Predicate Logic

Translation

PHIL 500

$\forall x((Px \lor Qx) \to Rx)$

Outline

- Four Important Statement Forms
- 2-Place Predicates
- Syntax for PL
  - Vocabulary
  - Grammar
  - Free and Bound Variables
  - Important Syntactic Features in PL

Four Important Statement Forms

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- Remember: any *quantified* claim in PL ($\forall x\ \mathscr{A}_x$ or $\exists x\ \mathscr{A}_x$) is made *relative to a domain*.

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  - ‘$\exists x\, \mathscr{A}_x$’ says *Something in the domain* makes $\mathscr{A}_x$ true

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Variants of ‘All $\mathcal{F}$s are $\mathcal{G}$s’

- All $\mathcal{F}$s are $\mathcal{G}$s
- Any $\mathcal{F}$ is a $\mathcal{G}$
- Every $\mathcal{F}$ is $\mathcal{G}$

Variants of ‘No $\mathscr{F}$s are $\mathscr{G}$s’

- No $\mathscr{F}$s are $\mathscr{G}$s
- No $\mathscr{F}$ is $\mathscr{G}$
- No $\mathscr{F}$ is a $\mathscr{G}$
- There are no $\mathscr{G}$ $\mathscr{F}$s

Variants of 'Some $\mathcal{F}$s are $\mathcal{G}$s'

- Some $\mathcal{F}$s are $\mathcal{G}$s
- Some $\mathcal{F}$s are $\mathcal{G}$
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- Some $\mathcal{F}$ is a $\mathcal{G}$
- There are $\mathcal{G}$ $\mathcal{F}$s

Variants of 'Some $\mathscr{F}$s are not $\mathscr{G}$s'

- Some $\mathscr{F}$s are not $\mathscr{G}$s
- Some $\mathscr{F}$s are not $\mathscr{G}$
- Some $\mathscr{F}$ is not $\mathscr{G}$
- Some $\mathscr{F}$ is a non-$\mathscr{G}$
- There are non-$\mathscr{G}$ $\mathscr{F}$s

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- Some of you may find one or more of these alternative translations more natural—if so, you should feel free to use them instead.

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▹ If you don't find any of these more natural, don't worry about it—just ignore this slide.

Examples

$$\begin{aligned} \text{domain} &: \text{all people} \\ F\_\_\_ &: \_\_\_\text{is funny} \\ S\_\_\_ &: \_\_\_\text{is shy} \\ T\_\_\_ &: \_\_\_\text{is tall} \\ Q\_\_\_ &: \_\_\_\text{is quirky} \end{aligned}$$

Everyone is funny :

Examples

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Everyone is funny :

$$\forall x\, Fx$$

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domain : all people

F___ : ___is funny

S___ : ___is shy

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Someone is quirky :

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Someone is quirky :

$$\exists y\, Qy$$

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$$\forall z\,(\mathscr{F}z \rightarrow \mathscr{G}z)$$

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Any shy quirky person is funny :

$$\forall x \, [\mathscr{F} z \longrightarrow \mathscr{G} z]$$

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Some tall people are shy :

$$\exists w\,(\mathscr{F}w \land \mathscr{G}w)$$

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No tall people are either funny or quirky :

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Some tall people are neither funny nor shy :

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$$\exists z[Tz \land \neg(Fz \lor Sz)]$$

Examples

$$\begin{aligned}
\text{domain} &: \text{all animals}\\
P___ &: ___\text{is a person}\\
V___ &: ___\text{is vegetarian}\\
C___ &: ___\text{is carnivorous}\\
D___ &: ___\text{is ferocious}\\
a &: \text{Albert}
\end{aligned}$$

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Some people are vegetarian :

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Some people are vegetarian :

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Some animals are vegetarian :

$$\exists x V x$$

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Some ferocious animals are not carnivorous :

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Some ferocious animals are not carnivorous :

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$$\exists x(Fx \land \neg Cx)$$

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Some people are vegetarians and some are not :

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Some people are vegetarians and some are not :

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If Albert is ferocious, then all people are ferocious :

Examples

$$\begin{aligned} \text{domain} &: \text{all animals} \\ P\_\_\_ &: \_\_\_\text{is a person} \\ V\_\_\_ &: \_\_\_\text{is vegetarian} \\ C\_\_\_ &: \_\_\_\text{is carnivorous} \\ D\_\_\_ &: \_\_\_\text{is ferocious} \\ a &: \text{Albert} \end{aligned}$$

If Albert is ferocious, then all people are ferocious :

$$Fa \rightarrow \forall x\,(\mathscr{F}x \rightarrow \mathscr{G}x)$$

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If Albert is ferocious, then all people are ferocious :

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Examples

$$\begin{aligned}
\text{domain} &: \text{all animals} \\
P___ &: ___\text{is a person} \\
V___ &: ___\text{is vegetarian} \\
C___ &: ___\text{is carnivorous} \\
D___ &: ___\text{is ferocious} \\
a &: \text{Albert}
\end{aligned}$$

Albert is ferocious if anyone is :

Examples

$$\begin{aligned} \text{domain} &: \text{all animals} \\ P\_\_\_ &: \_\_\_\text{is a person} \\ V\_\_\_ &: \_\_\_\text{is vegetarian} \\ C\_\_\_ &: \_\_\_\text{is carnivorous} \\ D\_\_\_ &: \_\_\_\text{is ferocious} \\ a &: \text{Albert} \end{aligned}$$

Albert is ferocious if anyone is :

$$\exists x(\mathscr{F}x \land \mathscr{G}x) \rightarrow Fa$$

Examples

$$\begin{aligned} \text{domain} &: \text{all animals} \\ P\_\_\_ &: \_\_\_\text{is a person} \\ V\_\_\_ &: \_\_\_\text{is vegetarian} \\ C\_\_\_ &: \_\_\_\text{is carnivorous} \\ D\_\_\_ &: \_\_\_\text{is ferocious} \\ a &: \text{Albert} \end{aligned}$$

Albert is ferocious if anyone is :

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Examples

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a &: \text{Albert}
\end{aligned}$$

If everyone is vegetarian, then no one is carnivorous :

Examples

$$\begin{aligned} \text{domain} &: \text{all animals} \\ P\_\_\_ &: \_\_\_\text{is a person} \\ V\_\_\_ &: \_\_\_\text{is vegetarian} \\ C\_\_\_ &: \_\_\_\text{is carnivorous} \\ D\_\_\_ &: \_\_\_\text{is ferocious} \\ a &: \text{Albert} \end{aligned}$$

If everyone is vegetarian, then no one is carnivorous :

$$\forall x(\mathscr{F}x \rightarrow \mathscr{G}x) \rightarrow \forall y(\mathscr{F}y \rightarrow \neg\mathscr{G}y)$$

Examples

$$\begin{aligned} \text{domain} &: \text{all animals} \\ P\_\_\_ &: \_\_\_\text{is a person} \\ V\_\_\_ &: \_\_\_\text{is vegetarian} \\ C\_\_\_ &: \_\_\_\text{is carnivorous} \\ D\_\_\_ &: \_\_\_\text{is ferocious} \\ a &: \text{Albert} \end{aligned}$$

If everyone is vegetarian, then no one is carnivorous :

$$\forall x(Px \to Vx) \to \forall y(Py \to \neg Cy)$$

Examples

domain : all animals
P___ : ___is a person
V___ : ___is vegetarian
C___ : ___is carnivorous
D___ : ___is ferocious
a : Albert

There are non-vegetarian people if and only if someone is ferocious :

Examples

$$\begin{aligned} \text{domain} &: \text{all animals} \\ P\_\_\_ &: \_\_\_\text{is a person} \\ V\_\_\_ &: \_\_\_\text{is vegetarian} \\ C\_\_\_ &: \_\_\_\text{is carnivorous} \\ D\_\_\_ &: \_\_\_\text{is ferocious} \\ a &: \text{Albert} \end{aligned}$$

There are non-vegetarian people if and only if someone is ferocious :

$$\exists x(\mathscr{F}x \land \neg \mathscr{G}x) \leftrightarrow \exists y(\mathscr{F}y \land \mathscr{G}y)$$

Examples

$$\begin{aligned} \text{domain} &: \text{all animals} \\ P\_\_\_ &: \_\_\_\text{is a person} \\ V\_\_\_ &: \_\_\_\text{is vegetarian} \\ C\_\_\_ &: \_\_\_\text{is carnivorous} \\ D\_\_\_ &: \_\_\_\text{is ferocious} \\ a &: \text{Albert} \end{aligned}$$

There are non-vegetarian people if and only if someone is ferocious :

$$\exists x(Px \wedge \neg Vx) \longleftrightarrow \exists y(Py \wedge Fy)$$

Examples

$$\begin{aligned} \text{domain} &: \text{all foods} \\ J\_\_\_ &: \_\_\_\text{is a jellybean} \\ B\_\_\_ &: \_\_\_\text{is black} \\ R\_\_\_ &: \_\_\_\text{is red} \\ D\_\_\_ &: \_\_\_\text{is delicious} \end{aligned}$$

Examples

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All black jellybeans are delicious, but no red jellybean is :

Examples

$$\begin{aligned} \text{domain} &: \text{all foods} \\ J\_\_\_ &: \_\_\_\text{is a jellybean} \\ B\_\_\_ &: \_\_\_\text{is black} \\ R\_\_\_ &: \_\_\_\text{is red} \\ D\_\_\_ &: \_\_\_\text{is delicious} \end{aligned}$$

All black jellybeans are delicious, but no red jellybean is :

$$\forall x[\mathcal{F}x \rightarrow \mathcal{G}x] \wedge \forall y[\mathcal{F}y \rightarrow \neg\mathcal{G}y]$$

Examples

$$\begin{aligned} \text{domain} &: \text{all foods} \\ J\_\_\_ &: \_\_\_\text{is a jellybean} \\ B\_\_\_ &: \_\_\_\text{is black} \\ R\_\_\_ &: \_\_\_\text{is red} \\ D\_\_\_ &: \_\_\_\text{is delicious} \end{aligned}$$

All black jellybeans are delicious, but no red jellybean is :

$$\forall x[(Bx \wedge Jx) \rightarrow \mathcal{G}x] \wedge \forall y[\mathcal{F}y \rightarrow \neg \mathcal{G}y]$$

Examples

$$\begin{aligned} \text{domain} &: \text{all foods} \\ J\_\_\_ &: \_\_\_\text{is a jellybean} \\ B\_\_\_ &: \_\_\_\text{is black} \\ R\_\_\_ &: \_\_\_\text{is red} \\ D\_\_\_ &: \_\_\_\text{is delicious} \end{aligned}$$

All black jellybeans are delicious, but no red jellybean is :

$$\forall x[(Bx \land Jx) \to Dx] \land \forall y[\mathcal{F}y \to \neg \mathcal{G}y]$$

Examples

$$\begin{aligned} \text{domain} &: \text{all foods} \\ J\_\_\_ &: \_\_\_\text{is a jellybean} \\ B\_\_\_ &: \_\_\_\text{is black} \\ R\_\_\_ &: \_\_\_\text{is red} \\ D\_\_\_ &: \_\_\_\text{is delicious} \end{aligned}$$

All black jellybeans are delicious, but no red jellybean is :

$$\forall x[(Bx \wedge Jx) \rightarrow Dx] \wedge \forall y[(Ry \wedge Jy) \rightarrow \neg \mathcal{G}y]$$

Examples

$$\begin{aligned} \text{domain} &: \text{all foods} \\ J\_\_\_ &: \_\_\_\text{is a jellybean} \\ B\_\_\_ &: \_\_\_\text{is black} \\ R\_\_\_ &: \_\_\_\text{is red} \\ D\_\_\_ &: \_\_\_\text{is delicious} \end{aligned}$$

All black jellybeans are delicious, but no red jellybean is :

$$\forall x[(Bx \wedge Jx) \rightarrow Dx] \wedge \forall y[(Ry \wedge Jy) \rightarrow \neg Dy]$$

Examples

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Black jellybeans are delicious :

$$\forall x \left[(Bx \wedge Jx) \rightarrow Dx \right]$$

Examples

$$\begin{aligned}
\text{domain} &: \text{all foods} \\
J___ &: ___ \text{is a jellybean} \\
B___ &: ___ \text{is black} \\
R___ &: ___ \text{is red} \\
D___ &: ___ \text{is delicious}
\end{aligned}$$

If some red jellybeans are delicious, then all black jellybeans are delicious :

Examples

$$\begin{aligned} \text{domain} &: \text{all foods} \\ J\_\_\_ &: \_\_\_\text{is a jellybean} \\ B\_\_\_ &: \_\_\_\text{is black} \\ R\_\_\_ &: \_\_\_\text{is red} \\ D\_\_\_ &: \_\_\_\text{is delicious} \end{aligned}$$

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$$\exists x[\mathcal{F}x \wedge \mathcal{G}x] \rightarrow \forall y[\mathcal{F}y \rightarrow \mathcal{G}y]$$

Examples

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If some red jellybeans are delicious, then all black jellybeans are delicious :

$$\exists x[(Rx \wedge Jx) \wedge \mathscr{G}x] \rightarrow \forall y[\mathscr{F}y \rightarrow \mathscr{G}y]$$

Examples

$$\begin{aligned} \text{domain} &: \text{all foods} \\ J\_\_\_ &: \_\_\_\text{is a jellybean} \\ B\_\_\_ &: \_\_\_\text{is black} \\ R\_\_\_ &: \_\_\_\text{is red} \\ D\_\_\_ &: \_\_\_\text{is delicious} \end{aligned}$$

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$$\exists x[(Rx \wedge Jx) \wedge Dx] \rightarrow \forall y[\mathscr{F}y \rightarrow \mathscr{G}y]$$

Examples

$$\begin{aligned} \text{domain} &: \text{all foods} \\ J\_\_\_ &: \_\_\_\text{is a jellybean} \\ B\_\_\_ &: \_\_\_\text{is black} \\ R\_\_\_ &: \_\_\_\text{is red} \\ D\_\_\_ &: \_\_\_\text{is delicious} \end{aligned}$$

If some red jellybeans are delicious, then all black jellybeans are delicious :

$$\exists x[(Rx \wedge Jx) \wedge Dx] \to \forall y[(By \wedge Jy) \to \mathscr{G}y]$$

Examples

$$\begin{aligned} \text{domain} &: \text{all foods} \\ J\_\_\_ &: \_\_\_\text{is a jellybean} \\ B\_\_\_ &: \_\_\_\text{is black} \\ R\_\_\_ &: \_\_\_\text{is red} \\ D\_\_\_ &: \_\_\_\text{is delicious} \end{aligned}$$

If some red jellybeans are delicious, then all black jellybeans are delicious :

$$\exists x[(Rx \wedge Jx) \wedge Dx] \rightarrow \forall y[(By \wedge Jy) \rightarrow Dy]$$

2-Place Predicates

Predicates

- a *predicate* is a *gappy statement*—it's a statement with a name (or names) missing.

Tammy loves Sammy.

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  - If a predicate has a single gap, then we'll call it a *1-place* predicate

Predicates

- a *predicate* is a *gappy statement*—it's a statement with a name (or names) missing.

  ______ loves ______

  - If a predicate has a single gap, then we'll call it a *1-place* predicate
  - If a predicate has two gaps, then we'll call it a *2-place* predicate

Predicates

- a *predicate* is a *gappy statement*—it's a statement with a name (or names) missing.

  ______ loves ______

  - If a predicate has a single gap, then we'll call it a *1-place* predicate
  - If a predicate has two gaps, then we'll call it a *2-place* predicate
  - If a predicate has *N* gaps, then we'll call it an *N-place* predicate

2-Place Predicates

$$\begin{aligned} \text{domain} &: \text{all people} \\ L\_\_\_\ \_\_\_ &: \_\_\_\text{loves}\ \_\_\_ \\ a &: \text{Abelard} \\ h &: \text{Heloise} \end{aligned}$$

2-Place Predicates

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Abelard loves Heloise :

2-Place Predicates

domain : all people

L___ ___ : ___loves ___

a : Abelard

h : Heloise

Abelard loves Heloise :

Heloise loves Abelard :

2-Place Predicates

$$\begin{aligned} \text{domain} &: \text{all people} \\ L\_\_\_\ \_\_\_ &: \_\_\_\text{loves}\ \_\_\_ \\ a &: \text{Abelard} \\ h &: \text{Heloise} \end{aligned}$$

Abelard loves Heloise :

Heloise loves Abelard :

- We need some way of saying *which* gap is which

2-Place Predicates

domain : all people

Lxy : $\_\_\__x$ loves $\_\_\__y$

a : Abelard

h : Heloise

Abelard loves Heloise :

Heloise loves Abelard :

- We need some way of saying *which* gap is which

2-Place Predicates

domain : all people

Lxy : $\_\_\__x$ loves $\_\_\__y$

a : Abelard

h : Heloise

Abelard loves Heloise : Lah

Heloise loves Abelard :

- We need some way of saying *which* gap is which

2-Place Predicates

domain : all people

Lxy : $\_\_\__x$ loves $\_\_\__y$

a : Abelard

h : Heloise

Abelard loves Heloise : Lah

Heloise loves Abelard : Lha

- We need some way of saying *which* gap is which

2-Place Predicates

$$
\begin{aligned}
\text{domain} &: \text{all people} \\
Lxy &: \_\_\__x \text{ loves } \_\_\__y \\
a &: \text{Abelard} \\
h &: \text{Heloise}
\end{aligned}
$$

2-Place Predicates

$$\begin{aligned} \text{domain} &: \text{all people} \\ Lxy &: \_\_\__x \text{ loves } \_\_\__y \\ a &: \text{Abelard} \\ h &: \text{Heloise} \end{aligned}$$

Everyone loves Abelard :

2-Place Predicates

domain : all people

Lxy : $\_\_\__x$ loves $\_\_\__y$

a : Abelard

h : Heloise

Everyone loves Abelard :

$$\forall x\, Lxa$$

2-Place Predicates

domain : all people

Lxy : $\_\_\__x$ loves $\_\_\__y$

a : Abelard

h : Heloise

Someone loves Heloise :

2-Place Predicates

$$
\begin{aligned}
\text{domain} &: \text{all people} \\
Lxy &: \_\_\__x \text{ loves } \_\_\__y \\
a &: \text{Abelard} \\
h &: \text{Heloise}
\end{aligned}
$$

Someone loves Heloise :

$$\exists z\, Lzh$$

2-Place Predicates

domain : all people

Lxy : $\_\_\__x$ loves $\_\_\__y$

a : Abelard

h : Heloise

Abelard loves Heloise if anyone does :

2-Place Predicates

domain : all people

Lxy : $\_\_\__x$ loves $\_\_\__y$

a : Abelard

h : Heloise

Abelard loves Heloise if anyone does :

$$\exists x\, Lxh \rightarrow Lah$$

2-Place Predicates

$$\begin{aligned} \text{domain} &: \text{all people} \\ Lxy &: \_\_\__x \text{ loves } \_\_\__y \\ a &: \text{Abelard} \\ h &: \text{Heloise} \end{aligned}$$

Everyone who loves Heloise loves Abelard, too. :

2-Place Predicates

$$\begin{aligned} \text{domain} &: \text{all people} \\ Lxy &: \_\_\__x \text{ loves } \_\_\__y \\ a &: \text{Abelard} \\ h &: \text{Heloise} \end{aligned}$$

Everyone who loves Heloise loves Abelard, too. :

$$\forall x\, (\mathscr{F}x \rightarrow \mathscr{G}x)$$

2-Place Predicates

$$\begin{aligned} \text{domain} &: \text{all people} \\ Lxy &: \_\_\__x \text{ loves } \_\_\__y \\ a &: \text{Abelard} \\ h &: \text{Heloise} \end{aligned}$$

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2-Place Predicates

$$\begin{aligned} \text{domain} &: \text{all people} \\ Lxy &: \_\_\__x \text{ loves } \_\_\__y \\ a &: \text{Abelard} \\ h &: \text{Heloise} \end{aligned}$$

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2-Place Predicates

$$\begin{aligned} \text{domain} &: \text{all people} \\ Lxy &: \_\_\__x \text{ loves } \_\_\__y \\ a &: \text{Abelard} \\ h &: \text{Heloise} \end{aligned}$$

Abelard loves himself :

2-Place Predicates

$$\begin{aligned} \text{domain} &: \text{all people} \\ Lxy &: \_\_\__x \text{ loves } \_\_\__y \\ a &: \text{Abelard} \\ h &: \text{Heloise} \end{aligned}$$

Abelard loves himself :

$$Laa$$

2-Place Predicates

$$\begin{aligned} \text{domain} &: \text{all people} \\ Lxy &: \_\_\__x \text{ loves } \_\_\__y \\ a &: \text{Abelard} \\ h &: \text{Heloise} \end{aligned}$$

Everyone loves themselves.

2-Place Predicates

$$\begin{aligned} \text{domain} &: \text{all people} \\ Lxy &: \_\_\__x \text{ loves } \_\_\__y \\ a &: \text{Abelard} \\ h &: \text{Heloise} \end{aligned}$$

Everyone loves themselves.

$$\forall z\, Lzz$$

2-Place Predicates

domain : all people

Mxy : $\_\_\__y$ loves $\_\_\__x$

a : Abelard

h : Heloise

2-Place Predicates

$$\begin{aligned} \text{domain} &: \text{all people} \\ Mxy &: \underline{\quad}_y \text{ loves } \underline{\quad}_x \\ a &: \text{Abelard} \\ h &: \text{Heloise} \end{aligned}$$

Everyone loves Abelard :

2-Place Predicates

$$
\begin{aligned}
\text{domain} &: \text{all people} \\
Mxy &: \_\_\__y \text{ loves } \_\_\__x \\
a &: \text{Abelard} \\
h &: \text{Heloise}
\end{aligned}
$$

Everyone loves Abelard :

$$\forall x\, Max$$

2-Place Predicates

domain : all people

Mxy : $\_\_\__y$ loves $\_\_\__x$

a : Abelard

h : Heloise

Someone loves Heloise :

2-Place Predicates

$$\begin{aligned} \text{domain} &: \text{all people} \\ Mxy &: \_\_\__y \text{ loves } \_\_\__x \\ a &: \text{Abelard} \\ h &: \text{Heloise} \end{aligned}$$

Someone loves Heloise :

$$\exists z\, Mhz$$

2-Place Predicates

domain : all people

Mxy : $\_\_\__y$ loves $\_\_\__x$

a : Abelard

h : Heloise

Abelard loves Heloise if anyone does :

2-Place Predicates

$$\begin{aligned} \text{domain} &: \text{all people} \\ Mxy &: \_\_\__y \text{ loves } \_\_\__x \\ a &: \text{Abelard} \\ h &: \text{Heloise} \end{aligned}$$

Abelard loves Heloise if anyone does :

$$\exists x\, Mhx \rightarrow Mha$$

2-Place Predicates

		domain	: everything in the office
j	: Jim	Lxy	: $\_\_\__x$ likes $\_\_\__y$
m	: Michael	Ex	: $\_\_\__x$ is easy going
p	: Pam	Txy	: $\_\_\__x$ is taller than $\_\_\__y$
s	: Stanley	Px	: $\_\_\__x$ is a person

2-Place Predicates

		domain	: everything in the office
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s	: Stanley	Px	: $\_\_\__x$ is a person

Everyone is easygoing :

2-Place Predicates

		domain	: everything in the office
j	: Jim	Lxy	: $\_\_\__x$ likes $\_\_\__y$
m	: Michael	Ex	: $\_\_\__x$ is easy going
p	: Pam	Txy	: $\_\_\__x$ is taller than $\_\_\__y$
s	: Stanley	Px	: $\_\_\__x$ is a person

Everyone is easygoing :

$$\forall x(\mathscr{F}x \rightarrow \mathscr{G}x)$$

2-Place Predicates

		domain	: everything in the office
j	: Jim	Lxy	: $\_\_\__x$ likes $\_\_\__y$
m	: Michael	Ex	: $\_\_\__x$ is easy going
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Everyone is easygoing :

$$\forall x(Px \rightarrow \mathcal{G}x)$$

2-Place Predicates

$$\begin{aligned} j &: \text{Jim} & \text{domain} &: \text{everything in the office} \\ m &: \text{Michael} & Lxy &: \_\_\__x \text{ likes } \_\_\__y \\ p &: \text{Pam} & Ex &: \_\_\__x \text{ is easy going} \\ s &: \text{Stanley} & Txy &: \_\_\__x \text{ is taller than } \_\_\__y \\ & & Px &: \_\_\__x \text{ is a person} \end{aligned}$$

Everyone is easygoing :

$$\forall x(Px \rightarrow Ex)$$

2-Place Predicates

		domain	: everything in the office
j	: Jim	Lxy	: $\_\_\__x$ likes $\_\_\__y$
m	: Michael	Ex	: $\_\_\__x$ is easy going
p	: Pam	Txy	: $\_\_\__x$ is taller than $\_\_\__y$
s	: Stanley	Px	: $\_\_\__x$ is a person

No one likes Michael :

2-Place Predicates

$$\begin{aligned} \text{domain} &: \text{everything in the office} \end{aligned}$$

j : Jim	Lxy : ___$_x$ likes ___$_y$
m : Michael	Ex : ___$_x$ is easy going
p : Pam	Txy : ___$_x$ is taller than ___$_y$
s : Stanley	Px : ___$_x$ is a person

No one likes Michael :

$$\forall x(\mathscr{F}x \rightarrow \neg \mathscr{G}x)$$

2-Place Predicates

$$\begin{aligned} &&\text{domain} &: \text{everything in the office} \\ j &: \text{Jim} & Lxy &: \_\_\__x \text{ likes } \_\_\__y \\ m &: \text{Michael} & Ex &: \_\_\__x \text{ is easy going} \\ p &: \text{Pam} & Txy &: \_\_\__x \text{ is taller than } \_\_\__y \\ s &: \text{Stanley} & Px &: \_\_\__x \text{ is a person} \end{aligned}$$

No one likes Michael :

$$\forall x(Px \rightarrow \neg \mathcal{G}x)$$

2-Place Predicates

		domain	: everything in the office
j	: Jim	Lxy	: $\_\_\__x$ likes $\_\_\__y$
m	: Michael	Ex	: $\_\_\__x$ is easy going
p	: Pam	Txy	: $\_\_\__x$ is taller than $\_\_\__y$
s	: Stanley	Px	: $\_\_\__x$ is a person

No one likes Michael :

$$\forall x(Px \rightarrow \neg Lxm)$$

2-Place Predicates

		domain	: everything in the office
j	: Jim	Lxy	: $\_\_\__x$ likes $\_\_\__y$
m	: Michael	Ex	: $\_\_\__x$ is easy going
p	: Pam	Txy	: $\_\_\__x$ is taller than $\_\_\__y$
s	: Stanley	Px	: $\_\_\__x$ is a person

Michael likes everyone :

2-Place Predicates

		domain	: everything in the office
j	: Jim	Lxy	: $\_\_\__x$ likes $\_\_\__y$
m	: Michael	Ex	: $\_\_\__x$ is easy going
p	: Pam	Txy	: $\_\_\__x$ is taller than $\_\_\__y$
s	: Stanley	Px	: $\_\_\__x$ is a person

Everyone is liked by Michael :

2-Place Predicates

$$\begin{aligned} & & \text{domain} &: \text{everything in the office} \\ j &: \text{Jim} & Lxy &: \_\_\__x \text{ likes } \_\_\__y \\ m &: \text{Michael} & Ex &: \_\_\__x \text{ is easy going} \\ p &: \text{Pam} & Txy &: \_\_\__x \text{ is taller than } \_\_\__y \\ s &: \text{Stanley} & Px &: \_\_\__x \text{ is a person} \end{aligned}$$

Everyone is liked by Michael :

$$\forall x(\mathscr{F}x \rightarrow \mathscr{G}x)$$

2-Place Predicates

$$\begin{aligned} &j : \text{Jim} & \text{domain} &: \text{everything in the office} \\ &m : \text{Michael} & Lxy &: \_\_\__x \text{ likes } \_\_\__y \\ &p : \text{Pam} & Ex &: \_\_\__x \text{ is easy going} \\ &s : \text{Stanley} & Txy &: \_\_\__x \text{ is taller than } \_\_\__y \\ & & Px &: \_\_\__x \text{ is a person} \end{aligned}$$

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\end{aligned}
$$

Everyone is liked by Michael :

$$\forall x(Px \rightarrow Lmx)$$

2-Place Predicates

$$
\begin{aligned}
&& \text{domain} &: \text{everything in the office} \\
j &: \text{Jim} & Lxy &: \underline{\quad}_x \text{ likes } \underline{\quad}_y \\
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s &: \text{Stanley} & Px &: \underline{\quad}_x \text{ is a person}
\end{aligned}
$$

Stanley doesn't like anyone :

2-Place Predicates

		domain	: everything in the office
j	: Jim	Lxy	: $\_\_\__x$ likes $\_\_\__y$
m	: Michael	Ex	: $\_\_\__x$ is easy going
p	: Pam	Txy	: $\_\_\__x$ is taller than $\_\_\__y$
s	: Stanley	Px	: $\_\_\__x$ is a person

No one is liked by Stanley :

2-Place Predicates

$$\begin{aligned} \text{domain} &: \text{everything in the office} \\ j : \text{Jim} \qquad Lxy &: \_\_\__x \text{ likes } \_\_\__y \\ m : \text{Michael} \qquad Ex &: \_\_\__x \text{ is easy going} \\ p : \text{Pam} \qquad Txy &: \_\_\__x \text{ is taller than } \_\_\__y \\ s : \text{Stanley} \qquad Px &: \_\_\__x \text{ is a person} \end{aligned}$$

No one is liked by Stanley :

$$\forall x(\mathscr{F}x \rightarrow \neg \mathscr{G}x)$$

2-Place Predicates

$$\begin{aligned} &j : \text{Jim} & \text{domain} &: \text{everything in the office} \\ &m : \text{Michael} & Lxy &: \_\_\__x \text{ likes } \_\_\__y \\ &p : \text{Pam} & Ex &: \_\_\__x \text{ is easy going} \\ &s : \text{Stanley} & Txy &: \_\_\__x \text{ is taller than } \_\_\__y \\ & & Px &: \_\_\__x \text{ is a person} \end{aligned}$$

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No one is liked by Stanley :

$$\forall x(Px \rightarrow \neg Lsx)$$

2-Place Predicates

		domain	: everything in the office
j	: Jim	Lxy	: $\_\_\__x$ likes $\_\_\__y$
m	: Michael	Ex	: $\_\_\__x$ is easy going
p	: Pam	Txy	: $\_\_\__x$ is taller than $\_\_\__y$
s	: Stanley	Px	: $\_\_\__x$ is a person

Someone likes Pam :

2-Place Predicates

$$
\begin{aligned}
&& \text{domain} &: \text{everything in the office} \\
j &: \text{Jim} & Lxy &: \_\_\__x \text{ likes } \_\_\__y \\
m &: \text{Michael} & Ex &: \_\_\__x \text{ is easy going} \\
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s &: \text{Stanley} & Px &: \_\_\__x \text{ is a person}
\end{aligned}
$$

Someone likes Pam :

$$\exists x(\mathcal{F}x \wedge \mathcal{G}x)$$

2-Place Predicates

$$\begin{aligned} \text{domain} &: \text{everything in the office} \\ j : \text{Jim} \qquad Lxy &: \_\_\__x \text{ likes } \_\_\__y \\ m : \text{Michael} \qquad Ex &: \_\_\__x \text{ is easy going} \\ p : \text{Pam} \qquad Txy &: \_\_\__x \text{ is taller than } \_\_\__y \\ s : \text{Stanley} \qquad Px &: \_\_\__x \text{ is a person} \end{aligned}$$

Someone likes Pam :

$$\exists x(Px \wedge \mathcal{G}x)$$

2-Place Predicates

$$\begin{aligned} &&\text{domain} &: \text{everything in the office} \\ j &: \text{Jim} & Lxy &: \_\_\__x \text{ likes } \_\_\__y \\ m &: \text{Michael} & Ex &: \_\_\__x \text{ is easy going} \\ p &: \text{Pam} & Txy &: \_\_\__x \text{ is taller than } \_\_\__y \\ s &: \text{Stanley} & Px &: \_\_\__x \text{ is a person} \end{aligned}$$

Someone likes Pam :

$$\exists x(Px \wedge Lxp)$$

2-Place Predicates

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Michael doesn't like anyone taller than him :

2-Place Predicates

		domain	: everything in the office
j	: Jim	Lxy	: $\_\_\__x$ likes $\_\_\__y$
m	: Michael	Ex	: $\_\_\__x$ is easy going
p	: Pam	Txy	: $\_\_\__x$ is taller than $\_\_\__y$
s	: Stanley	Px	: $\_\_\__x$ is a person

No one taller than Michael is liked by Michael :

2-Place Predicates

$$\begin{aligned} \text{domain} &: \text{everything in the office} \\ j : \text{Jim} \qquad Lxy &: \_\_\__x \text{ likes } \_\_\__y \\ m : \text{Michael} \qquad Ex &: \_\_\__x \text{ is easy going} \\ p : \text{Pam} \qquad Txy &: \_\_\__x \text{ is taller than } \_\_\__y \\ s : \text{Stanley} \qquad Px &: \_\_\__x \text{ is a person} \end{aligned}$$

No one taller than Michael is liked by Michael :

$$\forall x[\mathscr{F}x \to \neg \mathscr{G}x]$$

2-Place Predicates

$$\begin{aligned} \text{domain} &: \text{everything in the office} \\ j : \text{Jim} \qquad Lxy &: \_\_\__x \text{ likes } \_\_\__y \\ m : \text{Michael} \qquad Ex &: \_\_\__x \text{ is easy going} \\ p : \text{Pam} \qquad Txy &: \_\_\__x \text{ is taller than } \_\_\__y \\ s : \text{Stanley} \qquad Px &: \_\_\__x \text{ is a person} \end{aligned}$$

No one taller than Michael is liked by Michael :

$$\forall x[(Px \land Txm) \to \neg \mathcal{G}x]$$

2-Place Predicates

$$\begin{aligned} & & \text{domain} &: \text{everything in the office} \\ j &: \text{Jim} & Lxy &: \_\_\__x \text{ likes } \_\_\__y \\ m &: \text{Michael} & Ex &: \_\_\__x \text{ is easy going} \\ p &: \text{Pam} & Txy &: \_\_\__x \text{ is taller than } \_\_\__y \\ s &: \text{Stanley} & Px &: \_\_\__x \text{ is a person} \end{aligned}$$

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$$\forall x[(Px \land Txm) \rightarrow \neg Lmx]$$

2-Place Predicates

		domain	: everything in the office
j	: Jim	Lxy	: $\_\_\__x$ likes $\_\_\__y$
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Everyone likes everyone :

2-Place Predicates

		domain	: everything in the office
j : Jim		Lxy	: $\_\_\__x$ likes $\_\_\__y$
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Everyone likes everyone :

$$\forall x[Px \to \forall y(\mathscr{F}y \to \mathscr{G}y)]$$

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\end{array}
$$

Everyone likes everyone :

$$\forall x[Px \rightarrow \forall y(Py \rightarrow \mathcal{G}y)]$$

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		domain	: everything in the office
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Everyone likes someone :

$$\forall x[Px \rightarrow \exists y(\mathscr{F}y \land \mathscr{G}y)]$$

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2-Place Predicates

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j	: Jim		Lxy	:	$\_\_\__x$ likes $\_\_\__y$
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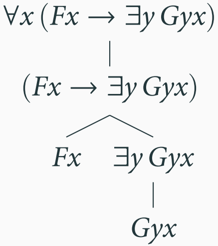
Syntax for PL

PHIL 500



Syntax for PL

Languages

SYNTAX — { 1. Vocabulary; 2. Grammar }

SEMANTICS — 3. Meaning

Syntax for PL

Vocabulary

Vocabulary

The vocabulary of PL includes the following symbols:

1. for each $N \geq 0$, N-place predicates (any capital letter—perhaps with subscripts)

$$
\begin{gathered}
A, B, C, D, E, \ldots, X, Y, Z \\
A_1, B_1, C_1, D_1, E_1, \ldots, X_1, Y_1, Z_1 \\
A_2, B_2, C_2, D_2, E_2, \ldots, X_2, Y_2, Z_2 \\
\vdots
\end{gathered}
$$

Vocabulary

2. *names* (any lowercase letter between a and v—perhaps with subscripts)

$$a, b, c, d, e, \ldots, t, u, v$$
$$a_1, b_1, c_1, d_1, e_1, \ldots, t_1, u_1, v_1$$
$$a_2, b_2, c_2, d_2, e_2, \ldots, t_2, u_2, v_2$$
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Vocabulary

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$$a_2, b_2, c_2, d_2, e_2, \ldots, t_2, u_2, v_2$$
$$\vdots$$

3. *variables* (lowercase w, x, y, and z—perhaps with subscripts)

$$w, x, y, z$$
$$w_1, x_1, y_1, z_1$$
$$w_2, x_2, y_2, z_2$$
$$\vdots$$

Vocabulary

Vocabulary

4. Logical operators

$$\neg, \vee, \wedge, \longrightarrow, \longleftrightarrow, \exists, \forall$$

Vocabulary

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Vocabulary

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5. parenthases

$$(,)$$

Vocabulary

4. Logical operators

$$\neg, \vee, \wedge, \longrightarrow, \longleftrightarrow, \exists, \forall$$

5. parenthases

$$(\,,\ )$$

Nothing else is included in the vocabulary of PL.

Vocabulary

- Let's call both names and variables *terms*. That is, both '*a*' and '*x*' are *terms* of PL.

Syntax for PL

Grammar

Grammar

- Any sequence of the symbols in the vocabulary of PL is an *expression* of PL.

Grammar

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- All of the following are expressions of PL:

$$Vx\neg((\to\to anv$$

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$$(\forall x\, Fxab \to \neg\exists y\, Pynst)$$

$$Nxy \vee \vee\neg\neg\exists xBx$$

Grammar

- Any sequence of the symbols in the vocabulary of PL is an *expression* of PL.
- All of the following are expressions of PL:

$$Vx\neg((\to\to anv$$

$$PQRST\neg\neg$$

$$(\forall x\, Fxab \to \neg\exists y\, Pynst)$$

$$Nxy \vee \vee\neg\neg\exists xBx$$

Grammar: Atomic Sentences

- If $\mathscr{R}$ is an *N*-place predicate and $t_1, t_2, \ldots, t_N$ are *N* terms, then

$$\mathscr{R}t_1t_2\ldots t_N$$

is an atomic sentence.

Grammar: Atomic sentences

- Let A be a 1-place predicate, B a 2-place predicate, C a 3-place predicate, and D a 4-place predicate

Grammar: Atomic sentences

- Let *A* be a 1-place predicate, *B* a 2-place predicate, *C* a 3-place predicate, and *D* a 4-place predicate
- Then, all of the following are atomic sentences of PL:

$$Az$$

$$Aa$$

$$Bwg$$

$$Cxzt$$

$$Dcccc$$

$$Dxaxa$$

Grammar: Sentences

$\mathscr{R}$) Every atomic sentence is a sentence

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¬) If '$\mathscr{A}$' is a sentence, then '$\neg\mathscr{A}$' is a sentence.

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$\neg$) If '$\mathscr{A}$' is a sentence, then '$\neg\mathscr{A}$' is a sentence.

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∨) If '$\mathscr{A}$' and '$\mathscr{B}$' are sentences, then '$(\mathscr{A} \vee \mathscr{B})$' is a sentence.

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$\forall$) If '$\mathscr{A}$' is a sentence and 'x' is a variable, then '$\forall x \mathscr{A}$' is a sentence.

Grammar: Sentences

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Grammar: Sentences

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$\exists$) If '$\mathscr{A}$' is a sentence and 'x' is a variable, then '$\exists x \mathscr{A}$' is a sentence.

–) Nothing else is a sentence.

Grammar

Note: none of '$\mathscr{A}$', '$\mathscr{B}$', 'x', or 't' appear in the vocabulary of PL. They are not *themselves* sentences of PL. Rather, we are using them here as META-VARIABLES ranging over the expressions of PL.

Grammar

- To show that ‘$(\forall y\, Fy \rightarrow \neg\exists x\, \exists z\, Gzx)$’ is a sentence of PL:

Grammar

- To show that '$(\forall y\, Fy \rightarrow \neg\exists x\, \exists z\, Gzx)$' is a sentence of PL:

a) 'Fy' is a sentence [from $(\mathscr{R})$]

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c) 'Gzx' is a sentence [from ($\mathscr{R}$)]

d) So, '$\exists z\, Gzx$' is a sentence [from (c) and ($\exists$)]

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d) So, '$\exists z\, Gzx$' is a sentence [from (c) and ($\exists$)]

e) So, '$\exists x\, \exists z\, Gzx$' is a sentence [from (d) and ($\exists$)]

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- To show that '$(\forall y\, Fy \rightarrow \neg\exists x\, \exists z\, Gzx)$' is a sentence of PL:

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b) So, '$\forall y\, Fy$' is a sentence [from (a) and ($\forall$)]

c) 'Gzx' is a sentence [from ($\mathscr{R}$)]

d) So, '$\exists z\, Gzx$' is a sentence [from (c) and ($\exists$)]

e) So, '$\exists x\, \exists z\, Gzx$' is a sentence [from (d) and ($\exists$)]

f) So, '$\neg\exists x\, \exists z\, Gzx$' is a sentence [from (e) and ($\neg$)]

g) So, '$(\forall y\, Fy \rightarrow \neg\exists x\, \exists z\, Gzx)$' is a sentence [from (b), (f), and ($\rightarrow$)]

Grammar

- Conventions:

Grammar

- Conventions:
  - Omit the outermost parenthases in a sentence of PL.

Grammar

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Grammar

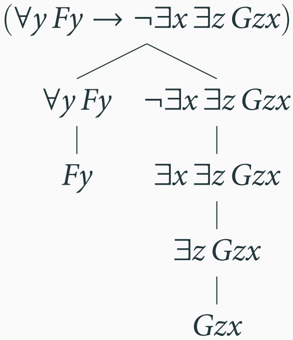
- Conventions:
  - Omit the outermost parenthases in a sentence of PL.
  - Allow ourselves to use square brackets '[,]' for readability
- So, rather than

$$(\forall y\, Fy \rightarrow \neg\exists x\, \exists z\, Gzx)$$

- we can write

$$\forall y\, Fy \rightarrow \neg\exists x\, \exists z\, Gzx$$

Syntax Trees



Syntactic Structure

Is it a sentence? (*F* is 2-place, *G* is 1-place)

- $\forall x(\exists y(\forall z\, Fab))$

Syntactic Structure

Is it a sentence? (F is 2-place, G is 1-place)

- $\forall x(\exists y(\forall z\, Fab))$ ×

Syntactic Structure

Is it a sentence? (*F* is 2-place, *G* is 1-place)

- $\forall x(\exists y(\forall z\, Fab))$ ×

$$\forall x\, \exists y\, \forall z\, Fab$$

$$|$$

$$\exists y\, \forall z\, Fab$$

$$|$$

$$\forall z\, Fab$$

$$|$$

$$Fab$$

Syntactic Structure

Is it a sentence? (F is 2-place, G is 1-place)

- $\forall x(\exists y(\forall z\, Fab))$ ×
- $\forall a\, Gaa$

Syntactic Structure

Is it a sentence? (F is 2-place, G is 1-place)

- $\forall x(\exists y(\forall z\, Fab))$ ×
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Syntactic Structure

Is it a sentence? (F is 2-place, G is 1-place)

- $\forall x(\exists y(\forall z\, Fab))$ ×
- $\forall a\, Gaa$ ×
- Fxy

Syntactic Structure

Is it a sentence? (F is 2-place, G is 1-place)

- $\forall x(\exists y(\forall z\, Fab))$ ×
- $\forall a\, Gaa$ ×
- Fxy ✓

Syntactic Structure

Is it a sentence? (F is 2-place, G is 1-place)

- $\forall x(\exists y(\forall z\, Fab))$ $\times$
- $\forall a\, Gaa$ $\times$
- Fxy $\checkmark$
- $\forall w\, Gx$

Syntactic Structure

Is it a sentence? (F is 2-place, G is 1-place)

- $\forall x(\exists y(\forall z\, Fab))$ ×
- $\forall a\, Gaa$ ×
- Fxy ✓
- $\forall w\, Gx$ ✓

Syntactic Structure

Is it a sentence? (F is 2-place, G is 1-place)

- $\forall x(\exists y(\forall z\, Fab))$ ✗
- $\forall a\, Gaa$ ✗
- Fxy ✓
- $\forall w\, Gx$ ✓

$$\forall w\, Gx$$
$$|$$
$$Gx$$

Syntactic Structure

Is it a sentence? (F is 2-place, G is 1-place)

- $\forall x(\exists y(\forall z\, Fab))$ ×
- $\forall a\, Gaa$ ×
- Fxy ✓
- $\forall w\, Gx$ ✓
- $\exists x\, \forall x\, Fxy$

Syntactic Structure

Is it a sentence? (F is 2-place, G is 1-place)

- $\forall x(\exists y(\forall z\, Fab))$ ✗
- $\forall a\, Gaa$ ✗
- Fxy ✓
- $\forall w\, Gx$ ✓
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Syntactic Structure

Is it a sentence? (F is 2-place, G is 1-place)

- $\forall x(\exists y(\forall z\, Fab))$ ×
- $\forall a\, Gaa$ ×
- Fxy ✓
- $\forall w\, Gx$ ✓
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$$\exists x\, \forall x\, Fxy$$
$$|$$
$$\forall x\, Fxy$$
$$|$$
$$Fxy$$

Syntactic Structure

Is it a sentence? (F is 2-place, G is 1-place)

- $\forall x(\exists y(\forall z\, Fab))$ ✗
- $\forall a\, Gaa$ ✗
- Fxy ✓
- $\forall w\, Gx$ ✓
- $\exists x\, \forall x\, Fxy$ ✓
- $\forall x\, Fxx \rightarrow (\exists z\, Gz \rightarrow Fab)$

Syntactic Structure

Is it a sentence? (F is 2-place, G is 1-place)

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  ✓

Syntactic Structure

Is it a sentence? (F is 2-place, G is 1-place)

- $\forall x(\exists y(\forall z\, Fab))$ ×
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- $\forall w\, Gx$ ✓
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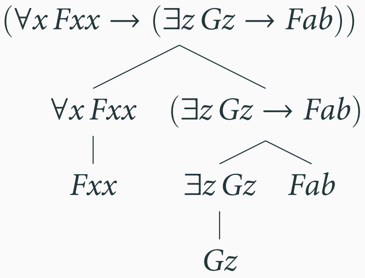
- $\forall x\, Fxx \rightarrow (\exists z\, Gz \rightarrow Fab)$ ✓



Subsentences

- '$\mathscr{B}$' is a *subsentence* of '$\mathscr{A}$' if and only if, in the course of building up '$\mathscr{A}$' by applying the rules for sentences, '$\mathscr{B}$' appears on a line before '$\mathscr{A}$'.

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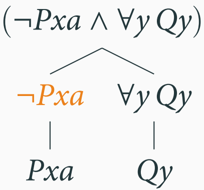

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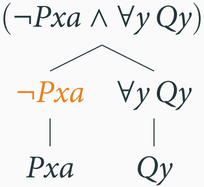

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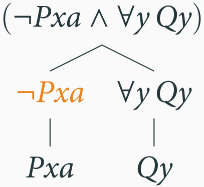
  - '$\neg Pxa$' is *not* a subsentence of '$\neg(Pxa \land \forall y\, Qy)$'



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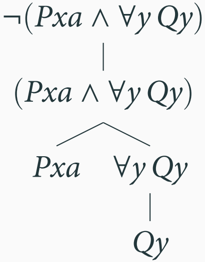

Main Operators

- The *main operator* in a (non-atomic) sentence is the operator which would be introduced *last*, if we were building the sentence up according to the rules for sentences.

Main Operators

- $Fab \rightarrow \exists y\, Ay$ main operator:

Main Operators

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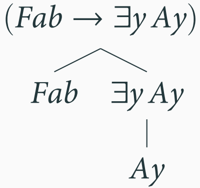
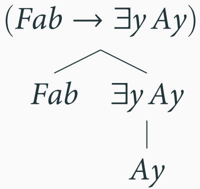


Main Operators

- $Fab \to \exists y\, Ay$ main operator: $\to$

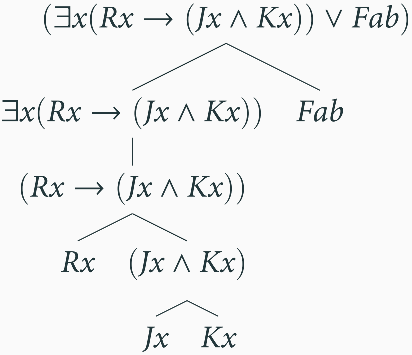


Main Operators

- $\exists x[Rx \rightarrow (Jx \wedge Kx)] \vee Fab$ main operator:

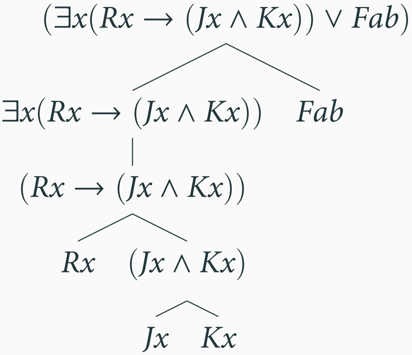
Main Operators

- $\exists x[Rx \rightarrow (Jx \land Kx)] \lor Fab$ main operator:



Main Operators

- $\exists x[Rx \rightarrow (Jx \wedge Kx)] \vee Fab$ main operator: $\vee$



Main Operators

- $\forall x\,(Fx \rightarrow Gx)$ main operator:

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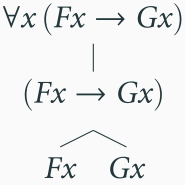

Main Operators

- $\forall x\,(Fx \rightarrow Gx)$ main operator: $\forall$

$$\forall x\,(Fx \rightarrow Gx)$$

$$(Fx \rightarrow Gx)$$

$$Fx \qquad Gx$$

Main Operators

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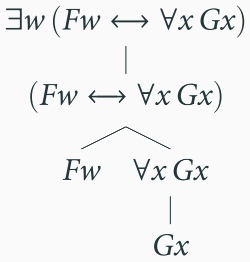
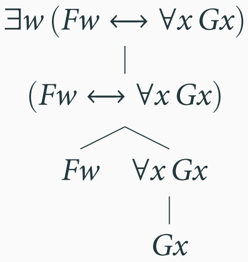


Main Operators

- $\exists w\,(Fw \leftrightarrow \forall x\, Gx)$ main operator: $\exists$



Main Operators

- A sentence whose main operator is '¬' is a *negation*

Main Operators

- A sentence whose main operator is '$\neg$' is a *negation*
- A sentence whose main operator is '$\wedge$' is a *conjunction*

Main Operators

- A sentence whose main operator is '¬' is a *negation*
- A sentence whose main operator is '∧' is a *conjunction*
- A sentence whose main operator is '∨' is a *disjunction*

Main Operators

- A sentence whose main operator is ‘$\neg$’ is a *negation*
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- A sentence whose main operator is '$\forall$' is a *universal* sentence
- A sentence whose main operator is '$\exists$' is an *existential* sentence

Scope

- The *scope* of an operator (in a sentence) is the sub-sentence for which that operator is the main operator

Scope

$$\forall x\, \exists y\, [\forall w\, Fwx \longleftrightarrow \forall z\, (Gxz \rightarrow Wzyx)]$$

Scope

$$\forall x\, \exists y\, [\forall w\, Fwx \leftrightarrow \forall z\, (Gxz \rightarrow Wzyx)]$$

- Scope of '$\forall z$':

Scope

$$\forall x\, \exists y\, [\forall w\, Fwx \leftrightarrow \forall z\, (Gxz \to Wzyx)]$$

- Scope of '$\forall z$': $\forall z\, (Gxz \to Wzyx)$

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$$\forall x\, \exists y\, [\forall w\, Fwx \leftrightarrow \forall z\, (Gxz \rightarrow Wzyx)]$$

- Scope of '$\forall z$': $\forall z\, (Gxz \rightarrow Wzyx)$
- Scope of '$\forall w$':

Scope

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- Scope of '$\forall z$': $\forall z\, (Gxz \rightarrow Wzyx)$
- Scope of '$\forall w$': $\forall w\, Fwx$

Scope

$$\forall x\, \exists y\, [\forall w\, Fwx \leftrightarrow \forall z\, (Gxz \rightarrow Wzyx)]$$

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- Scope of '$\forall w$': $\forall w\, Fwx$
- Scope of '$\exists y$':

Scope

$$\forall x\, \exists y\, [\forall w\, Fwx \leftrightarrow \forall z\, (Gxz \rightarrow Wzyx)]$$

- Scope of '$\forall z$': $\forall z\, (Gxz \rightarrow Wzyx)$
- Scope of '$\forall w$': $\forall w\, Fwx$
- Scope of '$\exists y$': $\exists y\, [Fx \leftrightarrow \forall z\, (Gaz \rightarrow Wzyx)]$

Scope

$$\forall x \, \exists y \, [\forall w \, Fwx \leftrightarrow \forall z \, (Gxz \rightarrow Wzyx)]$$

- Scope of '$\forall z$': $\forall z \, (Gxz \rightarrow Wzyx)$
- Scope of '$\forall w$': $\forall w \, Fwx$
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- Scope of '$\forall x$':

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- Scope of '$\exists y$': $\exists y\, [Fx \leftrightarrow \forall z\, (Gaz \rightarrow Wzyx)]$
- Scope of '$\forall x$': $\forall x\, \exists y\, [Fx \leftrightarrow \forall z\, (Gaz \rightarrow Wzyx)]$

Syntax for PL

Free and Bound Variables

Free and Bound Variables

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- The variables appearing in '$\forall x \forall y\, Fxy$' are BOUND.
- In '$\forall x\, Px \rightarrow Qx$', the first x is bound, whereas the second one is free.

Free and Bound Variables

A variable x in a sentence of PL is BOUND if and only if it occurs within the scope of a quantifier, $\forall x$ or $\exists x$, whose associated variable is x.

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$$\forall w(\exists y\, Lwy \rightarrow \exists w\, Aw)$$

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In a sentence of the form $\forall x \mathscr{A}$ or $\exists x \mathscr{A}$, the quantifier binds every *free* occurrence of x in $\mathscr{A}$. If an occurrence of x in $\mathscr{A}$ is already bound, then the quantifier does not bind it.

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- E.g., in

  $$\exists x \forall x Fx$$

  the variable 'x' is bound by the *universal* quantifier '$\forall x$'. It is *not* bound by the existential quantifier '$\exists x$'.

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- When translating into PL, we want our translations to be *closed*.

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$$\forall x\, Ax \wedge Bx$$

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$$\forall x\, Ax \wedge Bx$$

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Free and Bound Variables

$$\forall x\,\exists y\,[Rxy \rightarrow (Jz \wedge Kx)] \vee Ryx$$

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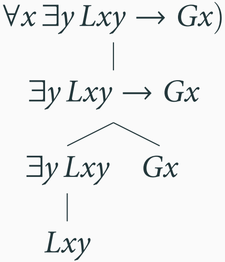
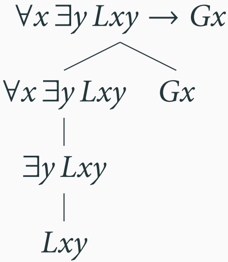
Syntax for PL

Important Syntactic Features in PL

Parenthases





Term Order

$$
\begin{array}{ccc}
\forall x\, \exists y\, Lxy & \qquad\qquad & \forall x\, \exists y\, Lyx \\
| & & | \\
\exists y\, Lxy & & \exists y\, Lyx \\
| & & | \\
Lxy & & Lyx
\end{array}
$$

Quantifier Order

$$\begin{array}{c} \exists y\, \forall x\, Lxy \\ | \\ \forall x\, Lxy \\ | \\ Lxy \end{array} \qquad\qquad \begin{array}{c} \forall x\, \exists y\, Lxy \\ | \\ \exists y\, Lxy \\ | \\ Lxy \end{array}$$