Predicate Logic

Syntax

PHIL 500

Outline

- Syntax for PL
  - Free and Bound Variables
  - Important Syntactic Features in PL
- Semantics for PL
  - Interpretations
  - Truth on an Interpretation
  - Four Important Statement Forms
  - Order of Quantifiers

Syntax for PL

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Free and Bound Variables

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Free and Bound Variables

$$\forall w(\exists y\, Lwy \rightarrow \exists w\, Aw)$$

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In a sentence of the form $\forall x \mathscr{A}$ or $\exists x \mathscr{A}$, the quantifier binds every *free* occurrence of x in $\mathscr{A}$. If an occurrence of x in $\mathscr{A}$ is already bound, then the quantifier does not bind it.

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- E.g., in

$$\exists x \forall x Fx$$

  the variable '*x*' is bound by the *universal* quantifier '$\forall x$'. It is *not* bound by the existential quantifier '$\exists x$'.

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- When translating into PL, we want our translations to be *closed*.

Free and Bound Variables

$$\exists x\, Lxy \wedge \forall y\, Lyx$$

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Free and Bound Variables

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Free and Bound Variables

$$\forall x\, Ax \land Bx$$

Free and Bound Variables

$$\forall x(Ax \wedge Bx) \wedge \forall y(Cx \wedge Dy)$$

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Free and Bound Variables

$$\forall x\, \exists y\, [Rxy \to (Jz \land Kx)] \lor Ryx$$

Free and Bound Variables

$$\forall x \, \exists y \, [Rxy \to (Jz \land Kx)] \lor Ryx$$

Free and Bound Variables

$$\forall x \, \exists y \, [Rxy \rightarrow (Jz \wedge Kx)] \vee Ryx$$

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Free and Bound Variables

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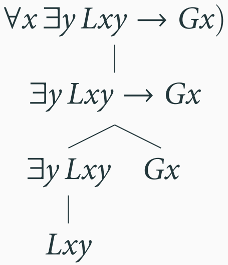
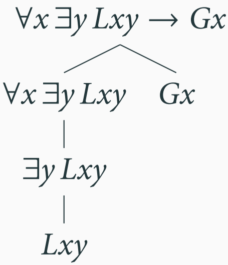
Syntax for PL

Important Syntactic Features in PL

Parenthases





Term Order

$$\forall x\, \exists y\, Lxy$$
$$|$$
$$\exists y\, Lxy$$
$$|$$
$$Lxy$$

$$\forall x\, \exists y\, Lyx$$
$$|$$
$$\exists y\, Lyx$$
$$|$$
$$Lyx$$

Quantifier Order

$$\exists y \, \forall x \, Lxy$$
$$|$$
$$\forall x \, Lxy$$
$$|$$
$$Lxy$$

$$\forall x \, \exists y \, Lxy$$
$$|$$
$$\exists y \, Lxy$$
$$|$$
$$Lxy$$

Predicate Logic

Semantics

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Semantics for PL

Semantics for PL

Interpretations

Semantics for SL

A VALUATION is an assignment of truth-value—either true or false—to every statement letter of SL.

- Given a valuation, we can determine whether sentences of SL are true or false.

Semantics for SL

- An argument of SL is ***an entailment*** iff every valuation which makes all the premises true makes the conclusion true as well

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- An argument of SL is ***an entailment*** iff every valuation which makes all the premises true makes the conclusion true as well
- A collection of sentences of SL are ***consistent*** iff there is some valution which makes all of the sentences true.
- A sentence is a ***tautology*** iff it is true on every valuation.

Semantics for PL

An INTERPRETATION, **I**, tells us:

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3) For every relevant predicate of PL, which things in the domain the predicate is true of

Semantics for PL

An INTERPRETATION, **I**, tells us:

1) What *domain* of things we are talking about
2) For each relevant term (name or variable), what thing in the domain it refers to
3) For every relevant predicate of PL, which things in the domain the predicate is true of

- Given an interpretation, we can determine whether sentences of PL are true or false.

An Interpretation

$$\forall z(Dz \rightarrow Hz),\ Do \quad \therefore Ho$$

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$$\begin{aligned} \text{Domain} &: \text{all people} \\ Hx &: \underline{\quad}_x \text{ is happy} \\ Dx &: \underline{\quad}_x \text{ has a dog} \\ o &: \text{Obama} \end{aligned}$$

An Interpretation

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$$\forall z(Dz \rightarrow Hz),\ Do \quad \therefore Ho$$

$$\begin{aligned} \text{Domain} &: \text{Adam, Betty, and Carl} \\ H &: \text{Adam and Betty} \\ D &: \text{Adam} \\ o &: \text{Adam} \end{aligned}$$

An Interpretation

$$\forall z(Dz \rightarrow Hz),\ Do \quad \therefore\ Ho$$

$$\begin{aligned} \text{Domain} &: 1, 2, \text{and } 3 \\ H &: 1 \text{ and } 2 \\ D &: 1 \\ o &: 1 \end{aligned}$$

Interpretations

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Interpretations

- There are two ways to specify what things in the domain a predicate of PL is true of:
  - provide a gappy English sentence, with the understanding that the predicate is true of whatever makes *that sentence* true.
  - provide a list of things from the domain, with the understanding that the predicate is true of those and only those things.
- Either method is acceptable, but if you use the former: ***make sure your grader knows which things in the domain make the sentence true***.

Interpretations

$$\exists z Hz \rightarrow Fa$$

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$$\begin{aligned} \text{Domain} &: \text{Sabeen and Matthew} \\ Hx &: \underline{\quad}_x \ \text{lives in New York} \\ Fx &: \underline{\quad}_x \ \text{lives in London} \\ a &: \text{Sabeen} \end{aligned}$$

2-place Predicates

- When you're specifying which things satisfy a 1-place predicate, you can just *list* them.

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- When you're specifying which things satisfy a 1-place predicate, you can just *list* them.
- When you're specifying which things satisfy a 2-place predicate, you need to list them, *in order*.

2-place Predicates

$$\forall x Lax \land \exists y Lya$$

2-place Predicates

$$\forall x Lax \wedge \exists y Lya$$

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2-place Predicates

$$\forall x Lax \wedge \exists y Lya$$

$$\begin{aligned} \text{Domain} &: 1 \text{ and } 2 \\ L &: \langle\ 1, 2\ \rangle, \langle\ 1, 1\ \rangle \\ a &: 1 \end{aligned}$$

Semantics for PL

Truth on an Interpretation

Atomic Truth

- An *atomic* sentence is an N-place predicate followed by N terms.

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An atomic sentence '$\mathscr{R} t_1 t_2 \ldots t_{\mathcal{N}}$' is true on an interpretation **I** iff, according to **I**, $\mathscr{R}$ is true of the objects referred to by $t_1, t_2, \ldots, t_{\mathcal{N}}$ (in that order).

Atomic Truth

Ha

Atomic Truth

$$Ha$$

$$\begin{aligned} \text{Domain} &: \text{Adam, Betty, and Carl} \\ H &: \text{Adam and Betty} \\ D &: \text{Adam} \\ a &: \text{Adam} \end{aligned}$$

Atomic Truth

Ha [T]

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$$\begin{aligned} \text{Domain} &: 1, 2, 3, 4, 5, \ldots \\ Lxy &: \_\_\__x \text{ is greater than } \_\_\__y \\ a &: 1 \\ x &: 3 \end{aligned}$$

Atomic Truth

Lxa [T]

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→) '$(\mathscr{A} \rightarrow \mathscr{B})$' is true on **I** iff either '$\mathscr{A}$' is false on **I** or '$\mathscr{B}$' is true on **I**

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- →) '$(\mathscr{A} \to \mathscr{B})$' is true on **I** iff either '$\mathscr{A}$' is false on **I** or '$\mathscr{B}$' is true on **I**
- ↔) '$(\mathscr{A} \leftrightarrow \mathscr{B})$' is true on **I** iff '$\mathscr{A}$' and '$\mathscr{B}$' have the same truth-value on **I**

Truth on an Interpretation

$$Lax \;\rightarrow\; \neg\, Laa$$

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Truth on an Interpretation

$$Lax \rightarrow \neg Laa$$

T

Domain : Sammy and Tammy

L : ⟨ Sammy, Tammy ⟩ , ⟨ Sammy, Sammy ⟩

a : Sammy

x : Tammy

Truth on an Interpretation

$$Lax \;\rightarrow\; \neg\, Laa$$

$$\color{green}{T} \qquad\qquad \color{green}{T}$$

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$$T \qquad\qquad F \quad T$$

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$$T$$

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T F

Domain : Adam, Betty, and Carl

H : Adam and Betty

D : Adam

a : Adam

c : Carl

Truth on an Interpretation

$$\begin{array}{ccccccccc} (Ha & \vee & Dc) & \leftrightarrow & (Ha & \rightarrow & Dc) \\ \color{green}{T} & & \color{red}{F} & & \color{green}{T} & & \end{array}$$

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Truth on an Interpretation

$$(Ha \vee Dc) \leftrightarrow (Ha \rightarrow Dc)$$

$$T \quad F \quad T \quad F$$

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Truth on an Interpretation

$$(Ha \quad \vee \quad Dc) \quad \leftrightarrow \quad (Ha \quad \rightarrow \quad Dc)$$

$$T \quad T \quad F \qquad\qquad T \qquad\quad F$$

Domain : Adam, Betty, and Carl

H : Adam and Betty

D : Adam

a : Adam

c : Carl

Truth on an Interpretation

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Modified Interpretations

The interpretation **I**:

$$\begin{aligned} \text{Domain} &: \text{Sammy and Tammy} \\ L &: \langle \text{ Sammy, Tammy } \rangle \text{ , } \langle \text{ Sammy, Sammy } \rangle \\ a &: \text{Sammy} \\ x &: \text{Tammy} \end{aligned}$$

Modified Interpretations

The interpretation $\mathbf{I}[x : \text{Sammy}]$

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Modified Interpretations

The interpretation **I**:

$$\begin{aligned} \text{Domain} &: 1, 2, 3, 4, \text{and } 5 \\ Rxy &: \langle\, 1, 2 \,\rangle, \langle\, 2, 5 \,\rangle, \langle\, 5, 1 \,\rangle, \langle\, 4, 4 \,\rangle \\ F &: 4, 5 \\ a &: 1 \end{aligned}$$

Modified Interpretations

The interpretation $\mathbf{I}[x : 3]$

$$\begin{aligned} \text{Domain} &: 1, 2, 3, 4, \text{and } 5 \\ Rxy &: \langle 1, 2 \rangle, \langle 2, 5 \rangle, \langle 5, 1 \rangle, \langle 4, 4 \rangle \\ F &: 4, 5 \\ a &: 1 \\ x &: 3 \end{aligned}$$

Modified Interpretations

The interpretation $\mathbf{I}[x : 3, y : 5]$:

$$
\begin{aligned}
\text{Domain} &: 1, 2, 3, 4, \text{and } 5 \\
Rxy &: \langle\, 1, 2 \,\rangle, \langle\, 2, 5 \,\rangle, \langle\, 5, 1 \,\rangle, \langle\, 4, 4 \,\rangle \\
F &: 4, 5 \\
a &: 1 \\
x &: 3 \\
y &: 5
\end{aligned}
$$

Modified Interpretations

If $\mathbf{I}$ is an interpretation, d is something in the domain of $\mathbf{I}$, and x is a variable, then the modified interpretation $\mathbf{I}[x : d]$ is the interpretation which is exactly like $\mathbf{I}$, except that the variable x refers to d.

Truth on an Interpretation

∃) '$\exists x\ \mathscr{A}$' is true on $\mathbf{I}$ iff '$\mathscr{A}$' is true on $\mathbf{I}[x : d]$ for ***some*** d in the domain of $\mathbf{I}$

Truth on an Interpretation

∃) '$\exists x\ \mathscr{A}$' is true on $\mathbf{I}$ iff '$\mathscr{A}$' is true on $\mathbf{I}[x : d]$ for ***some*** d in the domain of $\mathbf{I}$

∀) '$\forall x\ \mathscr{A}$' is true on $\mathbf{I}$ iff '$\mathscr{A}$' is true on $\mathbf{I}[x : d]$ for ***every*** d in the domain of $\mathbf{I}$

Truth on an Interpretation

$\exists w\, Fw$

$|$

Fw

I

Domain	: Amy and Bruce
F	: Bruce

Truth on an Interpretation

$$\exists w\, Fw$$
$$|$$
$$Fw$$

$\underline{\mathbf{I}[w : \text{Amy}]}$

Domain : Amy and Bruce

F : Bruce

w : Amy

Truth on an Interpretation

$\exists w\, Fw$

Fw [F]

$\underline{\mathbf{I}[w : \text{Amy}]}$

Domain : Amy and Bruce

F : Bruce

w : Amy

Truth on an Interpretation

$$\exists w\, Fw$$

$$|$$

$$Fw \quad [\text{F}]$$

$$\underline{\mathbf{I}[w : \text{ Bruce}]}$$

Domain : Amy and Bruce

F : Bruce

w : Bruce

Truth on an Interpretation

$$\exists w\, Fw$$

$$|$$

$$Fw\ [\mathrm{F}, \mathrm{T}]$$

$$\underline{\mathbf{I}[w :\ \text{Bruce}]}$$

$$\begin{aligned} \text{Domain} &: \text{Amy and Bruce} \\ F &: \text{Bruce} \end{aligned}$$

Truth on an Interpretation

$\exists w\, Fw$

Fw [F, T]

I

Domain : Amy and Bruce

F : Bruce

Truth on an Interpretation

$\exists w\, Fw$ [T]

|

Fw [F, T]

I

Domain : Amy and Bruce

F : Bruce

Truth on an Interpretation

$$\forall y\, Fy$$

$$|$$

$$Fy$$

I	
Domain	: Amy and Bruce
F	: Bruce

Truth on an Interpretation

$$\forall y\, Fy$$

$$|$$

$$Fy$$

$\mathbf{I}[y : \text{Amy}]$

Domain : Amy and Bruce

F : Bruce

y : Amy

Truth on an Interpretation

$$\forall y\, Fy$$

$$|$$

$$Fy \quad [\mathrm{F}]$$

$\underline{\mathbf{I}[y : \text{Amy}]}$

Domain : Amy and Bruce

F : Bruce

y : Amy

Truth on an Interpretation

$$\forall y\, Fy$$

$$|$$

$$Fy \quad [\text{F}]$$

$\underline{\mathbf{I}[y : \text{Bruce}]}$

Domain : Amy and Bruce

F : Bruce

y : Bruce

Truth on an Interpretation

$$\forall y\, Fy$$

$$|$$

$$Fy\ [\text{F}, \text{T}]$$

$$\underline{\mathbf{I}[y:\ \text{Bruce}]}$$

$$\text{Domain} : \text{Amy and Bruce}$$
$$F : \text{Bruce}$$

Truth on an Interpretation

$\forall y\, Fy$

|

Fy [F, T]

I

Domain : Amy and Bruce

F : Bruce

Truth on an Interpretation

$\forall y\, Fy$ [F]

|

Fy [F, T]

I

Domain : Amy and Bruce

F : Bruce

Predicate Logic

Semantics

PHIL 500

Outline

- Syntax for PL
  - Free and Bound Variables
  - Important Syntactic Features in PL
- Semantics for PL
  - Interpretations
  - Truth on an Interpretation
  - Four Important Statement Forms
  - Order of Quantifiers

Modified Interpretations

If $\mathbf{I}$ is an interpretation, d is something in the domain of $\mathbf{I}$, and x is a variable, then the modified interpretation $\mathbf{I}[x : d]$ is the interpretation which is exactly like $\mathbf{I}$, except that the variable x refers to d.

Truth on an Interpretation

∃) '$\exists x\ \mathscr{A}$' is true on $\mathbf{I}$ iff '$\mathscr{A}$' is true on $\mathbf{I}[x : d]$ for ***some*** d in the domain of $\mathbf{I}$

Truth on an Interpretation

∃) '$\exists x\ \mathscr{A}$' is true on $\mathbf{I}$ iff '$\mathscr{A}$' is true on $\mathbf{I}[x : d]$ for ***some*** d in the domain of $\mathbf{I}$

∀) '$\forall x\ \mathscr{A}$' is true on $\mathbf{I}$ iff '$\mathscr{A}$' is true on $\mathbf{I}[x : d]$ for ***every*** d in the domain of $\mathbf{I}$

Semantics for PL

Four Important Statement Forms

Truth on an Interpretation

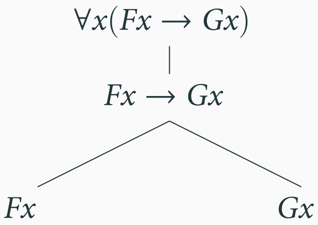
$$\forall x(Fx \rightarrow Gx)$$

$$Fx \rightarrow Gx$$

$$Fx \qquad\qquad Gx$$

Domain : Amy and Bruce
F : Bruce
G : Amy and Bruce

Truth on an Interpretation



Domain : Amy and Bruce

F : Bruce

G : Amy and Bruce

x : Amy

Truth on an Interpretation

$$\forall x(Fx \rightarrow Gx)$$

$$Fx \rightarrow Gx$$

Fx [F] $\qquad$ Gx [T]

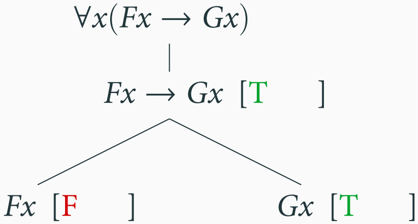
Domain : Amy and Bruce

F : Bruce

G : Amy and Bruce

x : Amy

Truth on an Interpretation



Domain : Amy and Bruce

F : Bruce

G : Amy and Bruce

x : Amy

Truth on an Interpretation

$$\forall x(Fx \rightarrow Gx)$$

$$Fx \rightarrow Gx \;\; [\text{T} \quad]$$

$$Fx \;\; [\text{F} \quad] \qquad\qquad Gx \;\; [\text{T} \quad]$$

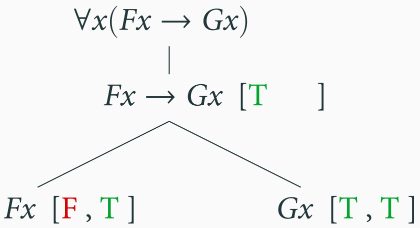
Domain : Amy and Bruce

F : Bruce

G : Amy and Bruce

x : Bruce

Truth on an Interpretation



Domain : Amy and Bruce

F : Bruce

G : Amy and Bruce

x : Bruce

Truth on an Interpretation

$\forall x(Fx \rightarrow Gx)$

$Fx \rightarrow Gx$ [T , T]

Fx [F , T]

Gx [T , T]

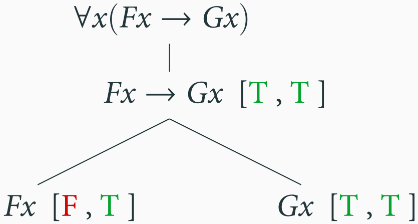
Domain : Amy and Bruce

F : Bruce

G : Amy and Bruce

x : Bruce

Truth on an Interpretation



Domain : Amy and Bruce

F : Bruce

G : Amy and Bruce

Truth on an Interpretation

$$\forall x(Fx \rightarrow Gx)\ \ [\mathrm{T}]$$

$$Fx \rightarrow Gx\ \ [\mathrm{T}, \mathrm{T}]$$

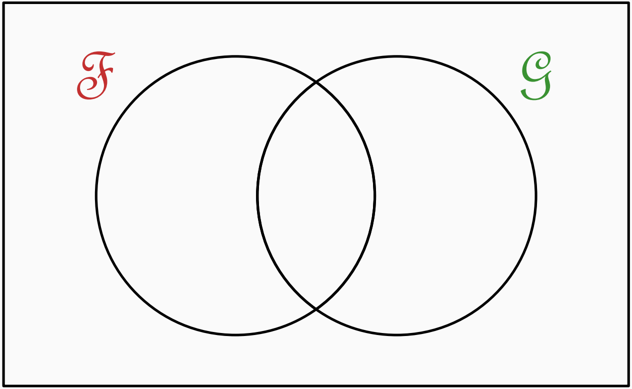
$$Fx\ \ [\mathrm{F}, \mathrm{T}] \qquad\qquad Gx\ \ [\mathrm{T}, \mathrm{T}]$$

Domain : Amy and Bruce

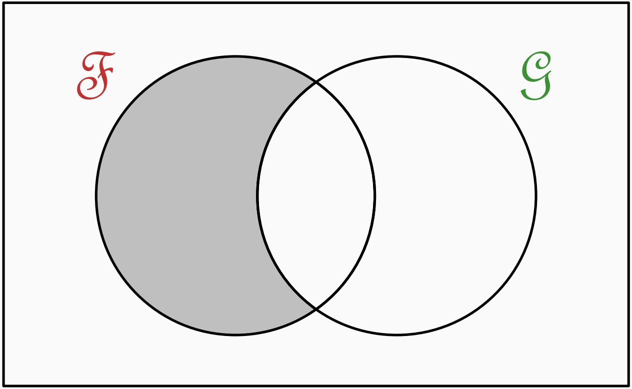
F : Bruce

G : Amy and Bruce

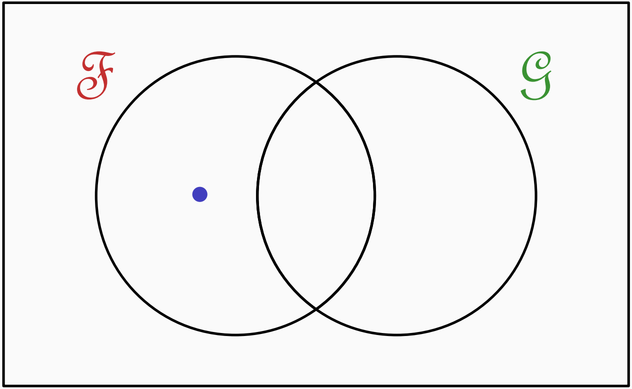
All $\mathscr{F}$s are $\mathscr{G}$s



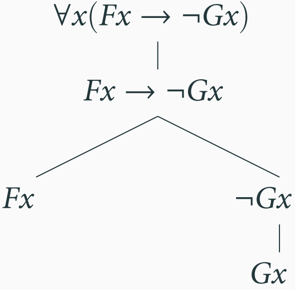
All $\mathcal{F}$s are $\mathcal{G}$s



All $\mathcal{F}$s are $\mathcal{G}$s

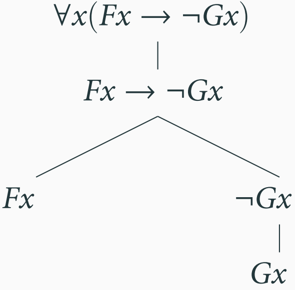


Truth on an Interpretation



Domain	:	Amy and Bruce
F	:	Bruce
G	:	Amy and Bruce

Truth on an Interpretation



Domain : Amy and Bruce

F : Bruce

G : Amy and Bruce

x : Amy

Truth on an Interpretation

$\forall x(Fx \rightarrow \neg Gx)$

$Fx \rightarrow \neg Gx$

Fx [F] $\neg Gx$ [F]

Gx [T]

Domain	: Amy and Bruce
F	: Bruce
G	: Amy and Bruce
x	: Amy

Truth on an Interpretation

$\forall x(Fx \rightarrow \neg Gx)$

$Fx \rightarrow \neg Gx$ [T]

Fx [F]

$\neg Gx$ [F]

Gx [T]

Domain : Amy and Bruce

F : Bruce

G : Amy and Bruce

x : Amy

Truth on an Interpretation

$\forall x(Fx \rightarrow \neg Gx)$

$Fx \rightarrow \neg Gx$ [T]

Fx [F]

$\neg Gx$ [F]

Gx [T]

Domain : Amy and Bruce

F : Bruce

G : Amy and Bruce

x : Bruce

Truth on an Interpretation

$\forall x(Fx \rightarrow \neg Gx)$

$Fx \rightarrow \neg Gx$ [T]

Fx [F , T]

$\neg Gx$ [F , F]

Gx [T , T]

Domain : Amy and Bruce

F : Bruce

G : Amy and Bruce

x : Bruce

Truth on an Interpretation

$$\forall x(Fx \rightarrow \neg Gx)$$

$$Fx \rightarrow \neg Gx \;\; [\mathrm{T}, \mathrm{F}]$$

$$Fx \;\; [\mathrm{F}, \mathrm{T}] \qquad\qquad \neg Gx \;\; [\mathrm{F}, \mathrm{F}]$$

$$Gx \;\; [\mathrm{T}, \mathrm{T}]$$

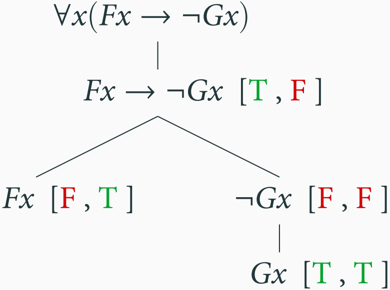
Domain : Amy and Bruce

F : Bruce

G : Amy and Bruce

x : Bruce

Truth on an Interpretation

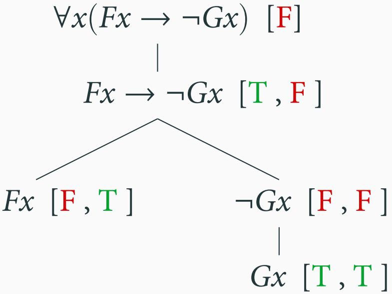


Domain : Amy and Bruce

F : Bruce

G : Amy and Bruce

Truth on an Interpretation

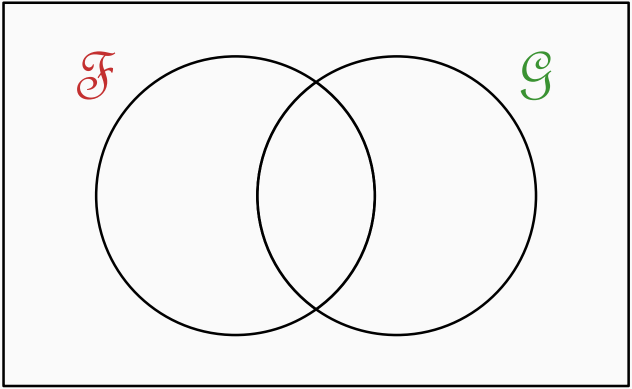


Domain : Amy and Bruce

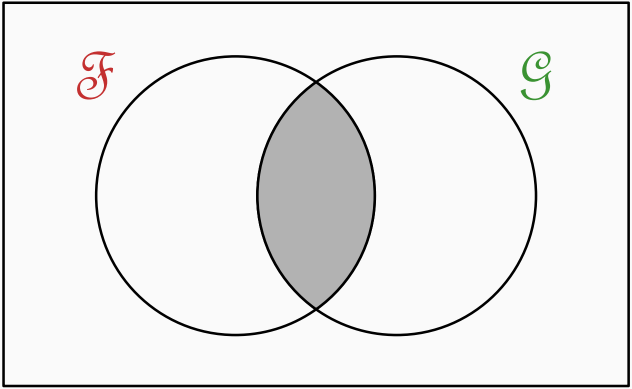
F : Bruce

G : Amy and Bruce

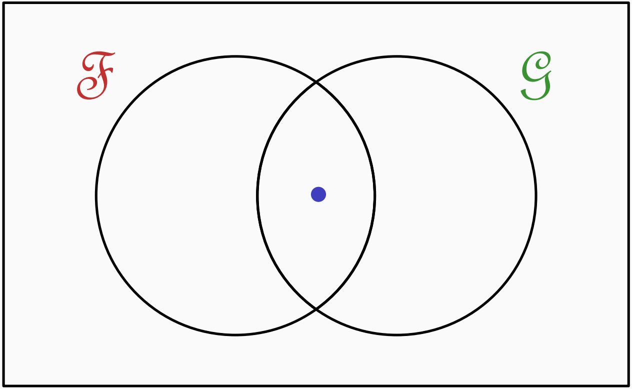
No $\mathscr{F}$s are $\mathscr{G}$s



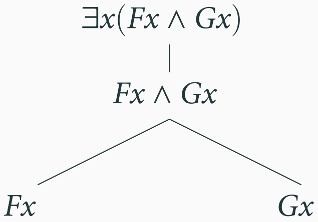
No $\mathscr{F}$s are $\mathscr{G}$s



No $\mathscr{F}$s are $\mathscr{G}$s



Truth on an Interpretation



Domain : Amy and Bruce

F : Bruce

G : Amy and Bruce

Truth on an Interpretation

$\exists x(Fx \land Gx)$

$Fx \land Gx$

Fx $\quad$ Gx

Domain : Amy and Bruce

F : Bruce

G : Amy and Bruce

x : Amy

Truth on an Interpretation

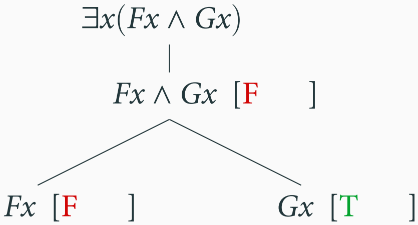
$\exists x(Fx \land Gx)$

$Fx \land Gx$

Fx [F] Gx [T]

Domain	: Amy and Bruce
F	: Bruce
G	: Amy and Bruce
x	: Amy

Truth on an Interpretation



Domain : Amy and Bruce

F : Bruce

G : Amy and Bruce

x : Amy

Truth on an Interpretation

$\exists x(Fx \wedge Gx)$

$Fx \wedge Gx$ [F]

Fx [F]

Gx [T]

Domain	:	Amy and Bruce
F	:	Bruce
G	:	Amy and Bruce
x	:	Bruce

Truth on an Interpretation

$$\exists x(Fx \wedge Gx)$$

$$Fx \wedge Gx \ [\mathrm{F} \quad]$$

$$Fx \ [\mathrm{F}, \mathrm{T}] \qquad\qquad Gx \ [\mathrm{T}, \mathrm{T}]$$

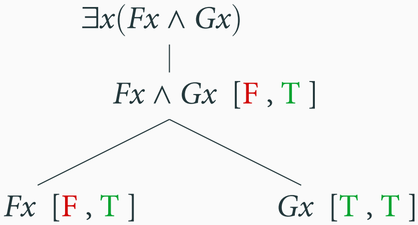
Domain : Amy and Bruce

F : Bruce

G : Amy and Bruce

x : Bruce

Truth on an Interpretation



Domain : Amy and Bruce

F : Bruce

G : Amy and Bruce

x : Bruce

Truth on an Interpretation

$\exists x(Fx \wedge Gx)$

$Fx \wedge Gx$ [F , T]

Fx [F , T]

Gx [T , T]

Domain : Amy and Bruce

F : Bruce

G : Amy and Bruce

Truth on an Interpretation

$\exists x(Fx \wedge Gx)$ [T]

$Fx \wedge Gx$ [F , T]

Fx [F , T]

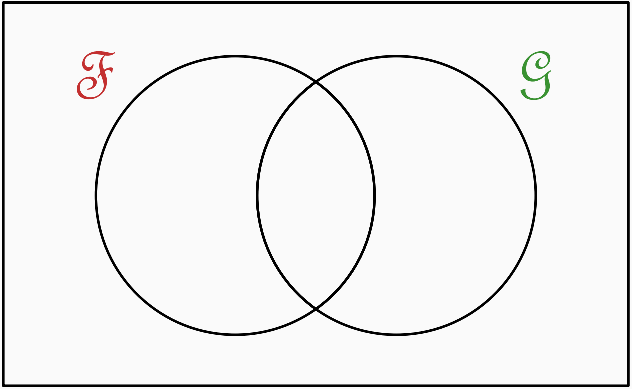
Gx [T , T]

Domain : Amy and Bruce

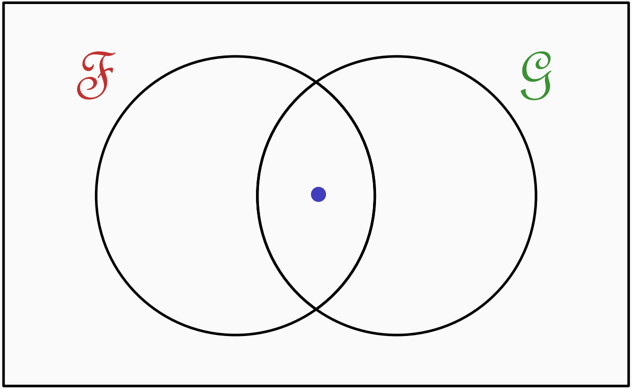
F : Bruce

G : Amy and Bruce

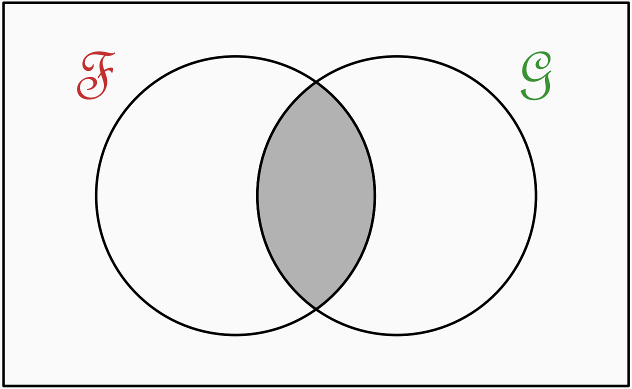
Some $\mathcal{F}$s are $\mathcal{G}$s



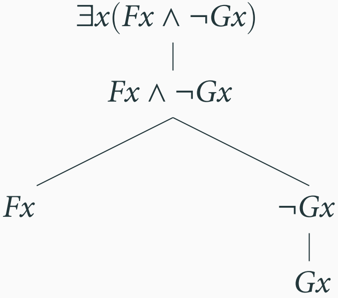
Some $\mathcal{F}$s are $\mathcal{G}$s



Some $\mathcal{F}$s are $\mathcal{G}$s



Truth on an Interpretation

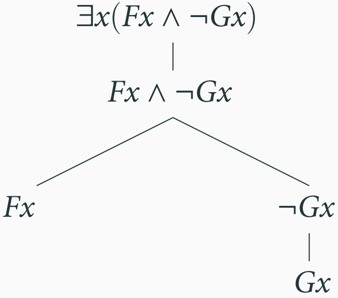


Domain : Amy and Bruce

F : Bruce

G : Amy and Bruce

Truth on an Interpretation



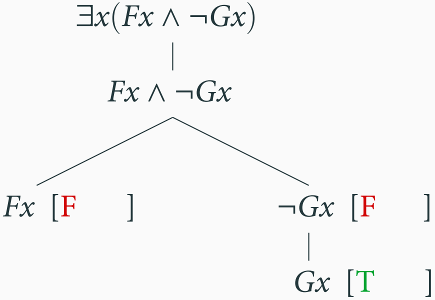
Domain : Amy and Bruce

F : Bruce

G : Amy and Bruce

x : Amy

Truth on an Interpretation



Domain : Amy and Bruce

F : Bruce

G : Amy and Bruce

x : Amy

Truth on an Interpretation

- $\exists x(Fx \wedge \neg Gx)$
  - $Fx \wedge \neg Gx$ [F]
    - Fx [F]
    - $\neg Gx$ [F]
      - Gx [T]

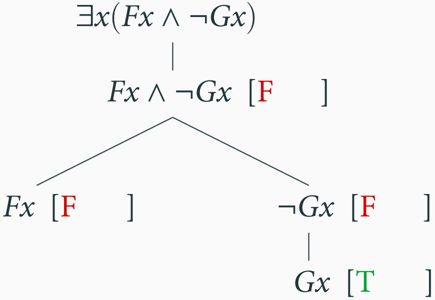
Domain : Amy and Bruce

F : Bruce

G : Amy and Bruce

x : Amy

Truth on an Interpretation



Domain : Amy and Bruce

F : Bruce

G : Amy and Bruce

x : Bruce

Truth on an Interpretation

$$\exists x(Fx \wedge \neg Gx)$$

$$Fx \wedge \neg Gx \quad [\mathrm{F} \qquad]$$

$$Fx \quad [\mathrm{F}, \mathrm{T}] \qquad\qquad \neg Gx \quad [\mathrm{F}, \mathrm{F}]$$

$$Gx \quad [\mathrm{T}, \mathrm{T}]$$

Domain : Amy and Bruce

F : Bruce

G : Amy and Bruce

x : Bruce

Truth on an Interpretation

$\exists x(Fx \wedge \neg Gx)$

$Fx \wedge \neg Gx$ [F , F]

Fx [F , T]

$\neg Gx$ [F , F]

Gx [T , T]

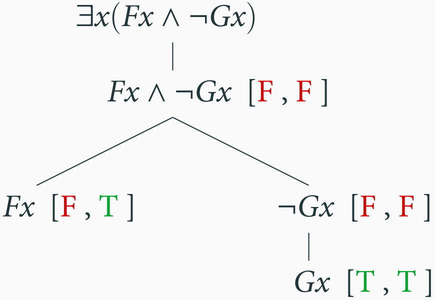
Domain : Amy and Bruce

F : Bruce

G : Amy and Bruce

x : Bruce

Truth on an Interpretation

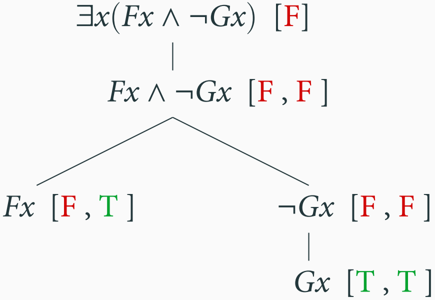


Domain : Amy and Bruce

F : Bruce

G : Amy and Bruce

Truth on an Interpretation

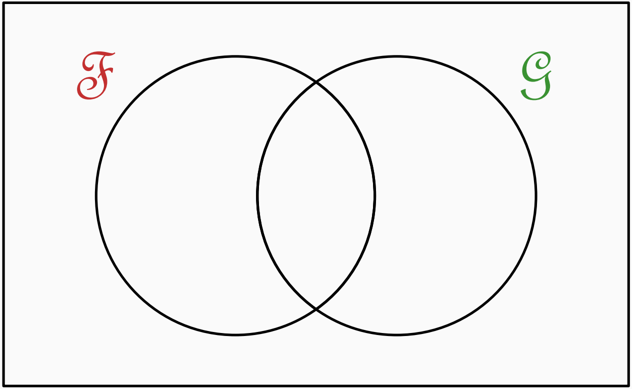


Domain : Amy and Bruce

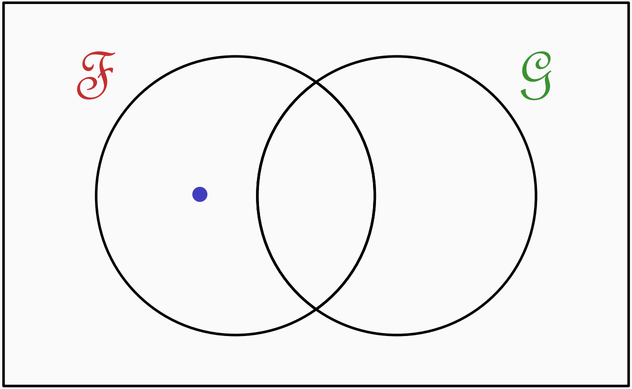
F : Bruce

G : Amy and Bruce

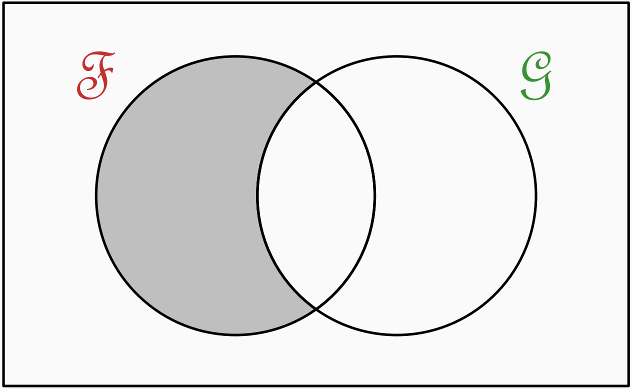
Some $\mathscr{F}$s are not $\mathscr{G}$s



Some $\mathcal{F}$s are not $\mathcal{G}$s



Some $\mathcal{F}$s are not $\mathcal{G}$s



Semantics for PL

Order of Quantifiers

Truth on an Interpretation

- $\exists x \mathscr{A}$ is true on $\mathbf{I}$ iff $\mathscr{A}$ is true on $\mathbf{I}[x : d]$ for ***some*** d in the domain of $\mathbf{I}$
- $\forall x \mathscr{A}$ is true on $\mathbf{I}$ iff $\mathscr{A}$ is true on $\mathbf{I}[x : d]$ for ***every*** d in the domain of $\mathbf{I}$

Order of Quantifiers

$$\begin{aligned} \text{domain} &: \text{all people} \\ L\,xy &: \_\_\__x \text{ loves } \_\_\__y \\ a &: \text{Abelard} \end{aligned}$$

Order of Quantifiers

$$\begin{aligned} \text{domain} &: \text{all people} \\ L\,xy &: \_\_\__x \text{ loves } \_\_\__y \\ a &: \text{Abelard} \end{aligned}$$

Abelard loves someone :

Order of Quantifiers

$$\begin{aligned} \text{domain} &: \text{all people} \\ Lxy &: \_\_\__x \text{ loves } \_\_\__y \\ a &: \text{Abelard} \end{aligned}$$

Abelard loves someone :

$$\exists y\, Lay$$

Order of Quantifiers

$$\begin{aligned} \text{domain} &: \text{all people} \\ L\,xy &: \_\_\__x \text{ loves } \_\_\__y \\ a &: \text{Abelard} \end{aligned}$$

Abelard loves someone :

$$\exists y\, Lay$$

Everyone loves someone :

Order of Quantifiers

$$\begin{aligned} \text{domain} &: \text{all people} \\ Lxy &: \_\_\__x \text{ loves } \_\_\__y \\ a &: \text{Abelard} \end{aligned}$$

Abelard loves someone :

$$\exists y\, Lay$$

Everyone loves someone :

$$\forall x\, \exists y\, Lxy$$

Order of Quantifiers

$$\begin{aligned} \text{domain} &: \text{all people} \\ L\,xy &: \underline{\quad\quad}_x \text{ loves } \underline{\quad\quad}_y \\ a &: \text{Abelard} \end{aligned}$$

Order of Quantifiers

$$\begin{aligned} \text{domain} &: \text{all people} \\ L\,xy &: \_\_\__x \text{ loves } \_\_\__y \\ a &: \text{Abelard} \end{aligned}$$

Abelard is loved by everyone :

Order of Quantifiers

$$\begin{aligned} \text{domain} &: \text{all people} \\ Lxy &: \_\_\__x \text{ loves } \_\_\__y \\ a &: \text{Abelard} \end{aligned}$$

Abelard is loved by everyone :

$$\forall x\, Lxa$$

Order of Quantifiers

$$\begin{aligned} \text{domain} &: \text{all people} \\ L\,xy &: \_\_\__x \text{ loves } \_\_\__y \\ a &: \text{Abelard} \end{aligned}$$

Abelard is loved by everyone :

$$\forall x\, Lxa$$

Someone is loved by everyone :

Order of Quantifiers

$$\begin{aligned} \text{domain} &: \text{all people} \\ L\,xy &: \_\_\__x \text{ loves } \_\_\__y \\ a &: \text{Abelard} \end{aligned}$$

Abelard is loved by everyone :

$$\forall x\, Lxa$$

Someone is loved by everyone :

$$\exists y\, \forall x\, Lxy$$

Order of Quantifiers

$$\begin{aligned} \text{domain} &: \text{all people} \\ Lxy &: \underline{\quad}_x \text{ loves } \underline{\quad}_y \\ a &: \text{Abelard} \end{aligned}$$

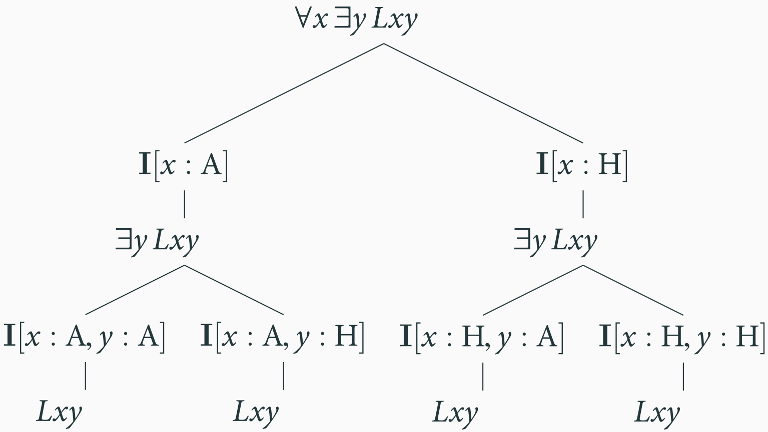
Everyone loves someone $\forall x \exists y Lxy$

Someone is loved by everyone $\exists y \forall x Lxy$

Order of Quantifiers

Domain : Abelard and Heloise

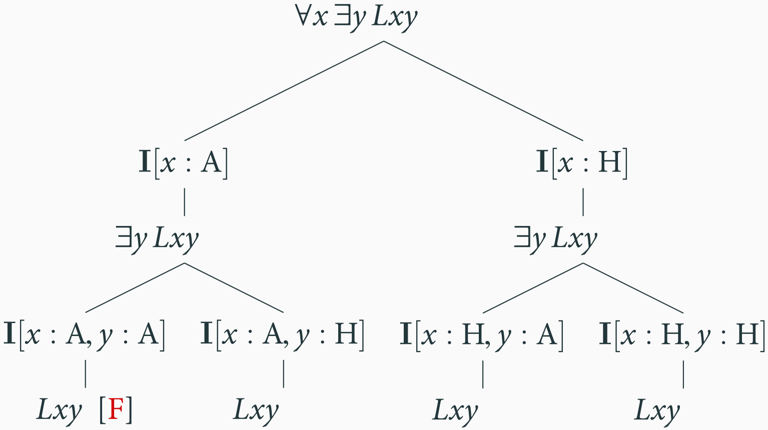
L : ⟨ Abelard, Heloise ⟩, ⟨ Heloise, Abelard ⟩



Order of Quantifiers

Domain : Abelard and Heloise

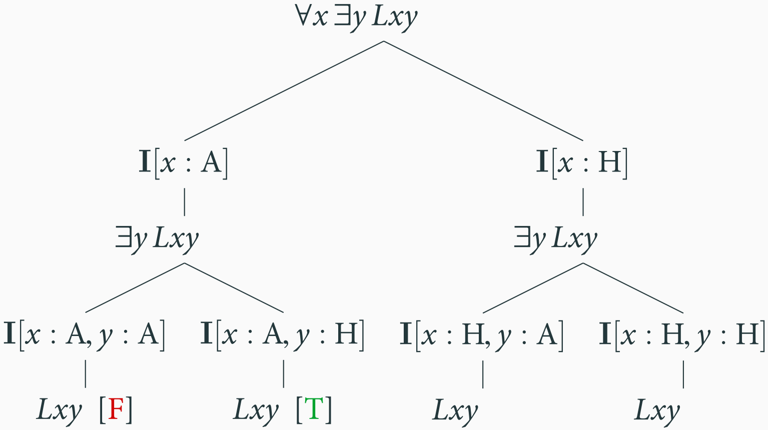
L : ⟨ Abelard, Heloise ⟩, ⟨ Heloise, Abelard ⟩



Order of Quantifiers

Domain : Abelard and Heloise

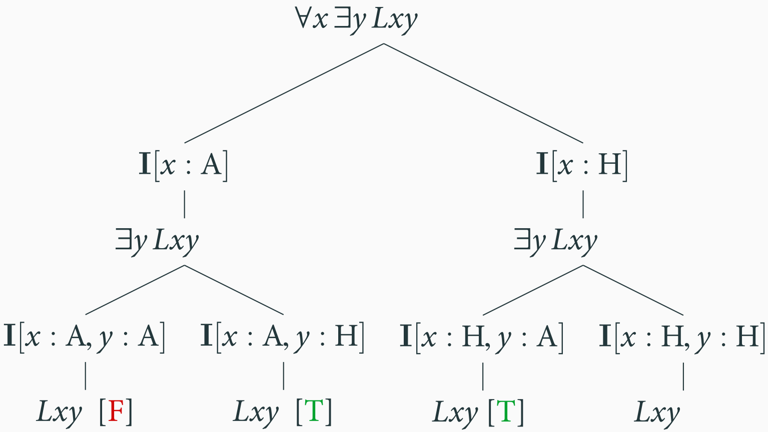
L : ⟨ Abelard, Heloise ⟩, ⟨ Heloise, Abelard ⟩



Order of Quantifiers

Domain : Abelard and Heloise

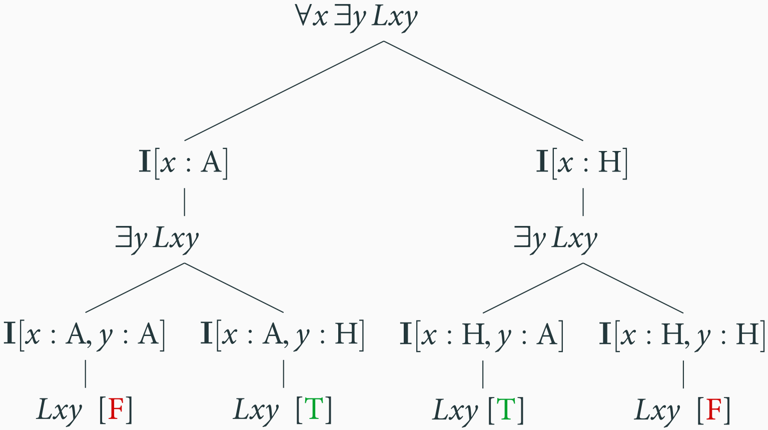
L : ⟨ Abelard, Heloise ⟩, ⟨ Heloise, Abelard ⟩



Order of Quantifiers

Domain : Abelard and Heloise

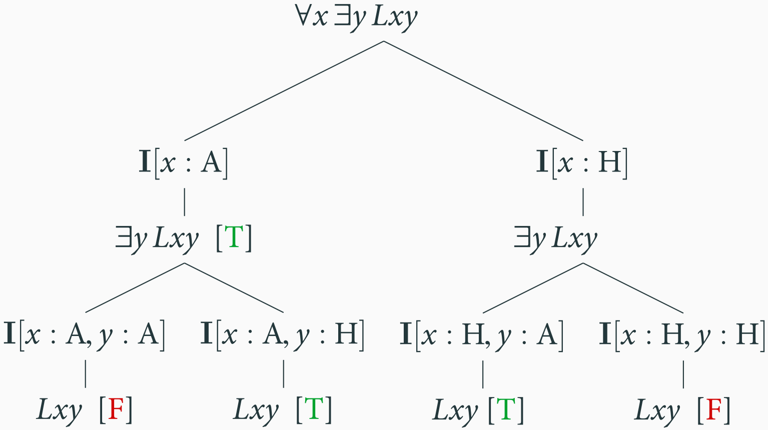
L : $\langle$ Abelard, Heloise $\rangle$, $\langle$ Heloise, Abelard $\rangle$



Order of Quantifiers

Domain : Abelard and Heloise

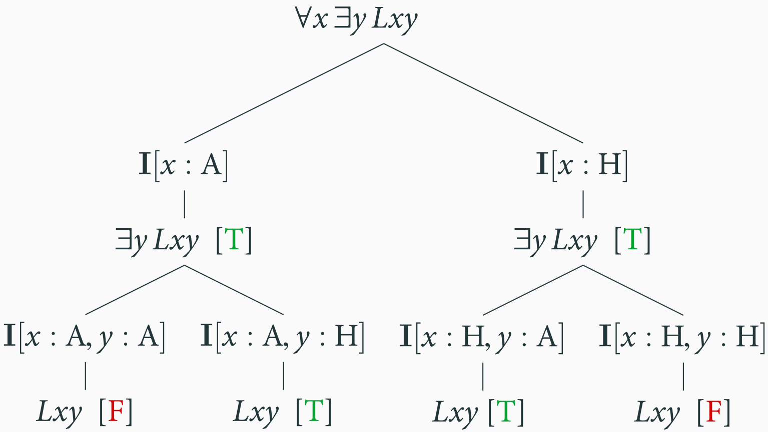
L : ⟨ Abelard, Heloise ⟩, ⟨ Heloise, Abelard ⟩



Order of Quantifiers

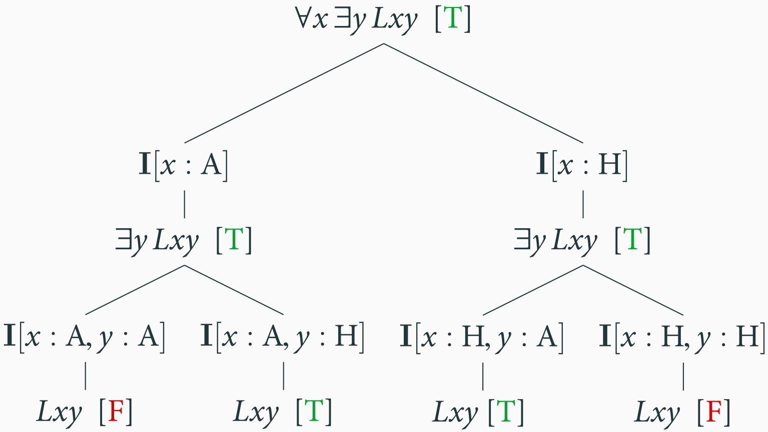
Domain : Abelard and Heloise

L : ⟨ Abelard, Heloise ⟩, ⟨ Heloise, Abelard ⟩



Order of Quantifiers

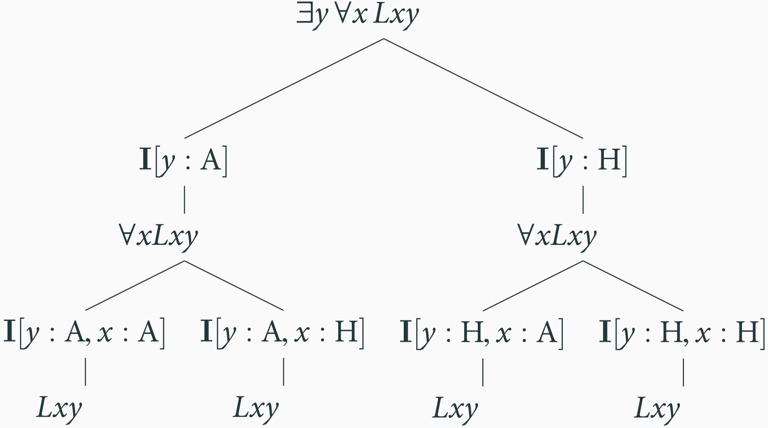
$$\begin{aligned} \text{Domain} &: \text{Abelard and Heloise} \\ L &: \langle \text{ Abelard, Heloise } \rangle, \langle \text{ Heloise, Abelard } \rangle \end{aligned}$$



Order of Quantifiers

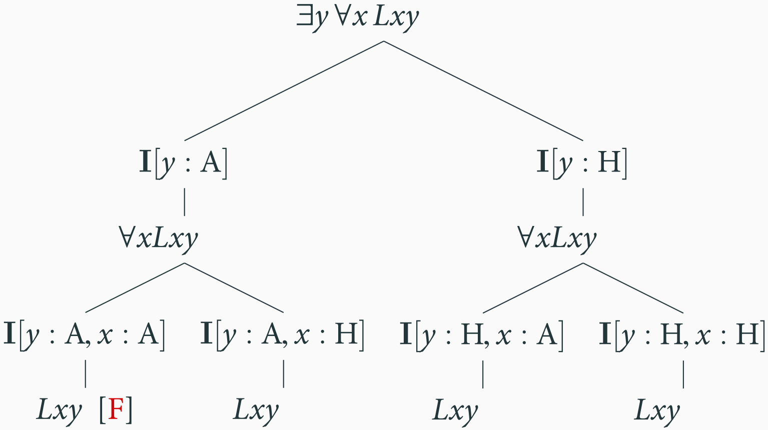
Domain : Abelard and Heloise

L : ⟨ Abelard, Heloise ⟩, ⟨ Heloise, Abelard ⟩



Order of Quantifiers

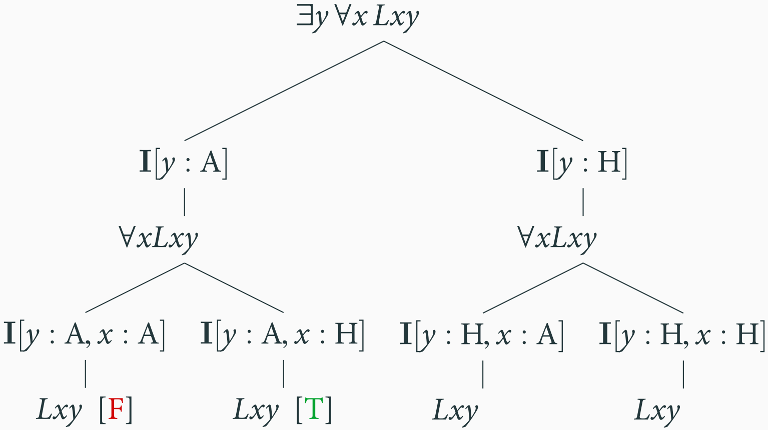
$$\begin{aligned} \text{Domain} &: \text{Abelard and Heloise} \\ L &: \langle\ \text{Abelard, Heloise}\ \rangle, \langle\ \text{Heloise, Abelard}\ \rangle \end{aligned}$$



Order of Quantifiers

Domain : Abelard and Heloise

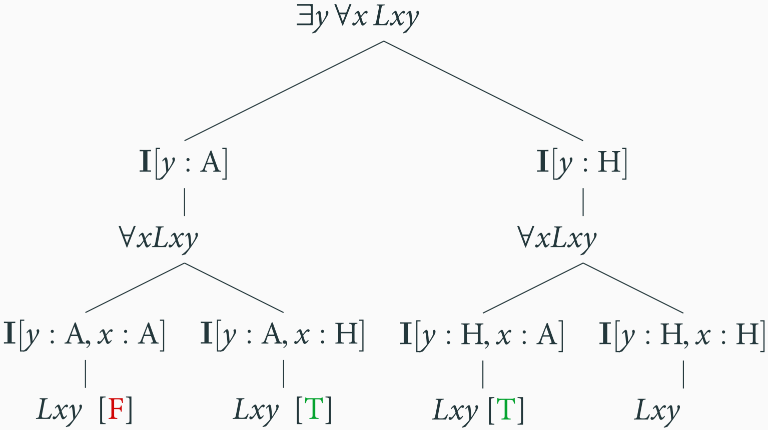
L : ⟨ Abelard, Heloise ⟩, ⟨ Heloise, Abelard ⟩



Order of Quantifiers

Domain : Abelard and Heloise

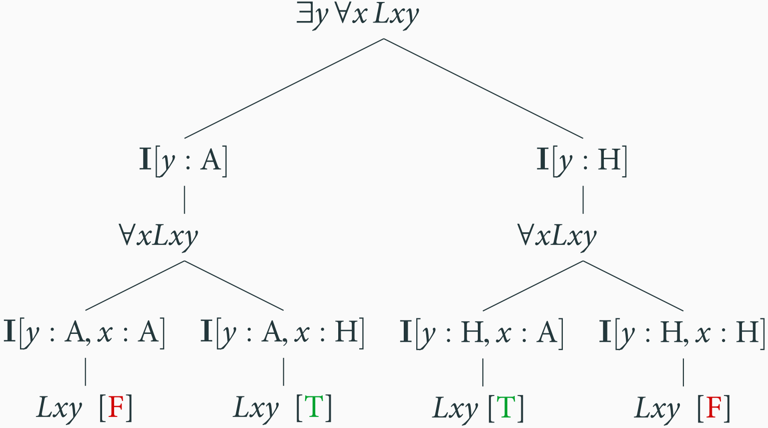
L : ⟨ Abelard, Heloise ⟩, ⟨ Heloise, Abelard ⟩



Order of Quantifiers

Domain : Abelard and Heloise

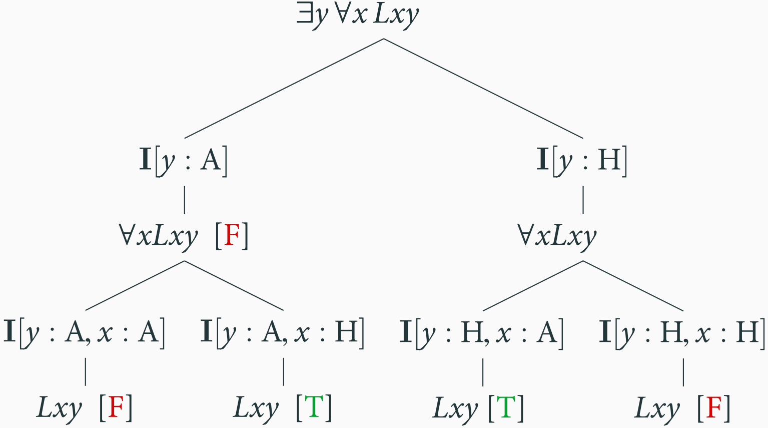
L : ⟨ Abelard, Heloise ⟩, ⟨ Heloise, Abelard ⟩



Order of Quantifiers

Domain : Abelard and Heloise

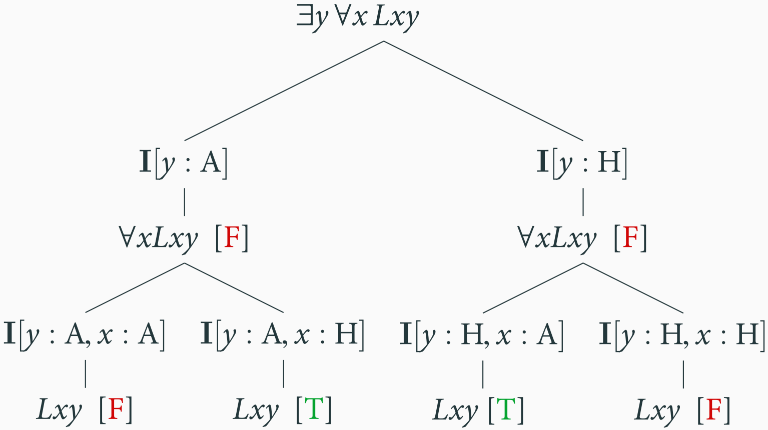
L : $\langle$ Abelard, Heloise $\rangle$, $\langle$ Heloise, Abelard $\rangle$



Order of Quantifiers

Domain : Abelard and Heloise

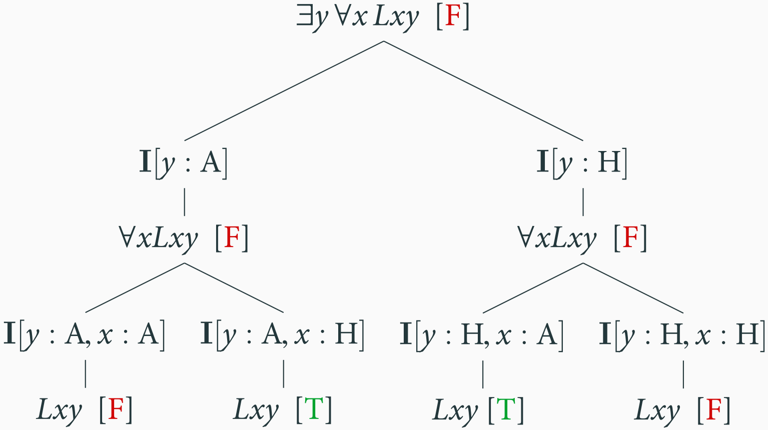
L : ⟨ Abelard, Heloise ⟩, ⟨ Heloise, Abelard ⟩



Order of Quantifiers

Domain : Abelard and Heloise

L : ⟨ Abelard, Heloise ⟩, ⟨ Heloise, Abelard ⟩



Order of Quantifiers

$$\begin{aligned} \text{domain} &: \text{all people} \\ Lxy &: \_\_\__x \text{ loves } \_\_\__y \\ a &: \text{Abelard} \end{aligned}$$

Order of Quantifiers

$$\begin{aligned} \text{domain} &: \text{all people} \\ Lxy &: \_\_\__x \text{ loves } \_\_\__y \\ a &: \text{Abelard} \end{aligned}$$

Abelard loves everyone :

Order of Quantifiers

$$\begin{aligned} \text{domain} &: \text{all people} \\ Lxy &: \_\_\__x \text{ loves } \_\_\__y \\ a &: \text{Abelard} \end{aligned}$$

Abelard loves everyone :

$$\forall x\, Lax$$

Order of Quantifiers

$$\begin{aligned} \text{domain} &: \text{all people} \\ Lxy &: \_\_\__x \text{ loves } \_\_\__y \\ a &: \text{Abelard} \end{aligned}$$

Abelard loves everyone :

$$\forall x\, Lax$$

Someone loves everyone :

Order of Quantifiers

$$\begin{aligned} \text{domain} &: \text{all people} \\ Lxy &: \_\_\__{x} \text{ loves } \_\_\__{y} \\ a &: \text{Abelard} \end{aligned}$$

Abelard loves everyone :

$$\forall x\, Lax$$

Someone loves everyone :

$$\exists y\, \forall x\, Lyx$$

Order of Quantifiers

$$\begin{aligned} \text{domain} &: \text{all people} \\ Lxy &: \_\_\__x \text{ loves } \_\_\__y \\ a &: \text{Abelard} \end{aligned}$$

Order of Quantifiers

$$\begin{aligned} \text{domain} &: \text{all people} \\ Lxy &: \_\_\__x \text{ loves } \_\_\__y \\ a &: \text{Abelard} \end{aligned}$$

Abelard is loved by someone :

Order of Quantifiers

$$\begin{aligned} \text{domain} &: \text{all people} \\ Lxy &: \_\_\__x \text{ loves } \_\_\__y \\ a &: \text{Abelard} \end{aligned}$$

Abelard is loved by someone :

$$\exists y Lya$$

Order of Quantifiers

$$\begin{aligned} \text{domain} &: \text{all people} \\ Lxy &: \_\_\__x \text{ loves } \_\_\__y \\ a &: \text{Abelard} \end{aligned}$$

Abelard is loved by someone :

$$\exists y Lya$$

Everyone is loved by someone :

Order of Quantifiers

$$\begin{aligned} \text{domain} &: \text{all people} \\ Lxy &: \_\_\__x \text{ loves } \_\_\__y \\ a &: \text{Abelard} \end{aligned}$$

Abelard is loved by someone :

$$\exists y L y a$$

Everyone is loved by someone :

$$\forall x \exists y\, Lyx$$

Order of Quantifiers

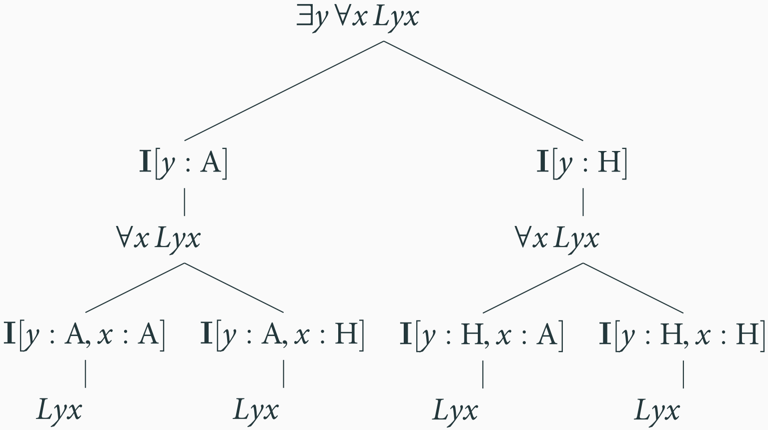
$$\begin{aligned} \text{domain} &: \text{all people} \\ Lxy &: \_\_\__x \text{ loves } \_\_\__y \\ a &: \text{Abelard} \end{aligned}$$

Someone loves everyone	$\exists y \forall x\, Lyx$
Everyone is loved by someone	$\forall x \exists y\, Lyx$

Order of Quantifiers

Domain : Abelard and Heloise

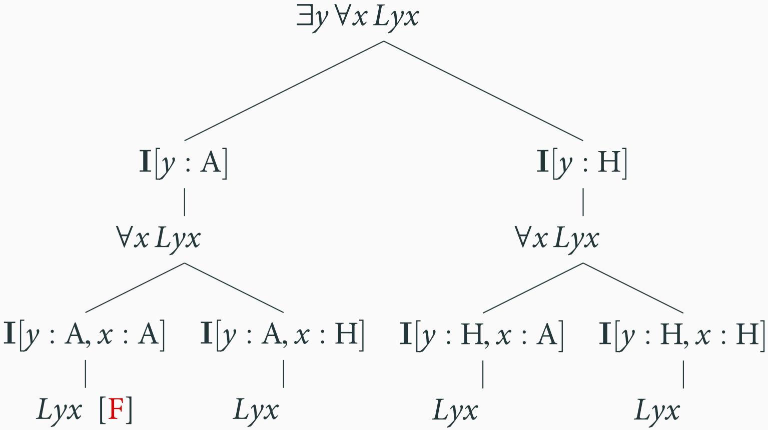
L : ⟨ Heloise, Abelard ⟩, ⟨ Abelard, Heloise ⟩



Order of Quantifiers

Domain : Abelard and Heloise

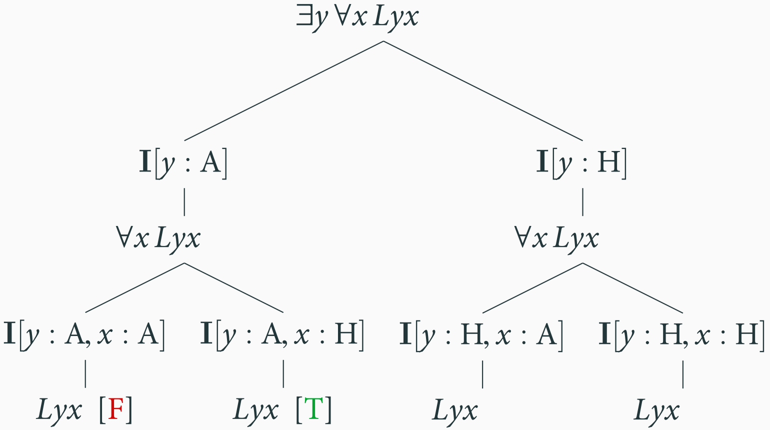
L : ⟨ Heloise, Abelard ⟩, ⟨ Abelard, Heloise ⟩



Order of Quantifiers

Domain : Abelard and Heloise

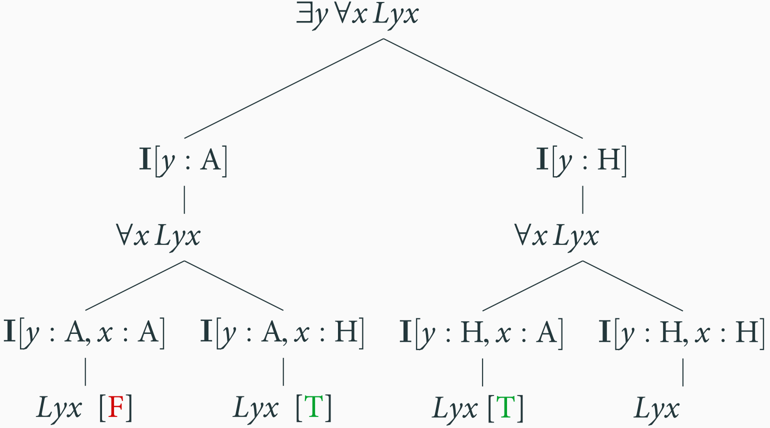
L : ⟨ Heloise, Abelard ⟩, ⟨ Abelard, Heloise ⟩



Order of Quantifiers

Domain : Abelard and Heloise

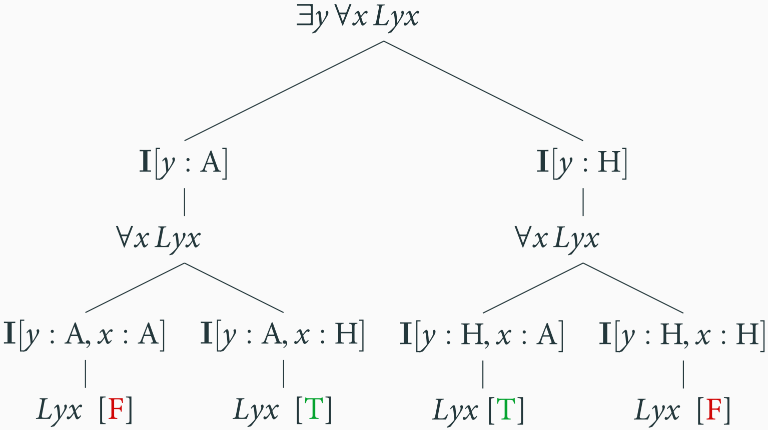
L : ⟨ Heloise, Abelard ⟩, ⟨ Abelard, Heloise ⟩



Order of Quantifiers

Domain : Abelard and Heloise

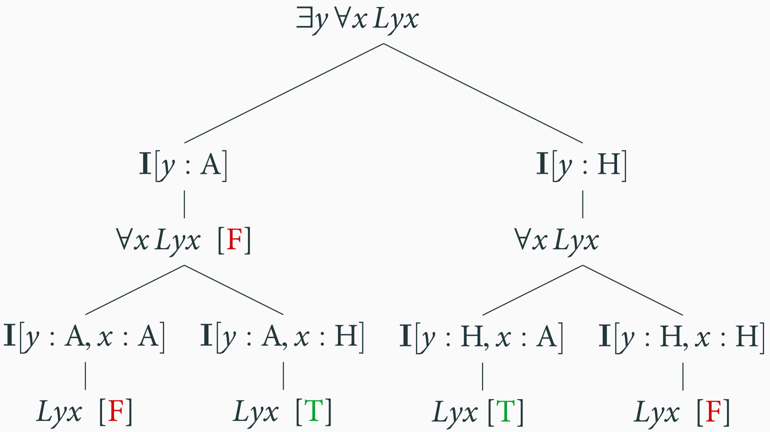
L : ⟨ Heloise, Abelard ⟩, ⟨ Abelard, Heloise ⟩



Order of Quantifiers

Domain : Abelard and Heloise

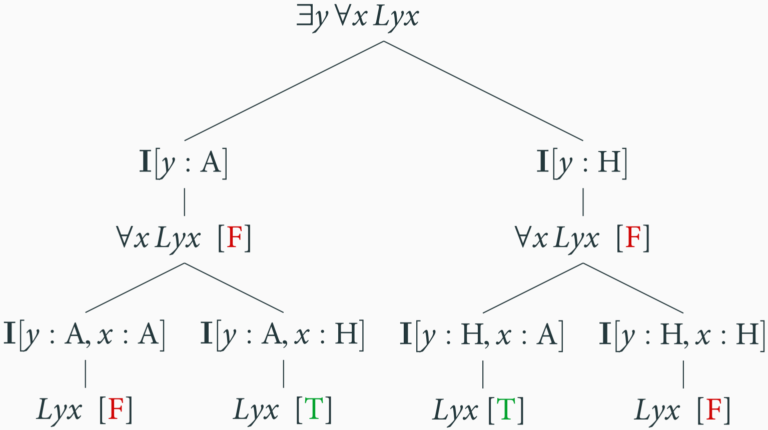
L : ⟨ Heloise, Abelard ⟩, ⟨ Abelard, Heloise ⟩



Order of Quantifiers

Domain : Abelard and Heloise

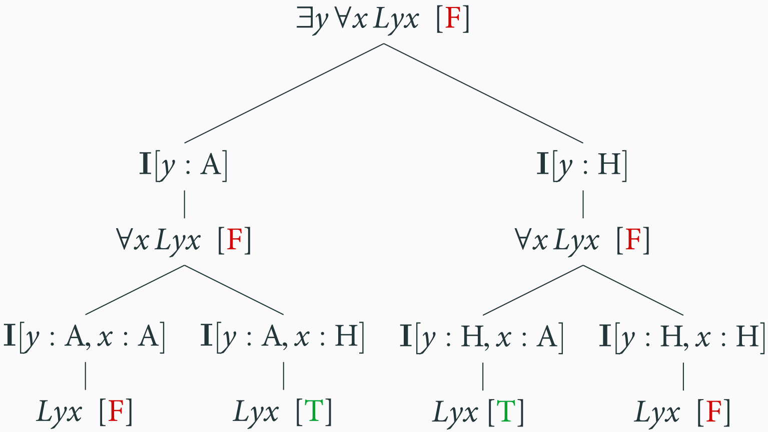
L : ⟨ Heloise, Abelard ⟩, ⟨ Abelard, Heloise ⟩



Order of Quantifiers

Domain : Abelard and Heloise

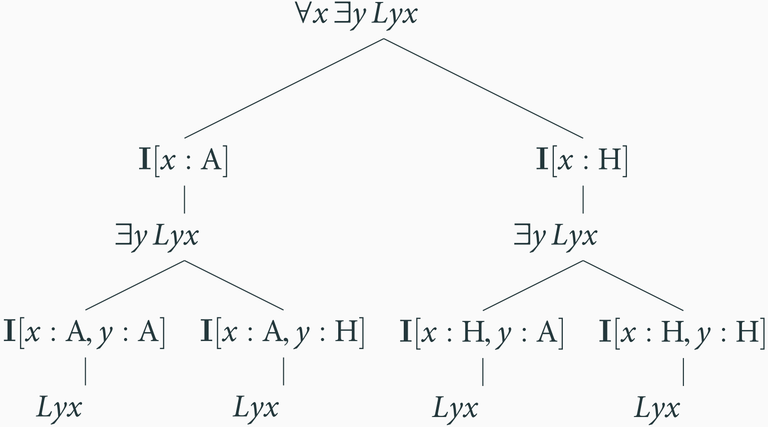
L : ⟨ Heloise, Abelard ⟩, ⟨ Abelard, Heloise ⟩



Order of Quantifiers

Domain : Abelard and Heloise

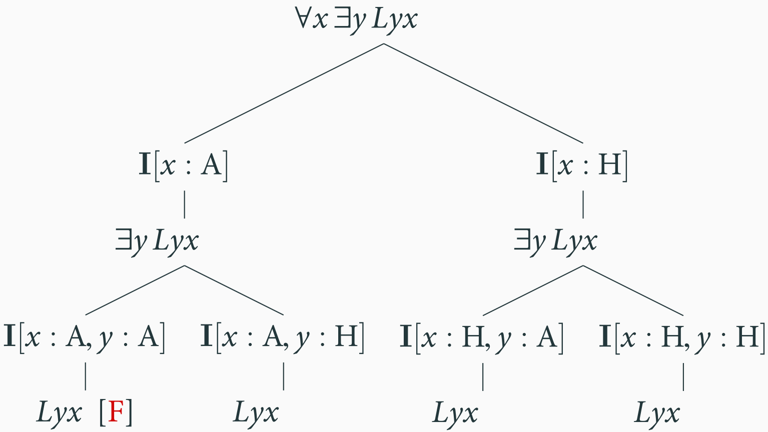
L : $\langle$ Heloise, Abelard $\rangle$, $\langle$ Abelard, Heloise $\rangle$



Order of Quantifiers

Domain : Abelard and Heloise

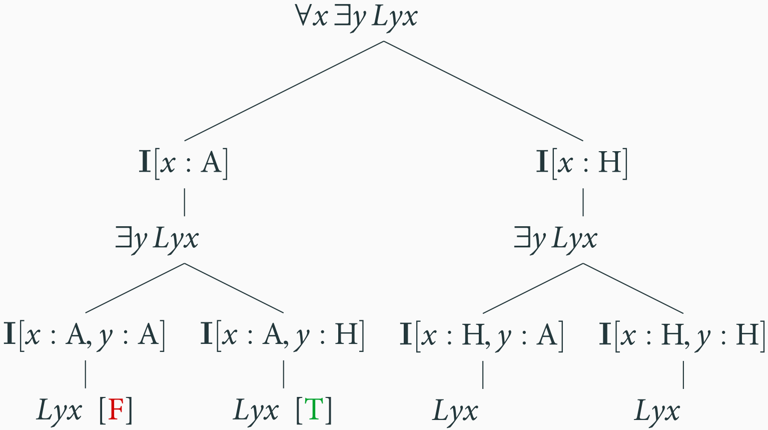
L : ⟨ Heloise, Abelard ⟩, ⟨ Abelard, Heloise ⟩



Order of Quantifiers

Domain : Abelard and Heloise

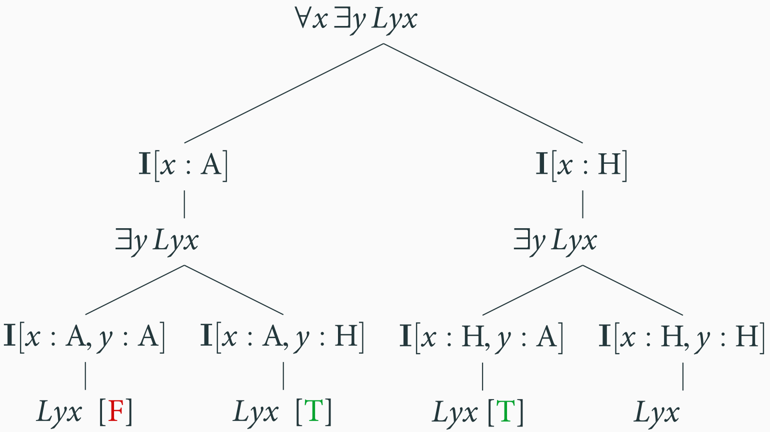
L : ⟨ Heloise, Abelard ⟩, ⟨ Abelard, Heloise ⟩



Order of Quantifiers

Domain : Abelard and Heloise

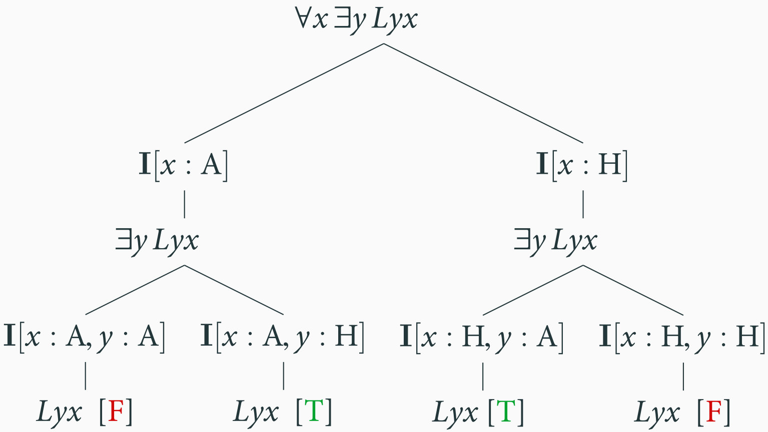
L : ⟨ Heloise, Abelard ⟩, ⟨ Abelard, Heloise ⟩



Order of Quantifiers

Domain : Abelard and Heloise

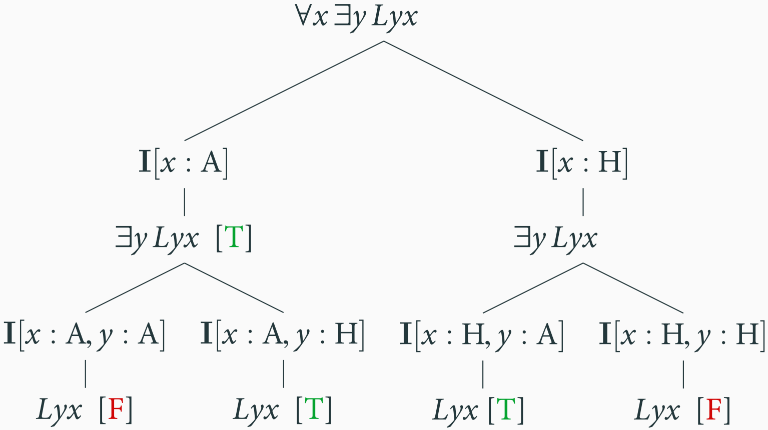
L : ⟨ Heloise, Abelard ⟩, ⟨ Abelard, Heloise ⟩



Order of Quantifiers

Domain : Abelard and Heloise

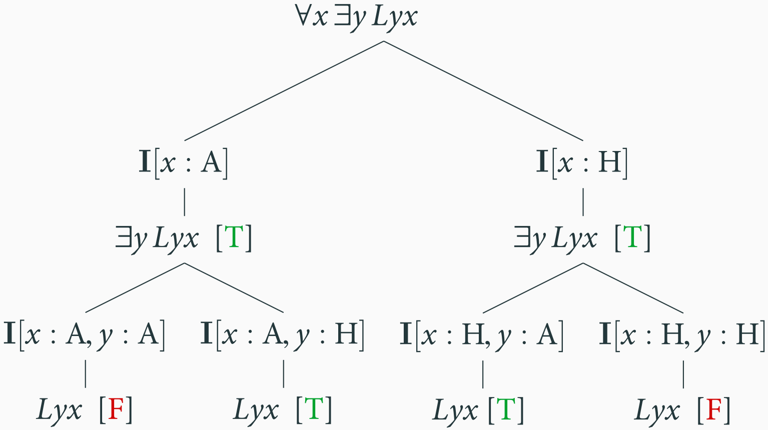
L : ⟨ Heloise, Abelard ⟩, ⟨ Abelard, Heloise ⟩



Order of Quantifiers

Domain : Abelard and Heloise

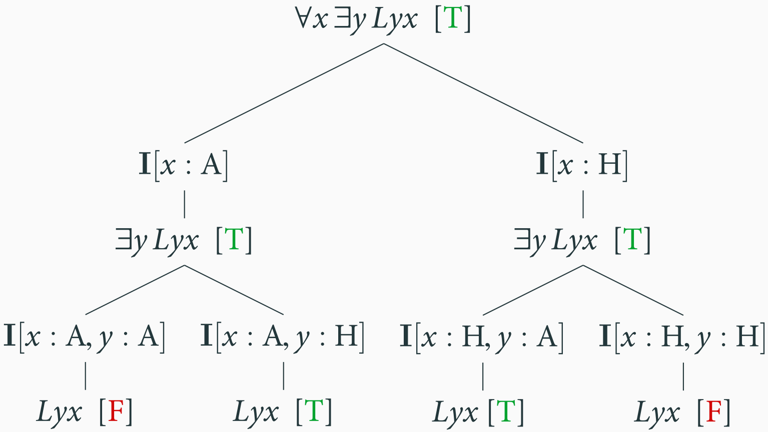
L : ⟨ Heloise, Abelard ⟩, ⟨ Abelard, Heloise ⟩



Order of Quantifiers

Domain : Abelard and Heloise

L : $\langle$ Heloise, Abelard $\rangle$, $\langle$ Abelard, Heloise $\rangle$



Order of Quantifiers

$$\begin{aligned} \text{domain} &: \text{all people} \\ Lxy &: \_\_\__{x} \text{ loves } \_\_\__{y} \\ a &: \text{Abelard} \end{aligned}$$

Everyone loves someone

Someone is loved by everyone

Someone loves everyone

Everyone is loved by someone

Order of Quantifiers

$$\begin{aligned} \text{domain} &: \text{all people} \\ Lxy &: \_\_\__x \text{ loves } \_\_\__y \\ a &: \text{Abelard} \end{aligned}$$

Everyone loves someone	$\forall x \exists y Lxy$
Someone is loved by everyone	
Someone loves everyone	
Everyone is loved by someone	

Order of Quantifiers

$$\begin{aligned} \text{domain} &: \text{all people} \\ Lxy &: \_\_\__x \text{ loves } \_\_\__y \\ a &: \text{Abelard} \end{aligned}$$

Everyone loves someone	$\forall x \exists y Lxy$
Someone is loved by everyone	$\exists y \forall x Lxy$
Someone loves everyone	
Everyone is loved by someone	

Order of Quantifiers

$$\begin{aligned} \text{domain} &: \text{all people} \\ Lxy &: \underline{\quad}_x \text{ loves } \underline{\quad}_y \\ a &: \text{Abelard} \end{aligned}$$

Everyone loves someone	$\forall x \exists y Lxy$
Someone is loved by everyone	$\exists y \forall x Lxy$
Someone loves everyone	$\exists y \forall x Lyx$
Everyone is loved by someone	

Order of Quantifiers

$$\begin{aligned} \text{domain} &: \text{all people} \\ Lxy &: \_\_\__x \text{ loves } \_\_\__y \\ a &: \text{Abelard} \end{aligned}$$

Everyone loves someone	$\forall x \, \exists y \, Lxy$
Someone is loved by everyone	$\exists y \, \forall x \, Lxy$
Someone loves everyone	$\exists y \, \forall x \, Lyx$
Everyone is loved by someone	$\forall x \, \exists y \, Lyx$

Order of Quantifiers

$$\begin{aligned} \text{domain} &: \text{all people} \\ Lxy &: \_\_\__x \text{ loves } \_\_\__y \\ a &: \text{Abelard} \end{aligned}$$

$\exists y \, \forall x \, Lyx$

$\exists y \, \forall x \, Lxy$

$\forall x \, \exists y \, Lyx$

$\forall x \, \exists y \, Lxy$

Order of Quantifiers

$$\begin{aligned} \text{domain} &: \text{all people} \\ Lxy &: \_\_\__x \text{ loves } \_\_\__y \\ a &: \text{Abelard} \end{aligned}$$

$\exists y \, \forall x \, Lyx$ Someone loves everyone

$\exists y \, \forall x \, Lxy$

$\forall x \, \exists y \, Lyx$

$\forall x \, \exists y \, Lxy$

Order of Quantifiers

$$\begin{aligned} \text{domain} &: \text{all people} \\ Lxy &: \_\_\__x \text{ loves } \_\_\__y \\ a &: \text{Abelard} \end{aligned}$$

$\exists y \forall x Lyx$	Someone loves everyone
$\exists y \forall x Lxy$	Someone is loved by everyone
$\forall x \exists y Lyx$	
$\forall x \exists y Lxy$	

Order of Quantifiers

$$\begin{aligned} \text{domain} &: \text{all people} \\ Lxy &: \underline{\quad}_x \text{ loves } \underline{\quad}_y \\ a &: \text{Abelard} \end{aligned}$$

$\exists y \forall x Lyx$	Someone loves everyone
$\exists y \forall x Lxy$	Someone is loved by everyone
$\forall x \exists y Lyx$	Everyone is loved by someone
$\forall x \exists y Lxy$	

Order of Quantifiers

$$\begin{aligned} \text{domain} &: \text{all people} \\ Lxy &: \_\_\__{x} \text{ loves } \_\_\__{y} \\ a &: \text{Abelard} \end{aligned}$$

$\exists y \forall x Lyx$	Someone loves everyone
$\exists y \forall x Lxy$	Someone is loved by everyone
$\forall x \exists y Lyx$	Everyone is loved by someone
$\forall x \exists y Lxy$	Everyone loves someone

Order of Quantifiers

Challenge: construct an interpretation on which:

$$\exists y\, \forall x\, Lyx \quad \text{T}$$

$$\forall x\, \exists y\, Lxy \quad \text{F}$$

Order of Quantifiers

Challenge: construct an interpretation on which:

$$\exists y \, \forall x \, Lxy \quad \text{T}$$

$$\forall x \, \exists y \, Lyx \quad \text{F}$$