

Correctness and Completeness of SL

PHIL 500

Arguments that SL is Not Correct

A Counterexample to Modus Ponens?

A Counterexample to Modus Tollens?

A Counterexample to Disjunctive Syllogism?

The Sorites Paradox

Why SL is Not Complete

A Quick Recap

A Quick Recap

An argument is **VALID** if and only if it is impossible for its premises to all be true while its conclusion is false.

A Quick Recap

- We decided to theorize about argument forms involving the following English constructions:

it is not the case that...

both ... and ...

either ... or ...

if ..., then ...

... if and only if ...

A Quick Recap

- We decided to theorize about argument forms involving the following English constructions:
 - it is not the case that...
 - both ... and ...
 - either ... or ...
 - if ..., then ...
 - ... if and only if ...
- We introduced a formal language SL, in which \neg , \wedge , \vee , \rightarrow , and \leftrightarrow are used to translate the expressions above.

A Quick Recap

- We decided to theorize about argument forms involving the following English constructions:
 - it is not the case that...
 - both ... and ...
 - either ... or ...
 - if ..., then ...
 - ... if and only if ...
- We introduced a formal language SL, in which \neg , \wedge , \vee , \rightarrow , and \leftrightarrow are used to translate the expressions above.
- We defined a notion of validity in SL—entailment—and used it to theorize about deductive validity

A Quick Recap

- We decided to theorize about argument forms involving the following English constructions:
 - it is not the case that...
 - both ... and ...
 - either ... or ...
 - if ..., then ...
 - ... if and only if ...
- We introduced a formal language SL, in which \neg , \wedge , \vee , \rightarrow , and \leftrightarrow are used to translate the expressions above.
- We defined a notion of validity in SL—entailment—and used it to theorize about deductive validity
- How well does SL do at its job?

Correctness and Completeness

- Two questions:

Correctness and Completeness

- Two questions:
 - Are there any entailments which are not valid?

Correctness and Completeness

- Two questions:
 - Are there any entailments which are not valid?
 - Are there any non-entailments which are valid?

Correctness and Completeness

- Two questions:
 - Are there any entailments which are not valid?
 - Are there any non-entailments which are valid?
- Two properties we might want our theory to have:

Correctness and Completeness

- Two questions:
 - Are there any entailments which are not valid?
 - Are there any non-entailments which are valid?
- Two properties we might want our theory to have:
 - **CORRECTNESS:** If SL tells us an argument is valid, then it is valid.

Correctness and Completeness

- Two questions:
 - Are there any entailments which are not valid?
 - Are there any non-entailments which are valid?
- Two properties we might want our theory to have:
 - **CORRECTNESS:** If SL tells us an argument is valid, then it is valid.
 - **COMPLETENESS:** If an argument's valid, then SL tells us it is valid

Correctness and Completeness

- Most logicians believe that SL is *correct*.

Correctness and Completeness

- Most logicians believe that SL is *correct*.
- However, no logician believes that SL is *complete*.

Correctness and Completeness

- Most logicians believe that SL is *correct*.
- However, no logician believes that SL is *complete*.
- Because SL is not complete, we will need to look at additional kinds of logical forms. This will be the task of *predicate logic*, PL.

Correctness and Completeness

- Most logicians believe that SL is *correct*.
- However, no logician believes that SL is *complete*.
- Because SL is not complete, we will need to look at additional kinds of logical forms. This will be the task of *predicate logic*, PL.
- First: let's consider why some think that SL is not even *correct*.

Arguments that SL is Not Correct

A Counterexample to Modus Ponens?

A Counterexample to Modus Tollens?

A Counterexample to Disjunctive Syllogism?

The Sorites Paradox

Why SL is Not Complete

Is SL Not Correct?

- A word of warning: these arguments are controversial (some more so than others); and many logicians are not moved by them to reject the correctness of SL.

Is SL Not Correct?

- A word of warning: these arguments are controversial (some more so than others); and many logicians are not moved by them to reject the correctness of SL.
- There's two components to our theory SL:

Is SL Not Correct?

- A word of warning: these arguments are controversial (some more so than others); and many logicians are not moved by them to reject the correctness of SL.
- There's two components to our theory SL:
 - the theory about which arguments *involving the sentences of* SL are valid; and

Is SL Not Correct?

- A word of warning: these arguments are controversial (some more so than others); and many logicians are not moved by them to reject the correctness of SL.
- There's two components to our theory SL:
 - the theory about which arguments *involving the sentences of* SL are valid; and
 - the translation guide from English into SL.

Is SL Not Correct?

- The first two arguments below are really objections to the *translation guide*

Is SL Not Correct?

- The first two arguments below are really objections to the *translation guide*
 - ‘ \rightarrow ’ is not a perfect translation of ‘if..., then...’

Is SL Not Correct?

- The first two arguments below are really objections to the *translation guide*
 - ‘ \rightarrow ’ is not a perfect translation of ‘if..., then...’
 - the first two arguments attempt to show that the differences between ‘ \rightarrow ’ and ‘if..., then...’ prevent the English ‘if..., then...’ from satisfying *modus ponens* ($\rightarrow E$) and *modus tollens*.

Is SL Not Correct?

- The first two arguments below are really objections to the *translation guide*
 - ‘ \rightarrow ’ is not a perfect translation of ‘if..., then...’
 - the first two arguments attempt to show that the differences between ‘ \rightarrow ’ and ‘if..., then...’ prevent the English ‘if..., then...’ from satisfying *modus ponens* ($\rightarrow E$) and *modus tollens*.
- The second two arguments object not to the translation guide, but rather to SL’s theory about which arguments *involving the sentences of SL* are valid.

Arguments that SL is Not Correct

A Counterexample to Modus Ponens?

A Counterexample to Modus Tollens?

A Counterexample to Disjunctive Syllogism?

The Sorites Paradox

Why SL is Not Complete

A Counterexample to Modus Ponens?

If Clinton doesn't win, then, if a Democrat wins, then Bernie wins.

Clinton doesn't win.

\therefore If a Democrat wins, then Bernie wins.

A Counterexample to Modus Ponens?

If Clinton doesn't win, then, if a Democrat wins, then Bernie wins.

Clinton doesn't win.

∴ If a Democrat wins, then Bernie wins.

- McGee: this is of the form *modus ponens*

if A then B

A

∴ B

A Counterexample to Modus Ponens?

If Clinton doesn't win, then, if a Democrat wins, then Bernie wins.

Clinton doesn't win.

∴ If a Democrat wins, then Bernie wins.

- McGee: this is of the form *modus ponens*

if A then B

A

∴ B

- However, in the run up to the 2016 presidential election, its premises were true yet its conclusion was false.

Exportation

- The following sentence forms are provably equivalent in SL:

$$(A \wedge B) \rightarrow C$$

$$A \rightarrow (B \rightarrow C)$$

A Counterexample to Modus Ponens?

If Clinton doesn't win and a Democrat wins, then Bernie wins.

Clinton doesn't win.

\therefore So, if a Democrat wins, then Bernie wins.

A Counterexample to Modus Ponens?

If Clinton doesn't win and a Democrat wins, then Bernie wins.

Clinton doesn't win.

∴ So, if a Democrat wins, then Bernie wins.

- If we accept *exportation* and *modus ponens*, then we will have to say that this is also valid.

A Counterexample to Modus Ponens?

If Clinton doesn't win and a Democrat wins, then Bernie wins.

Clinton doesn't win.

∴ So, if a Democrat wins, then Bernie wins.

- If we accept *exportation* and *modus ponens*, then we will have to say that this is also valid.

1		$(\neg C \wedge D) \rightarrow B$	
2		$\neg C$	
		—	
3		$\neg C \rightarrow (D \rightarrow B)$	1, <i>Exp</i>
4		$D \rightarrow B$	2, 3, $\rightarrow E$

A Counterexample to Modus Ponens?

- McGee shows that, if we accept both *modus ponens* and *exportation* for the English ‘if..., then...’, then the English ‘if..., then...’ will be logically indistinguishable from SL’s conditional \rightarrow .

A Counterexample to Modus Ponens?

- McGee shows that, if we accept both *modus ponens* and *exportation* for the English ‘if..., then...’, then the English ‘if..., then...’ will be logically indistinguishable from SL’s conditional \rightarrow .
- So, we’d have to accept the following argument as deductively valid:

A Counterexample to Modus Ponens?

- McGee shows that, if we accept both *modus ponens* and *exportation* for the English ‘if..., then...’, then the English ‘if..., then...’ will be logically indistinguishable from SL’s conditional \rightarrow .
- So, we’d have to accept the following argument as deductively valid:

Shakespeare wrote Hamlet.

∴ If Shakespeare didn’t write Hamlet, then Dan Brown did.
- McGee: this is unacceptable, so we must choose between *exportation* and *modus ponens* for the English ‘if..., then...’

A Counterexample to Modus Ponens?

- Actually,

Shakespeare wrote Hamlet.

∴ If Shakespeare didn't write Hamlet, then Dan Brown did.

gives us a reason, on its own, to doubt the adequacy of our translation guide, since this is an entailment, given our translation guide.

A Counterexample to Modus Ponens?

1		S	
		—	
2			
2			$\neg S$ Ass. ($\rightarrow I$)
			—
3			\perp $\perp I$ 1, 2
4			B $\perp E$ 3
5			$\neg S \rightarrow B$ $\rightarrow I$ 2-4

A Counterexample to Modus Ponens?

- McGee opts for *exportation* and rejects *modus ponens*.

A Counterexample to Modus Ponens?

- McGee opts for *exportation* and rejects *modus ponens*.
- Others opt for *modus ponens* and reject *exportation*.

A Counterexample to Modus Ponens?

- McGee opts for *exportation* and rejects *modus ponens*.
- Others opt for *modus ponens* and reject *exportation*.
- Still others accept both *exportation* and *modus ponens* and accept that the English ‘if..., then...’ is logically indistinguishable from \rightarrow

A Counterexample to Modus Ponens?

- McGee opts for *exportation* and rejects *modus ponens*.
- Others opt for *modus ponens* and reject *exportation*.
- Still others accept both *exportation* and *modus ponens* and accept that the English ‘if..., then...’ is logically indistinguishable from \rightarrow
 - they have stories to tell about why arguments like the ones above appear—falsely—to be invalid

Arguments that SL is Not Correct

A Counterexample to Modus Ponens?

A Counterexample to Modus Tollens?

A Counterexample to Disjunctive Syllogism?

The Sorites Paradox

Why SL is Not Complete

A Counterexample to Modus Tollens?

- Imagine that we have an urn which contains 100 marbles.

A Counterexample to Modus Tollens?

- Imagine that we have an urn which contains 100 marbles.

	blue	red
big	10	30
small	50	10

A Counterexample to Modus Tollens?

- Imagine that we have an urn which contains 100 marbles.

	blue	red
big	10	30
small	50	10

- Suppose that we have selected a marble at random from the urn, but that we do not yet know whether it is blue or red, or whether it is big or small.

A Counterexample to Modus Tollens?

- Imagine that we have an urn which contains 100 marbles.

	blue	red
big	10	30
small	50	10

- Suppose that we have selected a marble at random from the urn, but that we do not yet know whether it is blue or red, or whether it is big or small.

If the marble is big, then it's likely red.

The marble is not likely red.

\therefore The marble is not big.

A Counterexample to Modus Tollens?

If the marble is big, then it's likely red.

The marble is not likely red.

∴ The marble is not big.

A Counterexample to Modus Tollens?

If the marble is big, then it's likely red.

The marble is not likely red.

∴ The marble is not big.

- This is an instance of the argument form *modus tollens*

If A then B

It is not the case that B

∴ It is not the case that A

A Counterexample to Modus Tollens?

If the marble is big, then it's likely red.

The marble is not likely red.

∴ The marble is not big.

- This is an instance of the argument form *modus tollens*

If A then B

It is not the case that B

∴ It is not the case that A

- So, Yalcin contends, *modus tollens* is not valid for the English 'if..., then...'

A Counterexample to Modus Tollens?

If the marble is big, then it's likely red.

The marble is not likely red.

∴ The marble is not big.

A Counterexample to Modus Tollens?

If the marble is big, then it's likely red.

The marble is not likely red.

∴ The marble is not big.

- Some options:

A Counterexample to Modus Tollens?

If the marble is big, then it's likely red.

The marble is not likely red.

∴ The marble is not big.

- Some options:
 - We could contend that the proposed counterexample equivocates with respect to 'likely'

A Counterexample to Modus Tollens?

If the marble is big, then it's likely red.

The marble is not likely red.

∴ The marble is not big.

- Some options:
 - We could contend that the proposed counterexample equivocates with respect to 'likely'
 - in the first premise, it means "*likely given all the information that currently have, plus the information that the marble is big*"

A Counterexample to Modus Tollens?

If the marble is big, then it's likely red.

The marble is not likely red.

∴ The marble is not big.

- Some options:
 - We could contend that the proposed counterexample equivocates with respect to 'likely'
 - in the first premise, it means "likely *given all the information that currently have, plus the information that the marble is big*"
 - in the second premise, it means "likely, *given all the information that we currently have*".

A Counterexample to Modus Tollens?

If the marble is big, then it's likely red.

The marble is not likely red.

∴ The marble is not big.

- Some options:
 - We could contend that the proposed counterexample equivocates with respect to 'likely'
 - in the first premise, it means "likely *given all the information that currently have, plus the information that the marble is big*"
 - in the second premise, it means "likely, *given all the information that we currently have*".
 - We could contend that the first premise is equivalent to "it's likely that, if the marble is big, then it's red", so that "if..., then..." is not the main operator of the premise.

Arguments that SL is Not Correct

A Counterexample to Modus Ponens?

A Counterexample to Modus Tollens?

A Counterexample to Disjunctive Syllogism?

The Sorites Paradox

Why SL is Not Complete

A Counterexample to Disjunctive Syllogism?

- Consider the statement:

A Counterexample to Disjunctive Syllogism?

- Consider the statement:

This very statement is false.

A Counterexample to Disjunctive Syllogism?

- Consider the statement:
 $L := L$ is false.

A Counterexample to Disjunctive Syllogism?

- Consider the statement:

$L := L \text{ is false.}$

- Priest: L is both true and false. (Almost everyone else disagrees)

A Counterexample to Disjunctive Syllogism?

- Priest: A statement could be:

A Counterexample to Disjunctive Syllogism?

- Priest: A statement could be:
 - true (and not false) ‘ T ’

A Counterexample to Disjunctive Syllogism?

- Priest: A statement could be:
 - true (and not false) ' T '
 - false (and not true) ' F '; or

A Counterexample to Disjunctive Syllogism?

- Priest: A statement could be:
 - ▷ true (and not false) ' T '
 - ▷ false (and not true) ' F '; or
 - ▷ *both* true and false ' B '.

A Counterexample to Disjunctive Syllogism?

- Priest: A statement could be:
 - ▷ true (and not false) ' T '
 - ▷ false (and not true) ' F '; or
 - ▷ *both* true and false ' B '.
- updated truth-table for ' \neg ':

\mathcal{A}	$\neg\mathcal{A}$
T	F
F	T
B	B

A Counterexample to Disjunctive Syllogism?

- Updated truth-tables for \vee and \rightarrow :

$$A \vee B$$

		B		
		T	F	B
A	T	T	T	T
	F	T	F	B
	B	T	B	B

$$A \rightarrow B$$

		B		
		T	F	B
A	T	T	F	B
	F	T	T	T
	B	T	B	B

A Counterexample to Disjunctive Syllogism?

- Let $P :=$ Pigs can fly

A Counterexample to Disjunctive Syllogism?

- Let $P :=$ Pigs can fly

$$L \vee P$$

$$\neg L$$

$$\therefore P$$

$$L \rightarrow P$$

$$L$$

$$\therefore P$$

A Counterexample to Disjunctive Syllogism?

- Let $P :=$ Pigs can fly

$$L \vee P$$

$$\neg L$$

$$\therefore P$$

$$L \rightarrow P$$

$$L$$

$$\therefore P$$

- Priest: for each argument, the premises are all true (and also false), yet the conclusion is false.

A Counterexample to Disjunctive Syllogism?

- Let $P :=$ Pigs can fly

$$L \vee P$$

$$\neg L$$

$$\therefore P$$

$$L \rightarrow P$$

$$L$$

$$\therefore P$$

- Priest: for each argument, the premises are all true (and also false), yet the conclusion is false.
- Yet these arguments are of the form *disjunctive syllogism* and *modus ponens*

A Counterexample to Disjunctive Syllogism?

- Let $P :=$ Pigs can fly

$$L \vee P$$

$$\neg L$$

$$\therefore P$$

$$L \rightarrow P$$

$$L$$

$$\therefore P$$

- Priest: for each argument, the premises are all true (and also false), yet the conclusion is false.
- Yet these arguments are of the form *disjunctive syllogism* and *modus ponens*
- So, Priest concludes: both *disjunctive syllogism* and *modus ponens* are invalid.

A Counterexample to Disjunctive Syllogism?

- Almost everybody else is going to get off the boat by denying that L —or any other sentence, for that matter—is both true and false.

A Counterexample to Disjunctive Syllogism?

- Almost everybody else is going to get off the boat by denying that L —or any other sentence, for that matter—is both true and false.
- But then we have to say something about L .

A Counterexample to Disjunctive Syllogism?

- Almost everybody else is going to get off the boat by denying that L —or any other sentence, for that matter—is both true and false.
- But then we have to say something about L .
- Say that it's neither true nor false?

A Counterexample to Disjunctive Syllogism?

- Almost everybody else is going to get off the boat by denying that L —or any other sentence, for that matter—is both true and false.
- But then we have to say something about L .
- Say that it's neither true nor false?

$L' := L$ is not true.

Arguments that SL is Not Correct

A Counterexample to Modus Ponens?

A Counterexample to Modus Tollens?

A Counterexample to Disjunctive Syllogism?

The Sorites Paradox

Why SL is Not Complete

The Sorites Paradox

- Suppose that we have 10,000 tiles lined up in a row.

The Sorites Paradox

- Suppose that we have 10,000 tiles lined up in a row.
 - The first tile is unmistakably red.

The Sorites Paradox

- Suppose that we have 10,000 tiles lined up in a row.
 - The first tile is unmistakably red.
 - The next tile in the sequence is perceptually indistinguishable from the first, but its color has ever-so-slightly more yellow in it than the first.

The Sorites Paradox

- Suppose that we have 10,000 tiles lined up in a row.
 - The first tile is unmistakably red.
 - The next tile in the sequence is perceptually indistinguishable from the first, but its color has ever-so-slightly more yellow in it than the first.
 - The third tile has ever-so-slightly more yellow in it than the second

The Sorites Paradox

- Suppose that we have 10,000 tiles lined up in a row.
 - The first tile is unmistakably red.
 - The next tile in the sequence is perceptually indistinguishable from the first, but its color has ever-so-slightly more yellow in it than the first.
 - The third tile has ever-so-slightly more yellow in it than the second
 - So on and so forth.

The Sorites Paradox

- Suppose that we have 10,000 tiles lined up in a row.
 - The first tile is unmistakably red.
 - The next tile in the sequence is perceptually indistinguishable from the first, but its color has ever-so-slightly more yellow in it than the first.
 - The third tile has ever-so-slightly more yellow in it than the second
 - So on and so forth.
- Any pair of sequential tiles are perceptually indistinguishable.

The Sorites Paradox

- Suppose that we have 10,000 tiles lined up in a row.
 - The first tile is unmistakably red.
 - The next tile in the sequence is perceptually indistinguishable from the first, but its color has ever-so-slightly more yellow in it than the first.
 - The third tile has ever-so-slightly more yellow in it than the second
 - So on and so forth.
- Any pair of sequential tiles are perceptually indistinguishable.
- By the end of the sequence, we have a tile that is unmistakably orange.

The Sorites Paradox

1) The 1st tile is red.

The Sorites Paradox

- 1) The 1st tile is red.
- 2) The 1st tile is red \rightarrow the 2nd tile is red.

The Sorites Paradox

- 1) The 1st tile is red.
- 2) The 1st tile is red \rightarrow the 2nd tile is red.
- 3) The 2nd tile is red \rightarrow the 3rd tile is red.

The Sorites Paradox

- 1) The 1st tile is red.
- 2) The 1st tile is red \rightarrow the 2nd tile is red.
- 3) The 2nd tile is red \rightarrow the 3rd tile is red.
- 4) The 3rd tile is red \rightarrow the 4th tile is red.

The Sorites Paradox

- 1) The 1st tile is red.
- 2) The 1st tile is red \rightarrow the 2nd tile is red.
- 3) The 2nd tile is red \rightarrow the 3rd tile is red.
- 4) The 3rd tile is red \rightarrow the 4th tile is red.
- \vdots
- \vdots

10,000) The 9,999th tile is red \rightarrow the 10,000th tile is red.

The Sorites Paradox

- 1) The 1st tile is red.
- 2) The 1st tile is red \rightarrow the 2nd tile is red.
- 3) The 2nd tile is red \rightarrow the 3rd tile is red.
- 4) The 3rd tile is red \rightarrow the 4th tile is red.
- \vdots

10,000) The 9,999th tile is red \rightarrow the 10,000th tile is red.

10,001) The 10,000th tile is red.

- Something's gone wrong with this reasoning, and some people have been tempted to point the finger at *modus ponens*.

Arguments that SL is Not Correct

A Counterexample to Modus Ponens?

A Counterexample to Modus Tollens?

A Counterexample to Disjunctive Syllogism?

The Sorites Paradox

Why SL is Not Complete

Why SL is Not Complete

Johann knows Filipa.

∴ So, somebody knows Filipa.

Everyone who owns a Ford owns a car.

Rohan owns a Ford.

∴ So, Rohan owns a car.

Why SL is Not Complete

Johann knows Filipa.

\therefore So, somebody knows Filipa.

Everyone who owns a Ford owns a car.

Rohan owns a Ford.

\therefore So, Rohan owns a car.

- Both arguments are valid

Why SL is Not Complete

Johann knows Filipa.

\therefore So, somebody knows Filipa.

Everyone who owns a Ford owns a car.

Rohan owns a Ford.

\therefore So, Rohan owns a car.

- Both arguments are valid
- Neither arguments are entailments

Why SL is Not Complete

Johann knows Filipa.

\therefore So, somebody knows Filipa.

Everyone who owns a Ford owns a car.

Rohan owns a Ford.

\therefore So, Rohan owns a car.

- Both arguments are valid
- Neither arguments are entailments
 - $J \therefore S$

Why SL is Not Complete

Johann knows Filipa.

\therefore So, somebody knows Filipa.

Everyone who owns a Ford owns a car.

Rohan owns a Ford.

\therefore So, Rohan owns a car.

- Both arguments are valid
- Neither arguments are entailments
 - $J \therefore S$
 - $E, F \therefore C$