

Natural Deduction Proofs for SL

Derived Rules

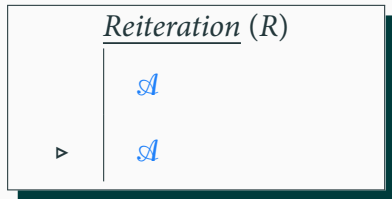
1	A	Ass. (LEM)
	—	
2	$A \wedge A$	$\wedge I$ 1
	—	
3	$\neg A$	Ass. (LEM)
	—	
4	$\neg A \vee A$	$\vee I$ 3
5	$(A \wedge A) \vee (\neg A \vee A)$	LEM 1-2, 3-4

Derived Rules

Derived Rules

- The plan: use the basic rules to establish additional rules

Reiteration



Reiteration

m | \mathcal{A}

Reiteration

m		\mathcal{A}	
k		$\mathcal{A} \wedge \mathcal{A}$	$\wedge I m$

Reiteration

m		\mathcal{A}	
k		$\mathcal{A} \wedge \mathcal{A}$	$\wedge I$ m
k+1		\mathcal{A}	$\wedge E$ k

Disjunctive Syllogism

Disjunctive Syllogism (DS)

$A \vee B$

$\neg A$

▷

B

$A \vee B$

$\neg B$

▷

A

Disjunctive Syllogism

m		$A \vee B$
n		$\neg A$

Disjunctive Syllogism

m		$A \vee B$	
n		$\neg A$	
k		A	Ass. ($\vee E$)

Disjunctive Syllogism

m		$A \vee B$	
n		$\neg A$	
		—	
k			A Ass. ($\vee E$)
			—
k+1			\perp $\perp I$ n, k

Disjunctive Syllogism

m		$A \vee B$	
n		$\neg A$	

k			Ass. ($\vee E$)
k+1			\perp $\perp I$ n, k

k+2			B $\perp E$ k+1

Disjunctive Syllogism

m		$A \vee B$	
n		$\neg A$	
		┌	
k			A Ass. ($\vee E$)
			┌
k+1			\perp $\perp I$ n, k
			└
k+2			B $\perp E$ k+1
			┌
k+3			B Ass. ($\vee E$)

Disjunctive Syllogism

m		$A \vee B$	
n		$\neg A$	
		—	
k			Ass. ($\vee E$)
k+1			\perp $\perp I$ n, k
			—
k+2			B $\perp E$ k+1
k+3			B Ass. ($\vee E$)
			—
k+4			B R k+3

Disjunctive Syllogism

m		$A \vee B$	
n		$\neg A$	
k			Ass. ($\vee E$)
k+1			\perp $\perp I$ n, k
k+2			B $\perp E$ k+1
k+3			B Ass. ($\vee E$)
k+4			B R k+3
k+5		B	$\vee E$ m, k-k+2, k+3-k+4

Double Negation Elimination

Double Negation Elimination (DNE)

▷

$\neg\neg A$

A

Double Negation Elimination

m $\left[\neg\neg A \right]$

Double Negation Elimination

m | $\neg\neg A$
 |_____
k | | $\neg A$ Ass. ($\neg E$)
 | |_____
 |_____|

Double Negation Elimination

m		$\neg\neg\mathcal{A}$	

k			Ass. ($\neg E$)
k+1			\perp $\perp I$ m, k
k+2		\mathcal{A}	$\neg E$ k-k+1

Modus Tollens

Modus Tollens (MT)

$$A \rightarrow B$$

$$\neg B$$

▷

$$\neg A$$

Modus Tollens

$$\begin{array}{l|l} \text{m} & A \rightarrow B \\ \text{n} & \neg B \end{array}$$

Modus Tollens

m		$A \rightarrow B$	
n		$\neg B$	
k		A	Ass. ($\neg I$)

Modus Tollens

m		$A \rightarrow B$	
n		$\neg B$	
k			Ass. ($\neg I$)
k+1			$\rightarrow E$ m, k

Modus Tollens

m		$A \rightarrow B$	
n		$\neg B$	
k		A	Ass. ($\neg I$)
k+1		B	$\rightarrow E$ m, k
k+2		\perp	$\perp I$ n, k+1

Modus Tollens

m		$A \rightarrow B$	
n		$\neg B$	
k		A	Ass. ($\neg I$)
k+1		B	$\rightarrow E$ m, k
k+2		\perp	$\perp I$ n, k+1
k+3		$\neg A$	$\neg I$ k-k+2

Law of Excluded Middle

Law of Excluded Middle (LEM)

A

\vdots

\mathcal{B}

$\neg A$

\vdots

\mathcal{B}

\triangleright

\mathcal{B}

Law of Excluded Middle

1

Law of Excluded Middle

$$1 \quad \left[\neg(A \vee \neg A) \quad \text{Ass. } (\neg E) \right]$$

Law of Excluded Middle

1		$\neg(\mathcal{A} \vee \neg\mathcal{A})$	Ass. ($\neg E$)

2		\mathcal{A}	Ass. ($\neg I$)

3		$\mathcal{A} \vee \neg\mathcal{A}$	$\vee I$ 2
4		\perp	$\perp I$ 1, 3

Law of Excluded Middle

1		$\neg(\mathcal{A} \vee \neg\mathcal{A})$	Ass. ($\neg E$)

2		\mathcal{A}	Ass. ($\neg I$)

3		$\mathcal{A} \vee \neg\mathcal{A}$	$\vee I$ 2
4		\perp	$\perp I$ 1, 3
5		$\neg\mathcal{A}$	$\neg I$ 2-4

Law of Excluded Middle

1		$\neg(\mathcal{A} \vee \neg\mathcal{A})$	Ass. ($\neg E$)

2		\mathcal{A}	Ass. ($\neg I$)

3		$\mathcal{A} \vee \neg\mathcal{A}$	$\vee I$ 2
4		\perp	$\perp I$ 1, 3
5		$\neg\mathcal{A}$	$\neg I$ 2-4
6		$\mathcal{A} \vee \neg\mathcal{A}$	$\vee I$ 5

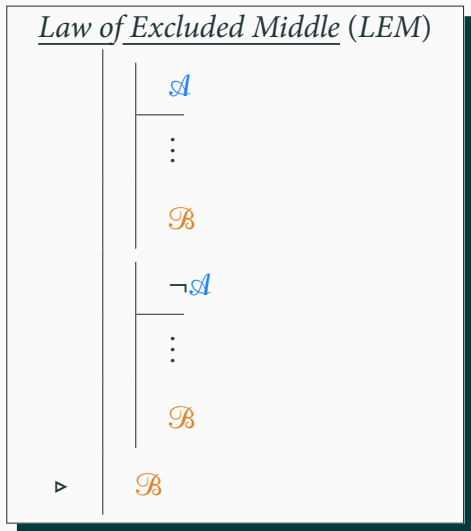
Law of Excluded Middle

1	$\neg(\mathcal{A} \vee \neg\mathcal{A})$	Ass. ($\neg E$)
2	\mathcal{A}	Ass. ($\neg I$)
3	$\mathcal{A} \vee \neg\mathcal{A}$	$\vee I$ 2
4	\perp	$\perp I$ 1, 3
5	$\neg\mathcal{A}$	$\neg I$ 2-4
6	$\mathcal{A} \vee \neg\mathcal{A}$	$\vee I$ 5
7	\perp	$\perp I$ 1, 6

Law of Excluded Middle

1		$\neg(A \vee \neg A)$	Ass. ($\neg E$)
		—	
2			
3			
4			
5			
6			
7			
8			

Law of Excluded Middle



DeMorgan's Rules

DeMorgan's Rules (DeM)

$$\neg(A \wedge B) \quad \Leftrightarrow \quad \neg A \vee \neg B$$

$$\neg(A \vee B) \quad \Leftrightarrow \quad \neg A \wedge \neg B$$

$$1 \quad \neg(A \wedge B)$$

1 | $\neg(A \wedge B)$
 |_____
2 | | A Ass. (LEM)
 | |_____
 |_____

1		$\neg(A \wedge B)$	

2			Ass. (LEM)
3			Ass. ($\neg I$)
4			$\wedge I$ 2, 3

1		$\neg(A \wedge B)$	

2			Ass. (LEM)
3			Ass. ($\neg I$)
4			$\wedge I$ 2, 3
5			$\perp I$ 1, 4

1		$\neg(A \wedge B)$	

2			Ass. (LEM)
3			Ass. ($\neg I$)
4			$\wedge I$ 2, 3
5			$\perp I$ 1, 4
6			$\neg I$ 3-5

1	$\neg(A \wedge B)$	
2	A	Ass. (LEM)
3	B	Ass. ($\neg I$)
4	$A \wedge B$	$\wedge I$ 2, 3
5	\perp	$\perp I$ 1, 4
6	$\neg B$	$\neg I$ 3-5
7	$\neg A \vee \neg B$	$\vee I$ 6

1	$\neg(A \wedge B)$	
2	A	Ass. (LEM)
3	B	Ass. ($\neg I$)
4	$A \wedge B$	$\wedge I$ 2, 3
5	\perp	$\perp I$ 1, 4
6	$\neg B$	$\neg I$ 3-5
7	$\neg A \vee \neg B$	$\vee I$ 6
8	$\neg A$	Ass. (LEM)

1	$\neg(A \wedge B)$	
2	A	Ass. (LEM)
3	B	Ass. ($\neg I$)
4	$A \wedge B$	$\wedge I$ 2, 3
5	\perp	$\perp I$ 1, 4
6	$\neg B$	$\neg I$ 3-5
7	$\neg A \vee \neg B$	$\vee I$ 6
8	$\neg A$	Ass. (LEM)
9	$\neg A \vee \neg B$	$\vee I$ 7

1	$\neg(A \wedge B)$	
2	A	Ass. (LEM)
3	B	Ass. ($\neg I$)
4	$A \wedge B$	$\wedge I$ 2, 3
5	\perp	$\perp I$ 1, 4
6	$\neg B$	$\neg I$ 3-5
7	$\neg A \vee \neg B$	$\vee I$ 6
8	$\neg A$	Ass. (LEM)
9	$\neg A \vee \neg B$	$\vee I$ 7
10	$\neg A \vee \neg B$	LEM 2-7, 8-9

$$1 \quad \neg A \vee \neg B$$

1 | $\neg A \vee \neg B$
 | ├──
2 | | $A \wedge B$ Ass. ($\neg I$)
 | └──

1	$\neg \mathcal{A} \vee \neg \mathcal{B}$	
2	$\mathcal{A} \wedge \mathcal{B}$	Ass. ($\neg I$)
3	\mathcal{A}	$\wedge E$ 2

1		$\neg \mathcal{A} \vee \neg \mathcal{B}$	

2			
2			Ass. ($\neg I$)
3			\mathcal{A} $\wedge E$ 2
4			\mathcal{B} $\wedge E$ 2

1	$\neg\mathcal{A} \vee \neg\mathcal{B}$	
2	$\mathcal{A} \wedge \mathcal{B}$	Ass. ($\neg I$)
3	\mathcal{A}	$\wedge E$ 2
4	\mathcal{B}	$\wedge E$ 2
5	$\neg\mathcal{A}$	Ass. ($\vee E$)

1		$\neg A \vee \neg B$	

2			
3			
4			
5			
6			

$\neg A \vee \neg B$

$A \wedge B$

A

B

$\neg A$

\perp

Ass. ($\neg I$)

$\wedge E$ 2

$\wedge E$ 2

Ass. ($\vee E$)

$\perp I$ 3, 5

1		$\neg A \vee \neg B$	
		┌	
2			
3			
4			
5			
6			
7			

 $\neg A \vee \neg B$
 $A \wedge B$

 Ass. ($\neg I$)

 A
 $\wedge E$ 2

 B
 $\wedge E$ 2

 $\neg A$

 Ass. ($\vee E$)

 \perp
 $\perp I$ 3, 5

 $\neg B$

 Ass. ($\vee E$)

1		$\neg A \vee \neg B$	
		—	
2			
2			Ass. ($\neg I$)
3			
3			A $\wedge E$ 2
4			
4			B $\wedge E$ 2
5			
5			$\neg A$ Ass. ($\vee E$)
6			
6			\perp $\perp I$ 3, 5
7			
7			$\neg B$ Ass. ($\vee E$)
8			
8			\perp $\perp I$ 4, 7

1		$\neg A \vee \neg B$	
2			Ass. ($\neg I$)
3			$A \wedge B$
4			A
			$\wedge E$ 2
5			B
			$\wedge E$ 2
6			
7			$\neg A$
			Ass. ($\vee E$)
8			\perp
			$\perp I$ 3, 5
9			
10			$\neg B$
			Ass. ($\vee E$)
11			\perp
			$\perp I$ 4, 7
12			\perp
			$\vee E$ 1, 5-6, 7-8

1	$\neg A \vee \neg B$	
2	$A \wedge B$	Ass. ($\neg I$)
3	A	$\wedge E$ 2
4	B	$\wedge E$ 2
5	$\neg A$	Ass. ($\vee E$)
6	\perp	$\perp I$ 3, 5
7	$\neg B$	Ass. ($\vee E$)
8	\perp	$\perp I$ 4, 7
9	\perp	$\vee E$ 1, 5-6, 7-8
10	$\neg(A \wedge B)$	$\neg I$ 2-9

$$1 \quad \neg(A \vee B)$$

1 $\neg(\mathcal{A} \vee \mathcal{B})$
2 \mathcal{A} Ass. (LEM)

1	$\neg(\mathcal{A} \vee \mathcal{B})$										
	<table style="border-collapse: collapse;"> <tr> <td style="padding-right: 10px;">2</td> <td style="border-left: 1px solid black; padding-left: 5px;"> \mathcal{A} </td> <td style="padding-left: 10px;">Ass. (LEM)</td> </tr> <tr> <td></td> <td style="border-left: 1px solid black; border-top: 1px solid black; padding-left: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 10px;">3</td> <td style="border-left: 1px solid black; padding-left: 5px;"> $\mathcal{A} \vee \mathcal{B}$ </td> <td style="padding-left: 10px;">$\vee I$ 2</td> </tr> </table> </td> <td></td> </tr> </table>	2	\mathcal{A}	Ass. (LEM)		<table style="border-collapse: collapse;"> <tr> <td style="padding-right: 10px;">3</td> <td style="border-left: 1px solid black; padding-left: 5px;"> $\mathcal{A} \vee \mathcal{B}$ </td> <td style="padding-left: 10px;">$\vee I$ 2</td> </tr> </table>	3	$\mathcal{A} \vee \mathcal{B}$	$\vee I$ 2		
2	\mathcal{A}	Ass. (LEM)									
	<table style="border-collapse: collapse;"> <tr> <td style="padding-right: 10px;">3</td> <td style="border-left: 1px solid black; padding-left: 5px;"> $\mathcal{A} \vee \mathcal{B}$ </td> <td style="padding-left: 10px;">$\vee I$ 2</td> </tr> </table>	3	$\mathcal{A} \vee \mathcal{B}$	$\vee I$ 2							
3	$\mathcal{A} \vee \mathcal{B}$	$\vee I$ 2									

1		$\neg(\mathcal{A} \vee \mathcal{B})$	
		┌	
2			Ass. (LEM)
3			\mathcal{A}
			└
3			$\mathcal{A} \vee \mathcal{B}$
			$\vee I$ 2
4			\perp
			$\perp I$ 1, 3

1		$\neg(A \vee B)$	
		┌	
2			Ass. (LEM)
3			$A \vee B$ $\vee I$ 2
4			\perp $\perp I$ 1, 3
5			$\neg A \wedge \neg B$ $\perp E$ 4

1		$\neg(A \vee B)$	
2			
2			Ass. (LEM)
3			
3			$A \vee B$ $\vee I$ 2
4			\perp $\perp I$ 1, 3
5			$\neg A \wedge \neg B$ $\perp E$ 4
6			$\neg A$ Ass. (LEM)

1		$\neg(A \vee B)$	
2			
2			Ass. (LEM)
3			
3			$A \vee B$ $\vee I$ 2
4			\perp $\perp I$ 1, 3
5			$\neg A \wedge \neg B$ $\perp E$ 4
6			$\neg A$ Ass. (LEM)
7			B Ass. ($\neg I$)

1		$\neg(A \vee B)$	
2			Ass. (LEM)
3			$\vee I$ 2
4			$\perp I$ 1, 3
5			$\perp E$ 4
6			Ass. (LEM)
7			Ass. ($\neg I$)
8			$\vee I$ 7

1		$\neg(A \vee B)$	
2			
3			
4			
5			
6			
7			
8			
9			

2		A	Ass. (LEM)
3		$A \vee B$	$\vee I$ 2
4		\perp	$\perp I$ 1, 3
5		$\neg A \wedge \neg B$	$\perp E$ 4
6		$\neg A$	Ass. (LEM)
7			
8			
9			

7		B	Ass. ($\neg I$)
8		$A \vee B$	$\vee I$ 7
9		\perp	$\perp I$ 1, 8

1	$\neg(A \vee B)$	
2	A	Ass. (LEM)
3	$A \vee B$	$\vee I$ 2
4	\perp	$\perp I$ 1, 3
5	$\neg A \wedge \neg B$	$\perp E$ 4
6	$\neg A$	Ass. (LEM)
7	B	Ass. ($\neg I$)
8	$A \vee B$	$\vee I$ 7
9	\perp	$\perp I$ 1, 8
10	$\neg B$	$\neg I$ 7-9

1		$\neg(A \vee B)$	
2			
3			
4			
5			
6			
7			
8			
9			
10			
11			

2		A	Ass. (LEM)
3		$A \vee B$	$\vee I$ 2
4		\perp	$\perp I$ 1, 3
5		$\neg A \wedge \neg B$	$\perp E$ 4
6		$\neg A$	Ass. (LEM)
7			
8			
9			
10			
11			

7		B	Ass. ($\neg I$)
8		$A \vee B$	$\vee I$ 7
9		\perp	$\perp I$ 1, 8
10		$\neg B$	$\neg I$ 7-9
11		$\neg A \wedge \neg B$	$\wedge I$ 6, 10

1	$\neg(A \vee B)$	
2	A	Ass. (LEM)
3	$A \vee B$	$\vee I$ 2
4	\perp	$\perp I$ 1, 3
5	$\neg A \wedge \neg B$	$\perp E$ 4
6	$\neg A$	Ass. (LEM)
7	B	Ass. ($\neg I$)
8	$A \vee B$	$\vee I$ 7
9	\perp	$\perp I$ 1, 8
10	$\neg B$	$\neg I$ 7-9
11	$\neg A \wedge \neg B$	$\wedge I$ 6, 10
12	$\neg A \wedge \neg B$	LEM 2-5, 6-11

$$1 \quad \neg A \wedge \neg B$$

1		$\neg A \wedge \neg B$	
2		$\neg A$	$\wedge E$ 1

1		$\neg A \wedge \neg B$	
		—	
2		$\neg A$	$\wedge E$ 1
3		$\neg B$	$\wedge E$ 1

1		$\neg A \wedge \neg B$	
		—	
2		$\neg A$	$\wedge E$ 1
3		$\neg B$	$\wedge E$ 1
4			
			—
			A
			Ass. (LEM)
5			\perp
			$\perp I$ 2, 4

1		$\neg \mathcal{A} \wedge \neg \mathcal{B}$	
		—	
2		$\neg \mathcal{A}$	$\wedge E$ 1
3		$\neg \mathcal{B}$	$\wedge E$ 1
4			
4			\mathcal{A}
			Ass. (LEM)
			—
5			\perp
			$\perp I$ 2, 4
6		$\neg(\mathcal{A} \vee \mathcal{B})$	$\perp E$ 5

1	$\neg A \wedge \neg B$	
2	$\neg A$	$\wedge E$ 1
3	$\neg B$	$\wedge E$ 1
4	A	Ass. (LEM)
5	\perp	$\perp I$ 2, 4
6	$\neg(A \vee B)$	$\perp E$ 5
7	$\neg A$	Ass. (LEM)

1		$\neg A \wedge \neg B$	
		├	
2		$\neg A$	$\wedge E$ 1
3		$\neg B$	$\wedge E$ 1
4			
4			A
			├
5			\perp
			$\perp I$ 2, 4
6			$\neg(A \vee B)$
			$\perp E$ 5
7			$\neg A$
			├
8			$A \vee B$
			$\text{Ass. } (\neg I)$

1		$\neg A \wedge \neg B$	
		┌	
2		$\neg A$	$\wedge E$ 1
3		$\neg B$	$\wedge E$ 1
4			
4			A
			Ass. (LEM)
			┌
5			\perp
			$\perp I$ 2, 4
6			$\neg(A \vee B)$
			$\perp E$ 5
7			$\neg A$
			Ass. (LEM)
			┌
8			$A \vee B$
			Ass. ($\neg I$)
			┌
9			A
			DS 3, 8

1		$\neg A \wedge \neg B$	
		┌	
2		$\neg A$	$\wedge E$ 1
3		$\neg B$	$\wedge E$ 1
4			
4			A
			Ass. (LEM)
			┌
5			\perp
			$\perp I$ 2, 4
6			$\neg(A \vee B)$
			$\perp E$ 5
7			$\neg A$
			Ass. (LEM)
			┌
8			$A \vee B$
			Ass. ($\neg I$)
9			A
			DS 3, 8
10			\perp
			$\perp I$ 7, 9

1	$\neg \mathcal{A} \wedge \neg \mathcal{B}$	
2	$\neg \mathcal{A}$	$\wedge E$ 1
3	$\neg \mathcal{B}$	$\wedge E$ 1
4	\mathcal{A}	Ass. (LEM)
5	\perp	$\perp I$ 2, 4
6	$\neg(\mathcal{A} \vee \mathcal{B})$	$\perp E$ 5
7	$\neg \mathcal{A}$	Ass. (LEM)
8	$\mathcal{A} \vee \mathcal{B}$	Ass. ($\neg I$)
9	\mathcal{A}	DS 3, 8
10	\perp	$\perp I$ 7, 9
11	$\neg(\mathcal{A} \vee \mathcal{B})$	$\neg I$ 8–10

1	$\neg \mathcal{A} \wedge \neg \mathcal{B}$	
2	$\neg \mathcal{A}$	$\wedge E$ 1
3	$\neg \mathcal{B}$	$\wedge E$ 1
4	\mathcal{A}	Ass. (LEM)
5	\perp	$\perp I$ 2, 4
6	$\neg(\mathcal{A} \vee \mathcal{B})$	$\perp E$ 5
7	$\neg \mathcal{A}$	Ass. (LEM)
8	$\mathcal{A} \vee \mathcal{B}$	Ass. ($\neg I$)
9	\mathcal{A}	DS 3, 8
10	\perp	$\perp I$ 7, 9
11	$\neg(\mathcal{A} \vee \mathcal{B})$	$\neg I$ 8-10
12	$\neg(\mathcal{A} \vee \mathcal{B})$	LEM 4-6, 7-11

A Sample Proof

$$\begin{array}{l|l} 1 & (P \rightarrow Q) \vee (Q \wedge R) \\ 2 & \neg Q \end{array}$$

A Sample Proof

1		$(P \rightarrow Q) \vee (Q \wedge R)$	
2		$\neg Q$	
3		$\neg Q \vee \neg R$	$\vee I_2$

A Sample Proof

1		$(P \rightarrow Q) \vee (Q \wedge R)$	
2		$\neg Q$	
		—	
3		$\neg Q \vee \neg R$	$\vee I$ 2
4		$\neg(Q \wedge R)$	DeM 3

A Sample Proof

1		$(P \rightarrow Q) \vee (Q \wedge R)$	
2		$\neg Q$	
		—	
3		$\neg Q \vee \neg R$	$\vee I$ 2
4		$\neg(Q \wedge R)$	DeM 3
5		$P \rightarrow Q$	DS 1, 4

A Sample Proof

1		$(P \rightarrow Q) \vee (Q \wedge R)$	
2		$\neg Q$	
3		$\neg Q \vee \neg R$	$\vee I$ 2
4		$\neg(Q \wedge R)$	DeM 3
5		$P \rightarrow Q$	DS 1, 4
6		$\neg P$	MT 2, 5

A Sample Proof

1		$(P \rightarrow Q) \vee (Q \wedge R)$	
2		$\neg Q$	
3		$\neg Q \vee \neg R$	$\vee I$ 2
4		$\neg(Q \wedge R)$	DeM 3
5		$P \rightarrow Q$	DS 1, 4
6		$\neg P$	MT 2, 5

A Sample Proof

$$1 \quad \left[P \rightarrow Q \right.$$

A Sample Proof

$$\begin{array}{l|l} 1 & P \rightarrow Q \\ \hline 2 & \boxed{\neg(\neg P \vee Q)} \quad \text{Ass. } (\neg E) \end{array}$$

A Sample Proof

1		$P \rightarrow Q$	
		—	
2			
2		$\neg(\neg P \vee Q)$	Ass. ($\neg E$)
3		$\neg\neg P \wedge \neg Q$	DeM 2
4		$\neg\neg P$	$\wedge E$ 3

A Sample Proof

1		$P \rightarrow Q$	
		—	
2			
2		$\neg(\neg P \vee Q)$	Ass. ($\neg E$)
3		$\neg\neg P \wedge \neg Q$	DeM 2
4		$\neg\neg P$	$\wedge E$ 3
5		P	DNE 4

A Sample Proof

1		$P \rightarrow Q$	
		—	
2			
3			
4			
5			
6			

		$\neg(\neg P \vee Q)$	Ass. ($\neg E$)
		—	
		$\neg\neg P \wedge \neg Q$	DeM 2
		$\neg\neg P$	$\wedge E$ 3
		P	DNE 4
		Q	$\rightarrow E$ 1, 5

A Sample Proof

1		$P \rightarrow Q$	

2			
2			$\neg(\neg P \vee Q)$ Ass. ($\neg E$)

3			$\neg\neg P \wedge \neg Q$ DeM 2

4			$\neg\neg P$ $\wedge E$ 3

5			P DNE 4

6			Q $\rightarrow E$ 1, 5

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A Sample Proof

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8	\perp	$\perp I$ 6, 7
9	$\neg P \vee Q$	$\neg E$ 2-8