

# Correctness and Completeness of SL

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PHIL 500

## Arguments that SL is Not Correct

A Counterexample to Modus Ponens?

A Counterexample to Modus Tollens?

A Counterexample to Disjunctive Syllogism?

The Sorites Paradox

## Why SL is Not Complete

# A Quick Recap

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An argument is **VALID** if and only if it is impossible for its premises to all be true while its conclusion is false.

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both ... and ...

either ... or ...

if ..., then ...

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- How well does SL do at its job?

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# Correctness and Completeness

- Two questions:
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- Two properties we might want our theory to have:
  - **CORRECTNESS:** If SL tells us an argument is valid, then it is valid.
  - **COMPLETENESS:** If an argument's valid, then SL tells us it is valid

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- First: let's consider why some think that SL is not even *correct*.

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- There's two components to our theory SL:
  - the theory about which arguments *involving the sentences of* SL are valid; and
  - the translation guide from English into SL.

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## Is SL Not Correct?

- The first two arguments below are really objections to the *translation guide*
  - ‘ $\rightarrow$ ’ is not a perfect translation of ‘if..., then...’
  - the first two arguments attempt to show that the differences between ‘ $\rightarrow$ ’ and ‘if..., then...’ prevent the English ‘if..., then...’ from satisfying *modus ponens* ( $\rightarrow E$ ) and *modus tollens*.
- The second two arguments object not to the translation guide, but rather to SL’s theory about which arguments *involving the sentences of SL* are valid.

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## A Counterexample to Modus Ponens?

If Clinton doesn't win, then, if a Democrat wins, then Bernie wins.

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if  $A$  then  $B$

$A$

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- However, in the run up to the 2016 presidential election, its premises were true yet its conclusion was false.

# Exportation

- The following sentence forms are provably equivalent in SL:

$$(A \wedge B) \rightarrow C$$

$$A \rightarrow (B \rightarrow C)$$

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1		$(\neg C \wedge D) \rightarrow B$	
2		$\neg C$	
		—	
3		$\neg C \rightarrow (D \rightarrow B)$	1, <i>Exp</i>
4		$D \rightarrow B$	2, 3, $\rightarrow E$

## A Counterexample to Modus Ponens?

- McGee shows that, if we accept both *modus ponens* and *exportation* for the English ‘if..., then...’, then the English ‘if..., then...’ will be logically indistinguishable from SL’s conditional  $\rightarrow$ .

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- McGee shows that, if we accept both *modus ponens* and *exportation* for the English ‘if..., then...’, then the English ‘if..., then...’ will be logically indistinguishable from SL’s conditional  $\rightarrow$ .
- So, we’d have to accept the following argument as deductively valid:

Shakespeare wrote Hamlet.

∴ If Shakespeare didn’t write Hamlet, then Dan Brown did.
- McGee: this is unacceptable, so we must choose between *exportation* and *modus ponens* for the English ‘if..., then...’

# A Counterexample to Modus Ponens?

- Actually,

Shakespeare wrote Hamlet.

∴ If Shakespeare didn't write Hamlet, then Dan Brown did.

gives us a reason, on its own, to doubt the adequacy of our translation guide, since this is an entailment, given our translation guide.



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- Others opt for *modus ponens* and reject *exportation*.
- Still others accept both *exportation* and *modus ponens* and accept that the English ‘if..., then...’ is logically indistinguishable from  $\rightarrow$ 
  - they have stories to tell about why arguments like the ones above appear—falsely—to be invalid

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## Why SL is Not Complete

## A Counterexample to Modus Tollens?

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- Suppose that we have selected a marble at random from the urn, but that we do not yet know whether it is blue or red, or whether it is big or small.

If the marble is big, then it's likely red.

The marble is not likely red.

$\therefore$  The marble is not big.

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If  $A$  then  $B$

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- So, Yalcin contends, *modus tollens* is not valid for the English 'if..., then...'

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    - in the first premise, it means "likely *given all the information that currently have, plus the information that the marble is big*"
    - in the second premise, it means "likely, *given all the information that we currently have*".
  - We could contend that the first premise is equivalent to "it's likely that, if the marble is big, then it's red", so that "if..., then..." is not the main operator of the premise.

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*This very statement is false.*

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- Consider the statement:

$L := L \text{ is false.}$

- Priest:  $L$  is both true and false. (Almost everyone else disagrees)

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  - ▷ *both* true and false ' $B$ '.
- updated truth-table for ' $\neg$ ':

$\mathcal{A}$	$\neg\mathcal{A}$
$T$	$F$
$F$	$T$
$B$	$B$

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- Updated truth-tables for  $\vee$  and  $\rightarrow$ :

$$A \vee B$$

		$B$		
		$T$	$F$	$B$
$A$	$T$	$T$	$T$	$T$
	$F$	$T$	$F$	$B$
	$B$	$T$	$B$	$B$

$$A \rightarrow B$$

		$B$		
		$T$	$F$	$B$
$A$	$T$	$T$	$F$	$B$
	$F$	$T$	$T$	$T$
	$B$	$T$	$B$	$B$

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$$L \vee P$$

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$$L \rightarrow P$$

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- Priest: for each argument, the premises are all true (and also false), yet the conclusion is false.
- Yet these arguments are of the form *disjunctive syllogism* and *modus ponens*
- So, Priest concludes: both *disjunctive syllogism* and *modus ponens* are invalid.

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- Say that it's neither true nor false?

$L' := L$  is not true.

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## Why SL is Not Complete

# The Sorites Paradox

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  - The third tile has ever-so-slightly more yellow in it than the second
  - So on and so forth.
- Any pair of sequential tiles are perceptually indistinguishable.
- By the end of the sequence, we have a tile that is unmistakably orange.

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- $\vdots$

10,000) The 9,999<sup>th</sup> tile is red  $\rightarrow$  the 10,000<sup>th</sup> tile is red.

10,001) The 10,000<sup>th</sup> tile is red.

- Something's gone wrong with this reasoning, and some people have been tempted to point the finger at *modus ponens*.

# Outline

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# Why SL is Not Complete

Johann knows Filipa.

∴ So, somebody knows Filipa.

Everyone who owns a Ford owns a car.

Rohan owns a Ford.

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- Both arguments are valid

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- Both arguments are valid
- Neither arguments are entailments

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  - $J \therefore S$
  - $E, F \therefore C$