \exists w\, Lzw \quad \therefore \quad \exists x\, Lxx$$

Proving an Argument isn't an Entailment

Everyone loves anyone who loves themselves. Someone is loved. So someone loves themselves.

$$\begin{aligned} \text{domain} &: 1, 2 \\ L &: \langle\, 1, 2 \,\rangle \end{aligned}$$

$$\forall x \quad (Lxx \rightarrow \forall y\, Lyx) \quad \exists z\, \exists w\, Lzw \quad \therefore \quad \exists x\, Lxx$$

Proving an Argument isn't an Entailment

Everyone loves anyone who loves themselves. Someone is loved. So someone loves themselves.

$$\begin{aligned} \text{domain} &: 1, 2 \\ L &: \langle\, 1, 2 \,\rangle \\ x &: 1 \end{aligned}$$

$$\forall x \quad (Lxx \rightarrow \forall y\, Lyx) \quad \exists z\, \exists w\, Lzw \quad \therefore \quad \exists x\, Lxx$$

Proving an Argument isn't an Entailment

Everyone loves anyone who loves themselves. Someone is loved. So someone loves themselves.

$$\begin{aligned} \text{domain} &: 1, 2 \\ L &: \langle 1, 2 \rangle \\ x &: 1 \end{aligned}$$

$$\forall x \quad (Lxx \rightarrow \forall y\, Lyx) \quad \exists z\, \exists w\, Lzw \quad \therefore \quad \exists x\, Lxx$$

$$T$$

Proving an Argument isn't an Entailment

Everyone loves anyone who loves themselves. Someone is loved. So someone loves themselves.

$$\text{domain} : 1, 2$$
$$L : \langle 1, 2 \rangle$$
$$x : 2$$

$$\forall x \quad (Lxx \rightarrow \forall y\, Lyx) \quad \exists z\, \exists w\, Lzw \quad \therefore \quad \exists x\, Lxx$$

T

Proving an Argument isn't an Entailment

Everyone loves anyone who loves themselves. Someone is loved. So someone loves themselves.

$$\begin{aligned} \text{domain} &: 1, 2 \\ L &: \langle\, 1, 2 \,\rangle \\ x &: 2 \end{aligned}$$

$$\forall x \quad (Lxx \rightarrow \forall y\, Lyx) \quad \exists z\, \exists w\, Lzw \quad \therefore \quad \exists x\, Lxx$$

T

T

Proving an Argument isn't an Entailment

Everyone loves anyone who loves themselves. Someone is loved. So someone loves themselves.

$$\text{domain} : 1, 2$$
$$L : \langle 1, 2 \rangle$$

$\forall x$	$(Lxx \rightarrow \forall y\, Lyx)$	$\exists z \exists w\, Lzw$	$\therefore$	$\exists x\, Lxx$
	T			
	T			

Proving an Argument isn't an Entailment

Everyone loves anyone who loves themselves. Someone is loved. So someone loves themselves.

$$\begin{aligned} \text{domain} &: 1, 2 \\ L &: \langle\, 1, 2 \,\rangle \end{aligned}$$

$$\begin{array}{lllll} \forall x & (Lxx \to \forall y\, Lyx) & \exists z\, \exists w\, Lzw & \therefore & \exists x\, Lxx \\ \color{green}{T} & \qquad\quad \color{green}{T} & & & \\ & \qquad\quad \color{green}{T} & & & \end{array}$$

Proving an Argument isn't an Entailment

Everyone loves anyone who loves themselves. Someone is loved. So someone loves themselves.

$$\begin{aligned} \text{domain} &: 1, 2 \\ L &: \langle\, 1, 2 \,\rangle \end{aligned}$$

$$\forall x \quad (Lxx \rightarrow \forall y\, Lyx) \quad \exists z\, \exists w\, Lzw \quad \therefore \quad \exists x\, Lxx$$

T

Proving an Argument isn't an Entailment

Everyone loves anyone who loves themselves. Someone is loved. So someone loves themselves.

$$\text{domain} : 1, 2$$
$$L : \langle 1, 2 \rangle$$

$$\forall x \quad (Lxx \rightarrow \forall y\, Lyx) \quad \exists z\, \exists w\, Lzw \quad \therefore \quad \exists x\, Lxx$$

T $\qquad$ T

Proving an Argument isn't an Entailment

Everyone loves anyone who loves themselves. Someone is loved. So someone loves themselves.

$$\begin{aligned} \text{domain} &: 1, 2 \\ L &: \langle\, 1, 2 \,\rangle \end{aligned}$$

$$\forall x \quad (Lxx \rightarrow \forall y\, Lyx) \quad \exists z\, \exists w\, Lzw \quad \therefore \quad \exists x\, Lxx$$

$$T \qquad\qquad\qquad\qquad T \qquad\qquad\qquad F$$

Proving an Argument isn't an Entailment

$$\forall x(Lxx \to \forall y Lyx)\ \exists z\, \exists w\, Lzw \nvDash \exists x\, Lxx$$

Proving an Argument isn't an Entailment

Everyone quirky is either funny or shy. Someone is quirky. So someone is funny.

Proving an Argument isn't an Entailment

Everyone quirky is either funny or shy. Someone is quirky. So someone is funny.

$$\begin{aligned} \text{domain} &: \text{all people} \\ Qx &: \underline{\quad}_x \text{ is quirky} \\ Fx &: \underline{\quad}_x \text{ is funny} \\ Sx &: \underline{\quad}_x \text{ is shy} \end{aligned}$$

Proving an Argument isn't an Entailment

Everyone quirky is either funny or shy. Someone is quirky. So someone is funny.

$$\begin{aligned} \text{domain} &: \text{all people} \\ Qx &: \underline{\quad}_x \text{ is quirky} \\ Fx &: \underline{\quad}_x \text{ is funny} \\ Sx &: \underline{\quad}_x \text{ is shy} \end{aligned}$$

$$\forall x[Qx \rightarrow (Fx \vee Sx)] \quad \exists y\, Qy \quad \therefore \quad \exists x\, Fx$$

Proving an Argument isn't an Entailment

Everyone quirky is either funny or shy. Someone is quirky. So someone is funny.

$$
\begin{aligned}
\text{domain} &: 1\\
Q &: 1\\
F &: \\
S &: 1
\end{aligned}
$$

$$\forall x[Qx \rightarrow (Fx \vee Sx)] \quad \exists y\, Qy \quad \therefore \quad \exists x\, Fx$$

Proving an Argument isn't an Entailment

Everyone quirky is either funny or shy. Someone is quirky. So someone is funny.

$$\begin{aligned} \text{domain} &: 1 \\ Q &: 1 \\ F &: \\ S &: 1 \end{aligned}$$

$$\forall x[Qx \rightarrow (Fx \vee Sx)] \quad \exists y\, Qy \quad \therefore \quad \exists x\, Fx$$

$$T$$

Proving an Argument isn't an Entailment

Everyone quirky is either funny or shy. Someone is quirky. So someone is funny.

$$\begin{aligned} \text{domain} &: 1 \\ Q &: 1 \\ F &: \\ S &: 1 \end{aligned}$$

$$\forall x[Qx \rightarrow (Fx \vee Sx)] \quad \exists y\, Qy \quad \therefore \quad \exists x\, Fx$$

$$T \qquad\qquad\qquad T$$

Proving an Argument isn't an Entailment

Everyone quirky is either funny or shy. Someone is quirky. So someone is funny.

$$\begin{aligned} \text{domain} &: 1 \\ Q &: 1 \\ F &: \\ S &: 1 \end{aligned}$$

$$\forall x[Qx \rightarrow (Fx \vee Sx)] \quad \exists y\, Qy \quad \therefore \quad \exists x\, Fx$$

$$T \qquad\qquad T \qquad\qquad F$$

Proving an Argument isn't an Entailment

$$\forall x[Qx \rightarrow (Fx \vee Sx)]\ \exists y\, Qy \not\models \exists x\, Fx$$

Predicate Logic

Satisfiability, Tautologies, and Contradictions

PHIL 500

Outline

PL Entailment

Satisfiability in PL

Tautologies and Contradictions in PL

Proving an Argument isn't an Entailment

Everyone is either quirky or funny. So everyone is quirky—unless everyone is funny.

Proving an Argument isn't an Entailment

Everyone is either quirky or funny. So everyone is quirky—unless everyone is funny.

$$\begin{aligned} \text{domain} &: \text{all people} \\ Qx &: \underline{\quad}_x \text{ is quirky} \\ Fx &: \underline{\quad}_x \text{ is funny} \end{aligned}$$

Proving an Argument isn't an Entailment

Everyone is either quirky or funny. So everyone is quirky—unless everyone is funny.

$$\begin{aligned} \text{domain} &: \text{all people} \\ Qx &: \underline{\quad}_x \text{ is quirky} \\ Fx &: \underline{\quad}_x \text{ is funny} \end{aligned}$$

$$\forall x(Qx \vee Fx) \quad \therefore \quad \forall x\, Qx \quad \vee \quad \forall x\, Fx$$

Proving an Argument isn't an Entailment

Everyone is either quirky or funny. So everyone is quirky—unless everyone is funny.

$$\begin{aligned} \text{domain} &: 1, 2 \\ Q &: 1 \\ F &: 2 \end{aligned}$$

$$\forall x(Qx \lor Fx) \quad \therefore \quad \forall x\, Qx \quad \lor \quad \forall x\, Fx$$

Proving an Argument isn't an Entailment

Everyone is either quirky or funny. So everyone is quirky—unless everyone is funny.

$$\begin{aligned} \text{domain} &: 1, 2 \\ Q &: 1 \\ F &: 2 \end{aligned}$$

$$\forall x(Qx \lor Fx) \quad \therefore \quad \forall x\, Qx \quad \lor \quad \forall x\, Fx$$

$$T$$

Proving an Argument isn't an Entailment

Everyone is either quirky or funny. So everyone is quirky—unless everyone is funny.

$$\begin{aligned} \text{domain} &: 1, 2 \\ Q &: 1 \\ F &: 2 \end{aligned}$$

$$\underset{T}{\forall x(Qx \vee Fx)} \quad \therefore \quad \underset{F}{\forall x\, Qx} \quad \vee \quad \underset{F}{\forall x\, Fx}$$

Proving an Argument isn't an Entailment

Everyone is either quirky or funny. So everyone is quirky—unless everyone is funny.

$$\begin{aligned} \text{domain} &: 1, 2 \\ Q &: 1 \\ F &: 2 \end{aligned}$$

$$\forall x(Qx \vee Fx) \quad \therefore \quad \forall x\, Qx \quad \vee \quad \forall x\, Fx$$

$$T \qquad\qquad\qquad F \qquad\qquad F$$

Proving an Argument isn't an Entailment

Everyone is either quirky or funny. So everyone is quirky—unless everyone is funny.

$$\text{domain} : 1, 2$$
$$Q : 1$$
$$F : 2$$

$$\forall x(Qx \lor Fx) \quad \therefore \quad \forall x\, Qx \quad \lor \quad \forall x\, Fx$$
$$T \qquad\qquad\qquad F \qquad F \qquad F$$

Proving an Argument isn't an Entailment

Everyone is either quirky or funny. So everyone is quirky—unless everyone is funny.

$$\begin{aligned} \text{domain} &: 1, 2 \\ Q &: 1 \\ F &: 2 \end{aligned}$$

$$\forall x(Qx \lor Fx) \quad \therefore \quad \forall x\, Qx \quad \lor \quad \forall x\, Fx$$

$$\quad\; T \qquad\qquad\qquad\qquad\qquad F$$

Proving an Argument isn't an Entailment

$$\forall x(Qx \vee Fx) \nvDash \forall x\, Qx \vee \forall x\, Fx$$

Proving an Argument isn't an Entailment

$$\forall x(Qx \lor Fx) \models \forall x\, Qx \lor \exists x\, Fx$$

Satisfiability in PL

Joint Possibility and Satisfiability

Joint Possibility

$\mathscr{A}_1, \mathscr{A}_2, \ldots \mathscr{A}_N$ are *jointly possible* iff there is some possibility in which $\mathscr{A}_1, \mathscr{A}_2, \ldots \mathscr{A}_N$ are all true.

Joint Possibility and Satisfiability

Joint Possibility

$\mathscr{A}_1, \mathscr{A}_2, \ldots \mathscr{A}_N$ are *jointly possible* iff there is some possibility in which $\mathscr{A}_1, \mathscr{A}_2, \ldots \mathscr{A}_N$ are all true.

Satisfiability in SL

$\mathscr{A}_1, \mathscr{A}_2, \ldots \mathscr{A}_N$ are *satisfiable* in SL iff there is some valuation on which $\mathscr{A}_1, \mathscr{A}_2, \ldots \mathscr{A}_N$ are all true.

Joint Possibility and Satisfiability

Joint Possibility

$\mathscr{A}_1, \mathscr{A}_2, \ldots \mathscr{A}_N$ are *jointly possible* iff there is some possibility in which $\mathscr{A}_1, \mathscr{A}_2, \ldots \mathscr{A}_N$ are all true.

Satisfiability in PL

$\mathscr{A}_1, \mathscr{A}_2, \ldots \mathscr{A}_N$ are *satisfiable* in PL iff there's some interpretation on which $\mathscr{A}_1, \mathscr{A}_2, \ldots \mathscr{A}_N$ are all true

Joint Possibility and Satisfiability

Joint Impossibility

$\mathscr{A}_1, \mathscr{A}_2, \ldots \mathscr{A}_N$ are *jointly impossible* iff there is no possibility in which $\mathscr{A}_1, \mathscr{A}_2, \ldots \mathscr{A}_N$ are all true.

Joint Possibility and Satisfiability

Joint Impossibility

$\mathscr{A}_1, \mathscr{A}_2, \ldots \mathscr{A}_N$ are *jointly impossible* iff there is no possibility in which $\mathscr{A}_1, \mathscr{A}_2, \ldots \mathscr{A}_N$ are all true.

Unsatisfiability in SL

$\mathscr{A}_1, \mathscr{A}_2, \ldots \mathscr{A}_N$ are *satisfiable* in SL iff there is no valuation on which $\mathscr{A}_1, \mathscr{A}_2, \ldots \mathscr{A}_N$ are all true.

Joint Possibility and Satisfiability

Joint Impossibility

$\mathscr{A}_1, \mathscr{A}_2, \ldots \mathscr{A}_N$ are *jointly impossible* iff there is no possibility in which $\mathscr{A}_1, \mathscr{A}_2, \ldots \mathscr{A}_N$ are all true.

Unsatisfiability in PL

$\mathscr{A}_1, \mathscr{A}_2, \ldots \mathscr{A}_N$ are *unsatisfiable* in PL iff there's no interpretation on which $\mathscr{A}_1, \mathscr{A}_2, \ldots \mathscr{A}_N$ are all true

Terminology

- For the remainder of the course, whenever we say ‘satisfiable’ or ‘unsatisfiable’, we will mean ‘un/satisfiable *in PL*’.

Joint (Im)possibility and (Un)satisfiability

- If $\mathscr{A}_1, \mathscr{A}_2, \ldots \mathscr{A}_N$ are jointly possible, then $\mathscr{A}_1, \mathscr{A}_2, \ldots \mathscr{A}_N$ are satisfiable

Joint (Im)possibility and (Un)satisfiability

- If $\mathscr{A}_1, \mathscr{A}_2, \ldots \mathscr{A}_N$ are jointly possible, then $\mathscr{A}_1, \mathscr{A}_2, \ldots \mathscr{A}_N$ are satisfiable
- Equivalently: if $\mathscr{A}_1, \mathscr{A}_2, \ldots \mathscr{A}_N$ unsatisfiable, then $\mathscr{A}_1, \mathscr{A}_2, \ldots \mathscr{A}_N$ are jointly impossible.

Joint (Im)possibility and (Un)satisfiability

- If $\mathscr{A}_1, \mathscr{A}_2, \ldots \mathscr{A}_N$ are jointly possible, then $\mathscr{A}_1, \mathscr{A}_2, \ldots \mathscr{A}_N$ are satisfiable
- Equivalently: if $\mathscr{A}_1, \mathscr{A}_2, \ldots \mathscr{A}_N$ unsatisfiable, then $\mathscr{A}_1, \mathscr{A}_2, \ldots \mathscr{A}_N$ are jointly impossible.
- However, just because a collection of sentences is satisfiable, it doesn't follow that they are jointly possible.

Joint Possibility and Satisfiability

Sabeen is taller than Luella

Luella is taller than Sabeen

Joint Possibility and Satisfiability

Tsl

Luella is taller than Sabeen

Joint Possibility and Satisfiability

Tsl

Ssl

Joint Possibility and Satisfiability

Tsl

Ssl

$$\begin{aligned} \text{domain} &: 1 \\ T &: \langle\, 1,1 \,\rangle \\ s &: 1 \\ l &: 1 \end{aligned}$$

Joint Possibility and Satisfiability

Tsl [T]

Ssl

$$\begin{aligned} \text{domain} &: 1 \\ T &: \langle\, 1,1 \,\rangle \\ s &: 1 \\ l &: 1 \end{aligned}$$

Joint Possibility and Satisfiability

Tsl [T]

Ssl [T]

$$\begin{aligned} \text{domain} &: 1 \\ T &: \langle 1,1 \rangle \\ s &: 1 \\ l &: 1 \end{aligned}$$

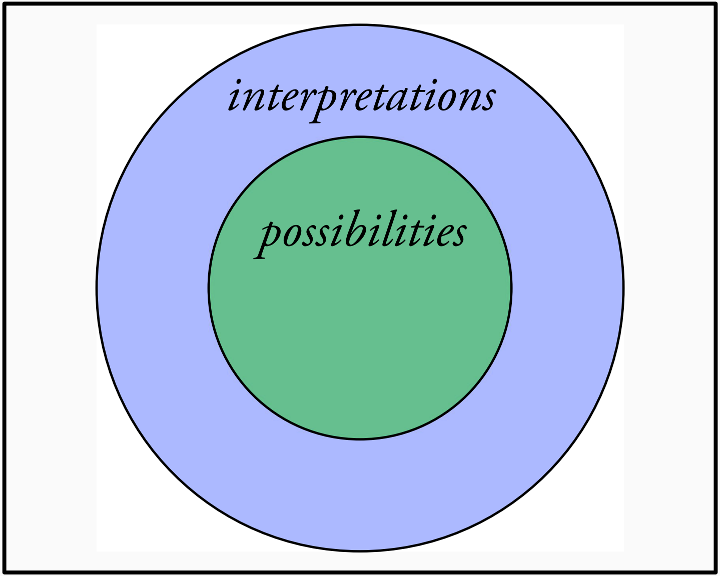
Joint Possibility and Satisfiability

- All *possibilities* are represented in some *interpretation*

Joint Possibility and Satisfiability

- All *possibilities* are represented in some *interpretation*
- But not all *interpretations* correspond to some *possibility*

Joint Possibility and Satisfiability



Joint Possibility and Satisfiability

- If we know something about *every* interpretation, then we know something about every possibility.

Joint Possibility and Satisfiability

- If we know something about *every* interpretation, then we know something about every possibility.
- But, just because we know something about *some* interpretation, that doesn't tell us that anything about any possibility.

Joint Possibility and Satisfiability

- Unsatisfiable $\Rightarrow$ Jointly Impossible

Joint Possibility and Satisfiability

- Unsatisfiable $\Longrightarrow$ Jointly Impossible
- Satisfiable $\not\Longrightarrow$ Jointly Possible

Proving that Some Sentences are Satisfiable

$$\neg \quad \forall x\, Lxx \quad , \quad \exists y \forall x\, Lxy$$

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$$\begin{aligned} \text{domain} &: 1, 2 \\ L &: \langle\, 1, 2 \,\rangle, \langle\, 2, 2 \,\rangle \end{aligned}$$

Proving that Some Sentences are Satisfiable

$$\neg \quad \forall x\, Lxx \quad , \quad \exists y \forall x\, Lxy$$

F

domain : 1, 2

L : $\langle$ 1, 2 $\rangle$, $\langle$ 2, 2 $\rangle$

Proving that Some Sentences are Satisfiable

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$T \qquad F$

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$$\begin{aligned} \text{domain} &: \text{Abelard, Heloise} \\ R &: \langle \text{A, H} \rangle , \langle \text{H, H} \rangle \end{aligned}$$

Proving that Some Sentences are Satisfiable

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$$\color{red}{F}$$

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Tautologies and Contradictions in PL

Necessary Truth and Tautology

Necessary Truth

A sentence $\mathscr{A}$ is a *necessary truth* iff it is true in every possibility.

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Tautology in SL

$\mathscr{A}$ is a *tautology* in SL iff it is true in every valuation.

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$\mathscr{A}$ is a *tautology* in PL iff it is true in every interpretation.

Necessary Falsehood and Contradiction

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Necessary Falsehood and Contradiction

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$\mathscr{A}$ is a *contradiction* in SL iff it is false in every valuation.

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A sentence $\mathscr{A}$ is a *necessary truth* iff it is false in every possibility.

Contradiction in PL

$\mathscr{A}$ is a *contradiction* in PL iff it is false in every interpretation.

Contingency

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A sentence $\mathscr{A}$ is a *contingency* iff it is true in some possibility and false in some possibility

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Neither a Tautology nor a Contradiction in SL

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Neither a Tautology nor a Contradiction in PL

A sentence $\mathscr{A}$ is *neither a tautology nor a contradiction* in PL iff it is true in some interpretation and false in some interpretation

Terminology

- For the remainder of the course, whenever we say ‘tautology’, ‘contradiction’, or ‘neither a tautology nor a contradiction’, we will mean *in PL*.

Tautology/Contradiction and Necessary Truth/Falsehood

- If $\mathscr{A}$ is a tautology, then it is a necessary truth

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Tautology/Contradiction and Necessary Truth/Falsehood

- If $\mathscr{A}$ is a tautology, then it is a necessary truth
- If $\mathscr{A}$ is a contradiction, then it is a necessary falsehood
- However, just because a sentence is neither a tautology nor a contradiction, it doesn't follow that it is a contingency.

Tautology/Contradiction and Necessary Truth/Falsehood

Someone is taller than themselves.

Tautology/Contradiction and Necessary Truth/Falsehood

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$$\begin{aligned} \text{domain} &: \text{people} \\ Txy &: \_\_\__x \text{ is taller than } \_\_\__y \end{aligned}$$

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$$\exists x Txx$$

Tautology/Contradiction and Necessary Truth/Falsehood

Someone is taller than themselves.

$$\begin{aligned} \text{domain} &: 1 \\ T &: \langle\, 1{,}1 \,\rangle \end{aligned}$$

$$\exists x Txx$$

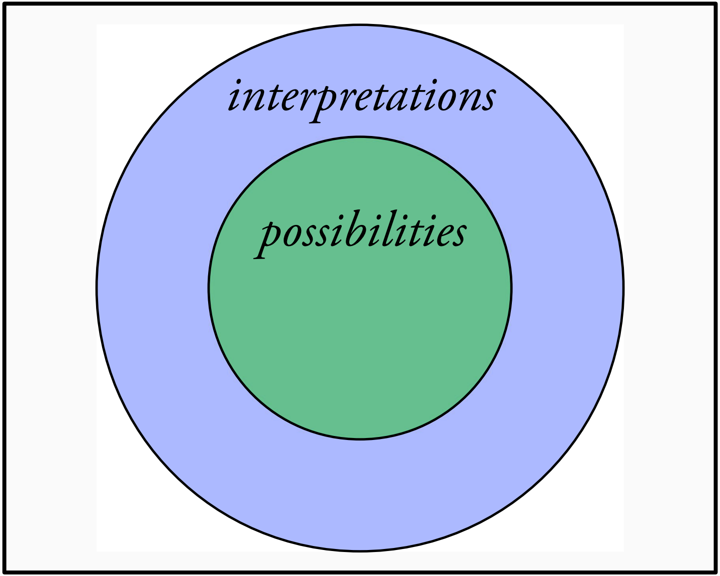
Tautology/Contradiction and Necessary Truth/Falsehood

Someone is taller than themselves.

$$\begin{aligned} \text{domain} &: 1 \\ T &: \langle\, 1{,}1 \,\rangle \end{aligned}$$

$$\exists x T x x \qquad [\mathrm{T}]$$

Tautology/Contradiction and Necessary Truth/Falsehood



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- Tautology $\Rightarrow$ Necessary Truth
- Contradiction $\Rightarrow$ Necessary Falsehood
- Neither a tautology nor a contradiction $\not\Rightarrow$ Contingency

Showing Neither a Tautology Nor a Contradiction

Someone hates everyone who loves them

Showing Neither a Tautology Nor a Contradiction

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$$\begin{aligned} \text{domain} &: \text{people} \\ Lxy &: \underline{\hspace{2em}}_x \text{ loves } \underline{\hspace{2em}}_y \\ Hxy &: \underline{\hspace{2em}}_x \text{ hates } \underline{\hspace{2em}}_y \end{aligned}$$

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$$\exists x \forall y (Lyx \rightarrow Hxy)$$

Showing Neither a Tautology Nor a Contradiction

Someone hates everyone who loves them

$$\begin{aligned} \text{domain} &: 1, 2 \\ L &: \langle\ 2,1\ \rangle, \langle\ 1, 1\ \rangle \\ H &: \langle\ 1, 2\ \rangle, \langle\ 1,1\ \rangle \end{aligned}$$

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$\exists x \forall y(Lyx \rightarrow Hxy)$ [T]

Showing Neither a Tautology Nor a Contradiction

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Showing Neither a Tautology Nor a Contradiction

Every barber shaves all and only those who don't shave themselves

Showing Neither a Tautology Nor a Contradiction

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$$\begin{aligned} \text{domain} &: \text{people} \\ Bx &: \underline{\quad}_x \text{ is a barber} \\ Sxy &: \underline{\quad}_x \text{ shaves } \underline{\quad}_y \end{aligned}$$

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$$\forall x\,[Bx \rightarrow \forall y(Sxy \leftrightarrow \neg Syy)]$$

Showing Neither a Tautology Nor a Contradiction

Every barber shaves all and only those who don't shave themselves

$$\begin{aligned} \text{domain} &: \text{Billy, Susie} \\ B &: \text{Billy} \\ S &: \langle \text{ Billy, Susie } \rangle \end{aligned}$$

$$\forall x \, [Bx \longrightarrow \forall y (Sxy \longleftrightarrow \neg Syy)]$$

Showing Neither a Tautology Nor a Contradiction

Every barber shaves all and only those who don't shave themselves

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Contradiction

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Contradiction

Some barber shaves all and only those who don't shave themselves

$$\begin{aligned} \text{domain} &: 1 \\ B &: \\ S &: \end{aligned}$$

$$\exists x[Bx \wedge \forall y(Sxy \longleftrightarrow \neg Syy)]$$

Contradiction

Some barber shaves all and only those who don't shave themselves

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$$
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S &: \\
x &: 1
\end{aligned}
$$

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B : 1

S :

x : 1

y : 1

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Contradiction

Some barber shaves all and only those who don't shave themselves

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Natural Deduction for PL

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Natural Deduction for PL

- To show that...
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- we will provide a *natural deduction proof* (in PL)

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- The natural deduction system for PL includes *all the rules from SL*

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Natural Deduction for PL

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  - Existential Introduction ($\exists I$)
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